Chapter 5

As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.

Berlin 1922
A. Einstein

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5.1 Systems of Linear Equations in 2 Variables

1.1 Introduction

Imagine you go to the store and buy 1 Tilapia fish and 1 Salmon fish and they cost \$12. The next day, you go to the same store and buy 2 Tilapia fish and 1 Salmon fish. This time your total is \$17. The question is how we can find the price for each fish.
Let x the price of 1 Tilapia fish and y the price of 1 Salmon fish. Therefore, translating from English to mathematics, we would get:

$$\begin{cases} 1 \text{ Tilapia} + 1 \text{ Salmon} = 12 \\ 2 \text{ Tilapia} + 1 \text{ Salmon} = 17 \end{cases} \implies \begin{cases} 1x + 1y = 12 \\ 2x + 1y = 17 \end{cases}$$

We obtain two equations with two unknowns. These two equations are happening at the same time. This is called a system of equations with 2 unknowns.
In general, if we have n number of linear equations, $n > 2$, each with n number of unknowns, which are occurring at the same time, then we have a system of linear equations. For example, following equations create a system of linear equation in three variables,

$$\begin{cases} x - 2y + z = 5 \\ 2x - 5y - 3z = 9 \\ x + 4y - 2z = -2 \end{cases}$$

In this section, we will focus on a system that has only two equations with 2 unknowns. The **solution for a system of linear equations** is an ordered pair that satisfies **both equations** in the system.

Example 1.1. Determine whether $(-3, 4)$ is a solution to the following system.

$$\begin{cases} 2x + 2y = 6 \\ 3x - 4y = 7 \end{cases}$$

We will begin by substituting $(-3, 4)$ into the first equation to see whether the ordered pair is a solution to the first equation

$$\begin{aligned} 2(-3) - 2(4) &\stackrel{?}{=} 5 \\ -6 + 12 &\stackrel{?}{=} 5 \\ 5 &= 5 \qquad \checkmark \end{aligned}$$

Likewise, we will substitute $(-3, 4)$ into the second equation. Thus,

$$\begin{aligned} 3(-3) - 4(4) &\stackrel{?}{=} 5 \\ -9 - 16 &\stackrel{?}{=} 5 \\ -25 &= 5 \qquad ✗ \end{aligned}$$

Ordered pair $(-3, 4)$ satisfies the first equation but not the second equation. Therefore, this ordered pair is not the solution of the system.

Note 1.1. As you observed in previous example, the ordered pair should satisfies both equations. If it only satisfies one of them, it is not the solution of our system.

1.2 Solving a system of linear equation

There are many ways to find the solution for a system of linear equations. We will only discuss 3 major methods in this section:

- Graphing
- Substitution
- Addition (or Elimination)

1.2.1 Graphing method

Each equation in a system of equation with two variables is representing a line in the xy-plane. If you graph each of them, there will be three possibilities:

I. **Two lines intersect each other at one point:** if two lines intersect, the point of intersection lies on both lines. Thus, the point of intersection is our only solution. Such a system is said to have a unique solution.

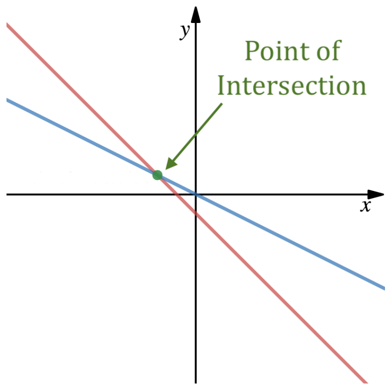

Figure 1.1: Intersection of two lines.

II. **Two lines don't intersect each other at all:** This happens when two lines are parallel to each other. Since they don't intersect each other, we do not have any solution. Such a system is said to be inconsistent.

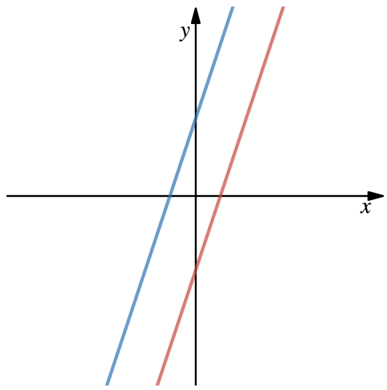

Figure 1.2: Parallel lines.

III. **Two lines are actually same line:** Another possibility is that when we graph each equation in the system, we obtain one line. In such a case, there is an infinite number of solutions. Because any point that lies on the first line will also lie on the second line. Such a system is said to have dependent equations.

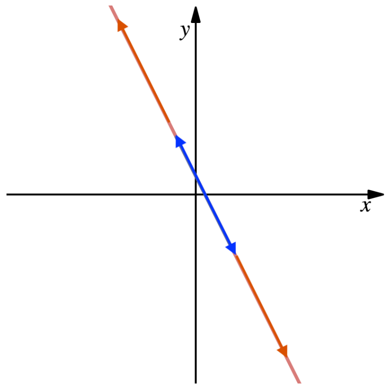

Figure 1.3: Same lines.

1.2.2 Substitution method

When solving a system by graphing has several limitations. First, it requires the graph to be perfectly drawn, if the lines are not straight we may arrive at the wrong answer. Second, graphing is not a great method to use if the answer is really large, over 100 for example, or if the answer is a decimal the that graph will not help us find, 3.2134 for example. For these reasons we will rarely use graphing to solve our systems. Instead, an algebraic approach will be used. The first algebraic approach is called substitution.

Substitution Method

1. Choose one of the equations and solve for one variable in terms of the other variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the equation from Step 2. (There will be one equation with one variable).
4. Substitute the solution from Step 3 into either of the original equations. This will give the value of the other variable.

Common Mistakes to Avoid:

- Remember that a system of linear equations is not completely solved until values for both x and y are found. To avoid this mistake, write all answers as an ordered pair.
- Remember that all ordered pairs are stated with the x variable first and the y variable second; namely, (x, y).
- If the first equation is used to solve for the variable, you must substitute it into the second equation. Otherwise, this will incorrectly lead

to the statement $0 = 0$.

Example 1.2. Solve.

$$\begin{cases} 2x + y = 5 \\ 3x + 2y = -8 \end{cases}$$

Notice, the first equation can be solved easily for y, giving us

$$\begin{aligned} 2x + y &= 5 && \text{Subtract } 2x \text{ from both sides} \\ \boxed{y = 5 - 2x} & && y \text{ in terms of other veriable} \end{aligned}$$

This is what we will now substitute into the y variable in our second equations. This gives us:

$$\begin{aligned} 3x + 2(5 - 2x) &= -8 && \text{Distribute} \\ 3x + 10 - 4x &= -8 && \text{Combine like terms} \\ 10 - x &= -8 && \text{Subtract 10 from both sides} \\ -x &= -18 && \text{Divide by -1} \\ x &= 18 && \text{Our } x \end{aligned}$$

Next, we need to find the value of our y variable by substituting $x = 18$ into either equation. Since we already know that $y = 5 - 2x$, substituting in this equation gives us:

$$\begin{aligned} y &= 5 - 2(18) \\ y &= 5 - 36 \\ y &= -31 \end{aligned}$$

So our solution is $(18, -31)$.

Example 1.3. Solve.

$$\begin{cases} 2x + 3y = 5 \\ x - 4y = 6 \end{cases}$$

Notice, the second equation can be solved easily for x, so

$$\begin{aligned} x - 4y &= 6 && \text{Add } 4y \text{ to both sides} \\ \boxed{x = 6 + 4y} & && x \text{ in terms of other variable} \end{aligned}$$

We will now substitute this into the x variable in our first equation

$$\begin{aligned} 2(6 + 4y) + 3y &= 5 && \text{Distribute} \\ 12 + 8y + 3y &= 5 && \text{Combine like terms on LHS} \\ 12 + 11y &= 5 && \text{Subtract 12 from both sides} \\ 11y &= -7 && \text{Divide by 11} \end{aligned}$$

$$y = \frac{-7}{11} \qquad \text{Our } y$$

Finally, we need to solve for x variable by substituting $y = -\frac{7}{11}$ into one of our equations. Since we already know that $x = 6 + 4y$ substituting into this equation, giving us

$$\begin{aligned} x &= 6 + 4\left(\frac{-7}{11}\right) \\ x &= 6 - \frac{28}{11} \\ x &= \frac{38}{11} \end{aligned}$$

So our answer is $\left(\frac{38}{11}, \frac{-7}{11}\right)$.

Note 1.2. During solving a system with two variables, if you get a false statement, means that we have no solution. On the other hand, if you reach to a true statement, like $0 = 0$, thus you have infinite solutions.

Example 1.4. Solve.

$$\begin{cases} 2x - y = 3 \\ -6x + 3y = 9 \end{cases}$$

Notice, the first equation can be solved quickly for y,

$$\begin{aligned} 2x - y &= 3 && \text{Subtract } 2x \text{ from both sides} \\ -y &= 3 - 2x && \text{Divide both sides by } -1 \\ \boxed{y = -3 + 2x} & && y \text{ in terms of other variable} \end{aligned}$$

We now substitute this into the y variable in our second equation

$$\begin{aligned} -6x + 3(-3 + 2x) &= 9 && \text{Distribute} \\ -6x - 9 + 6x &= 9 && \text{Combine like terms} \\ -9 &= 9 \ \text{✗} && \text{A false statement} \end{aligned}$$

Since this is a false statement, the system is inconsistent. Therefore, there is no solution.

1.2.3 Elimination method

When solving systems, we have found that graphing is very limited when solving equations. We then considered a second method known as substitution. This is probably the most used idea in solving systems in various areas of algebra. However, substitution can get ugly if we don't have a lone variable. This leads us to our second method for solving systems of equations. This method is known as either Elimination or Addition.

Elimination Method

1. Line up the variables and constants.
2. Multiply one or both equations by appropriate numbers (use LCM if you cannot find that magic numbers) so that the coefficient of one of the variables are opposites.
3. Add the two equations from step 2 to remove one of the variable
4. Solve the resulting equation for the remaining variable
5. Substitute the value you found from previous step into one of the original equations and find the other variable.

Common Mistakes to Avoid:

- Remember that a system of linear equations is not completely solved until values for both x and y are found. To avoid this mistake, write all answers as ordered pairs.
- Remember that all ordered pairs are stated with the x variable first and the y variable second; namely, (x, y).

Example 1.5. Solve

$$\begin{cases} 6x - 5y = 25 \\ 4x + 15y = 13 \end{cases}$$

If we multiply the first equation by 3 and leave the second equation alone, we will eliminate y.

$$\begin{cases} 3(6x - 5y) & = 3(25) \\ 4x + 15y & = 13 \end{cases} \implies \begin{cases} 18x - 15y & = 75 \\ 4x + 15y & = 13 \end{cases}$$

Add these two equations together we get

$$\begin{aligned} 18x \cancel{- 15y} &= 75 \\ 4x \cancel{+ 15y} &= 13 \\ \hline 22x &= 88 \\ x &= 4 \end{aligned}$$

Now we must find the value for y by substituting $x = 4$ into one of the two original equations. Substituting into the first equation gives us:

$$6(4) - 5y = 25$$

$$\begin{aligned} 24 - 5y &= 25 \\ -5y &= 1 \\ y &= -\frac{1}{5} \end{aligned}$$

So the answer is $(4, -\frac{1}{5})$

Example 1.6. Solve

$$\begin{cases} 8x + 9y = 13 \\ 6x - 5y = 45 \end{cases}$$

To eliminate the x variable, find the LCM of the coefficient of x variables. The LCM of 8 and 6 is 24. To create 24, we will multiply the first equation by -3 and the second equation by 4. Recall that their coefficient should be opposite. That's why we multiply one of them by -3.

$$\begin{cases} -3(8x + 9y) &= -3(13) \\ 4(6x - 5y) &= 4(45) \end{cases} \implies \begin{cases} -24x - 27y &= -39 \\ 24x - 20y &= 180 \end{cases}$$

Add these two equations together yields

$$\begin{aligned} \cancel{-24x} - 27y &= -39 \\ \cancel{24x} - 20y &= 180 \\ \hline -47y &= 141 \\ y &= -3 \end{aligned}$$

Next, we need to find the value for x by substituting $y = -3$ into either of the two original equations. If we substituting into the first equation, we get:

$$\begin{aligned} 8x + 9(-3) &= 13 \\ 8x - 27 &= 13 \\ 8x &= 40 \\ x &= 5 \end{aligned}$$

So the answer is $(5, -3)$

Example 1.7. Solve

$$\begin{cases} -6x + 9y = 12 \\ \quad 2x - 3y = -4 \end{cases}$$

If we multiply the second equation by 3, we can eliminate x

$$\begin{cases} -6x + 9y &= 12 \\ 3(2x - 3y) &= 3(-4) \end{cases} \implies \begin{cases} -6x + 9y &= 12 \\ 6x - 9y &= -12 \end{cases}$$

When we add these two equations together, we get

$$\begin{array}{r} -6x + 9y = 12 \\ 6x - 9y = -12 \\ \hline 0 = 0 \end{array}$$

Since we get a true statement, this system is dependent. Therefore, there are an infinite number of solutions.

5.2 Systems of Linear Equations in 3 Variables

2.1 Introduction

Solving systems of equations with three variables is very similar to how we solve systems with two variables. When we had two variables, we reduced the system down to one with only one variable (by substitution or addition). In a system with two variables, each equation represent a line in xy-plane. If the lines intersect each other, we had *one unique solution*. However, when they don't hit each other, the system have *no solution*. There were also one more possibility: two lines were actually the same line and hit each other at many points. Thus, in this case we had *infinite solutions*.
Similar story occurs in a system with three variables. Since we have three variables, x, y and z, each equation represent a plane in xyz-space. There are three possibilities:

- All three planes intersect each other at exactly one point. We will have one unique solution.
- All three planes are parallel and not hitting each other. In this case, we have no solution.
- All three planes might intersect at one or more lines or some of the planes might be coincident (same plane). In this case, we have infinite solution.

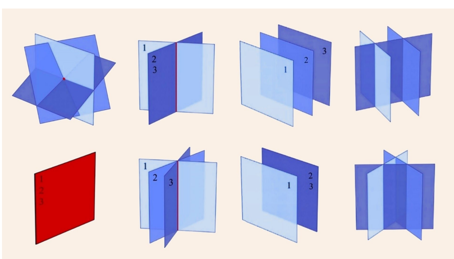

Figure 2.1: Intersection of three planes: all possibilities

2.2 Solving systems with 3 variables

With three variables we will reduce the system down to one with two variables (usually by addition), which we can then be solved by either addition or substitution. To reduce from three variables down to two it is very important to keep the work organized.

Solve a system with three variables

1. Choose one variable you want to eliminate. This is entirely up to you.
2. Choose two equations and use addition to eliminate the chosen variable. We will call this new equations (A).
3. Then we will use a different pair of equations and use addition to eliminate the same variable. We will call this second new equation (B).
4. We will have two equations (A) and (B) with the same two variables that we can be solved using either method.
5. Find one variable, substitute back into either (A) or (B) to find the other unknown. Then substitute all variables you found in one of the original equations, to find the last unknown.

Note 2.1. In systems with two variables, we had an ordered pair (x, y). In three variables, however, we have an ordered triplet (x, y, z) which represent a point in xyz-space

Example 2.1. Solve the system of equations.

$$\begin{cases} 3x + 2y - z = -1 \\ -2x - 2y + 3z = 5 \\ 5x + 2y - z = 3 \end{cases}$$

As a first step, Let's eliminate y using two different pairs of equations. Using the first two equations

$$\begin{cases} x + 2y - z = -1 \\ -2x - 2y + 3z = 5 \end{cases}$$

Notice the coefficients of y's are opposite of each other. So we don't need to multiply either equation by a number. So add them:

$$\begin{array}{rl} 3x + 2y - z = -1 & \\ -2x - 2y + 3z = 5 & \\ \hline x + 2z = 4 & \qquad \text{Equation (A)} \end{array}$$

Using the second two equations, we can find the equation (B). The coefficients of both ys are opposite, so just add them to remove y

$$\begin{array}{rl} -2x - 2y + 3z = 5 & \\ 5x + 2y - z = 3 & \\ \hline 3x + 2z = 8 & \qquad \text{Equation (B)} \end{array}$$

Using equation (A) and (B), we can find x and z. We will solve by using Addition. To remove z multiply (A) by -1

$$\begin{array}{rl} -x - 2z = -4 & \qquad (A) \\ 3x + 2z = 8 & \qquad (B) \\ \hline 2x = 4 & \qquad \text{Divide both sides by 2} \\ x = 2 & \end{array}$$

We now have x! Plug this into either (A) or (B). We plug it into (A), and solve the equation for z

$$\begin{array}{rl} x + 2z = 4 & \qquad \text{Plug } x = 2 \\ 2 + 2z = 4 & \qquad \text{Subtract 2 from both sides} \\ 2z = 2 & \qquad \text{Divide both sides by 2} \\ z = 1 & \end{array}$$

We now have z! Plug this and x into any original equation. We use the first equation and solve it for y

$$\begin{array}{rl} 3x + 2y - z = -1 & \qquad \text{Plug } x = 2 \text{ and } z = 1 \\ 3(2) + 2y - (1) = -1 & \qquad \text{Simplify} \\ 5 + 2y = -1 & \qquad \text{Subtract 5 from both sides} \\ 2y = -6 & \qquad \text{Divide both sides by 2} \\ y = -3 & \end{array}$$

We now have y! Therefore, our solution is $(2, -3, 1)$

Note 2.2. In this above problem, y was easily eliminated using the addition method. However, sometimes we may have to do a bit of work to get a variable to eliminate. Just as with addition of two equations, we may have to multiply equations by something on both sides to get the opposites we want so a variable eliminates. As we do this remember it is important to eliminate the same variable both times using two different pairs of equations.

Example 2.2. Solve the system of equations.

$$\begin{cases} 4x - 3y + 2z = -29 \\ 6x + 2y - z = -16 \\ -8x - y + 3z = 23 \end{cases}$$

No variable will easily removed. We could choose any variable, so we chose z. Using the first two equations

$$\begin{aligned} 4x - 3y + 2z &= -29 & \qquad 4x - 3y + 2z &= -29 \\ 6x + 2y - z &= -16 \text{ (multiply by 2)} \rightarrow & \qquad 12x + 4y - 2z &= -32 \\ & & \hline 16x + y &= -61 \ (A) \end{aligned}$$

We found equation (A). Now use the second two equations (a different pair of equations)

$$\begin{aligned} 6x + 2y - z &= -16 \text{ (multiply by 3)} \rightarrow & \qquad 18x + 6y - 3z &= -48 \\ -8x - y + 3z &= 23 & \qquad -8x - y + 3z &= 23 \\ & & \hline 10x + 5y &= -25 \ (B) \end{aligned}$$

Using equation (A) and (B), we can find x and y

$$\begin{aligned} 16x + y &= -61 \text{ (multiply by -5)} \rightarrow & \qquad -80x - 5y &= 305 \\ 10x + 5y &= -25 & \qquad 10x + 5y &= -25 \\ & & \hline -70x &= 280 \\ & & x &= -4 \end{aligned}$$

We have our x! You can plug this into either (A) or (B). We plug this into (A)

$$\begin{aligned} 16x + y &= -61 & \qquad &\text{Plug } x = -4 \\ 16(-4) + y &= -61 & \qquad &\text{Solve for } y \\ -64 + y &= -61 & \qquad &\text{Add 64 from both sides} \end{aligned}$$

$$y = 3$$

We found y! Plug this and y into any original equations to find z. We plug it into first equation

$$\begin{aligned} 4x - 3y + 2z &= -29 && \text{Plug } x = -4 \text{ and } y = 3 \\ 4(-4) - 3(3) + 2z &= -29 && \text{Simplify} \\ -25 + 2z &= -29 && \text{Add 13 from both sides} \\ 2z &= -4 && \text{Divide both sides by 2} \\ z &= -2 \end{aligned}$$

We now have z! Therefore, our solution is $(-4, 3, -2)$

Note 2.3. If we get a true statement in the middle of solving the system, such as $0 = 0$, that system have infinite solutions. On the other hand, if we get a false statement, such as $0 = 1$, the system have no solution (Identical to system with two variables).

Example 2.3. Solve the system of equations.

$$\begin{cases} 5x - 4y + 3z = -4 \\ -10x + 8y - 6z = 8 \\ 15x - 12y + 9z = -12 \end{cases}$$

We will eliminate z. Start with first two equations

$$\begin{array}{lr} 5x - 4y + 3z = -4 \text{ (multiply by 2)} \rightarrow & 10x - 8y + 6z = -8 \\ -10x + 8y - 6z = 8 & -10x + 8y - 6z = 8 \\ \hline & 0 = 0 \end{array}$$

Since we get a true statement, therefore we have many many solutions (infinite solutions).

Example 2.4. Solve the system of equations.

$$\begin{cases} 3x - 4y + z = 2 \\ -9x + 12y - 3z = -5 \\ 4x + 2y - z = 3 \end{cases}$$

We will eliminate z, start with first two equations

$$\begin{array}{lr} 3x - 4y + z = 2 \text{ (multiply by 3)} \rightarrow & 9x - 12y + 3z = 6 \\ -9x + 12y - 3z = -5 & -9x + 12y - 3z = -5 \\ \hline & 0 = 1 \end{array}$$

Since we get a false statement (a contradiction), therefore we have no solution.

5.3 Systems of Inequalities

3.1 A linear inequality

A linear inequality in two variables is similar to linear equation in two variables. However, instead of $=$ sign we might have one of the following signs $<$, $>$, $\leq$ and $\geq$. Therefore, it can be written as

$$Ax + By < C$$
$$Ax + By > C$$
$$Ax + By \leq C$$
$$Ax + By \geq C$$

examples are $x + 2y < -3$, $4x + y \geq \frac{2}{3}$, and $\frac{4}{3}y - 2\sqrt{3} \leq 0$.

3.2 Graphing a linear inequality

To graph a linear equality, you first need to find the boundary line. The boundary line is the line that divide the xy-plane into two planes. One of the half-plane will be our answer. To find out which sections is our answer, we use a test point within one of them; If it satisfies the inequality, thus that area containing the test point is our solution. Otherwise, the other area is solution.

If we have one of these inequalities $\leq$ or $\geq$, it means the boundary line itself is included in our solution. So we use a solid line. If we have other inequalities $<$ or $>$, then the boundary line is not included in our answer. We indicate this using dashed line.

For example, consider $x + y > 2$. The boundary line is $x + y = 2$. This boundary line divide the whole xy-plane into two sections. If we choose $(0, 0)$ as our test point, we'll see that this test point does not satisfy the inequality

$$0 + 0 > 2 \quad ✗$$

Therefore, the half-plane containing $(0, 0)$ is not our solution and the other one will be our final solution. The following graph shows the boundary line, correct area (shaded in green), and the wrong area (shaded in red). When solving linear inequality, we only need to shade the correct section.

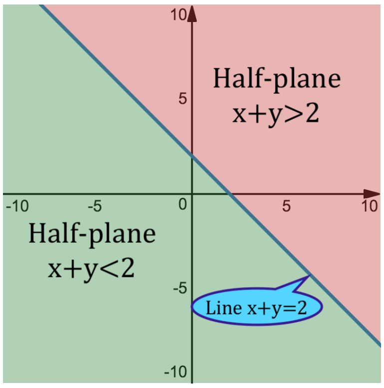

Figure 3.1: The boundary line of $x + y > 2$. The correct area is shaded in green

Graphing a linear inequality

1. Replace the inequality sign with $=$. This equation is our boundary line.
2. Draw the boundary line. Draw a dashed line, if the original inequality contains $<$ or $>$. Draw a solid line, if the original inequality contains $\leq$ or $\geq$.
3. Choose a test point from one of the half-planes.
4. Plug the test point into original inequality.
5. If the test point satisfies the inequality, then shade the half-plane contains this point. Otherwise, shade the other half-plane.

Example 3.1. Graph $y - 2x \leq -4$.

The boundary line is $y - 2x = -4$. Draw the line and since we have $\leq$ sign, therefore make it solid. Using $(0, 0)$ as our test point, we'll see

$$\begin{aligned} 0 - 2(0) &\leq -4 \\ 0 &\leq -4 \quad ✗ \end{aligned}$$

Therefore, we should shade the other half-plane which does not contain $(0, 0)$.

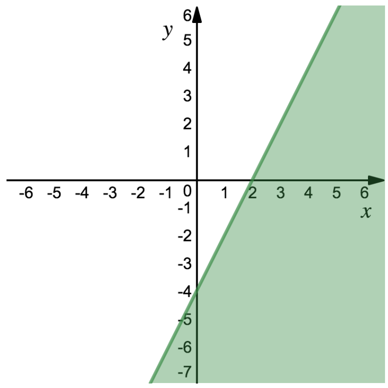

Figure 3.2: The graph of $y - 2x \leq -4$.

Example 3.2. Graph $y - 3x > 1$.

Here the boundary line is $y - 3x = 1$. Draw a dashed line because of $>$ sign. Using $(0, 0)$ as our test point, we get

$$\begin{aligned} 0 - 3(0) &> 1 \\ 0 &> 1 \quad ✗ \end{aligned}$$

Therefore, we shade the half-plane not containing the point $(0, 0)$.

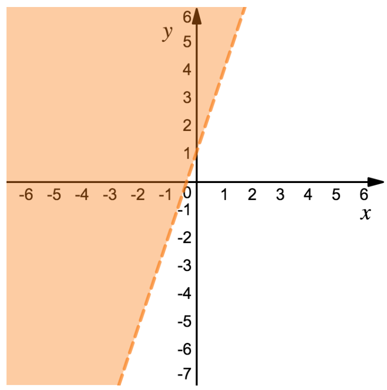

Figure 3.3: The graph of $y - 3x > 1$.

3.3 Graphing systems of linear inequalities

Two linear inequalities create a system of linear inequalities. Our previous work in this chapter dealt with finding the solution set of a system of linear equations. That solution set represented the points of intersection of the graphs of the equations in the system.
In this section, we extend that idea to include systems of linear inequalities. In this case, the solution set is all ordered pairs that satisfy each inequality.

The graph of the solution set of a system of linear inequalities is then the intersection of the graphs of the individual inequalities.

Example 3.3. Graph the solution set of the systems:

$$\begin{cases} x + y > 4 \\ x - y < 2 \end{cases}$$

The boundary lines of each equation are

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

Draw the first one in dashed. Next we will choose $(0, 0)$ as a test point.Since (0,0) does not satisfy the inequality, $0 + 0 > 4$ ✗ ,and we must shade the region where (0,0) is not included. That's why we shade the area above the line.

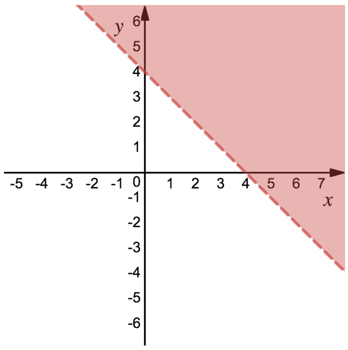

Figure 3.4: The graph of $x + y > 4$.

Graph the second boundary line, $x - y = 2$, use $(0, 0)$ as a test point. You'll see this point satisfy the inequality, $0 - 0 < 2$ ✓. So shade the area containing this point. Notice the boundary line is a dashed line because it's not included in our answer.

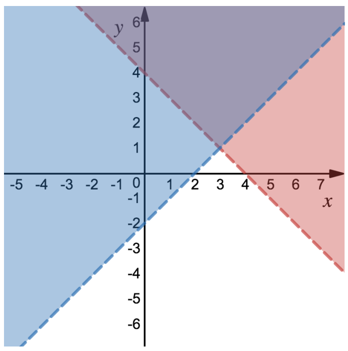

Figure 3.5: The graph of $x - y < 2$ in blue and $x + y > 4$ in red.

The darker shaded region is the intersection of two graphs. Thus, the solution to the system of two inequality is the darker shaded region and its boundary lines.

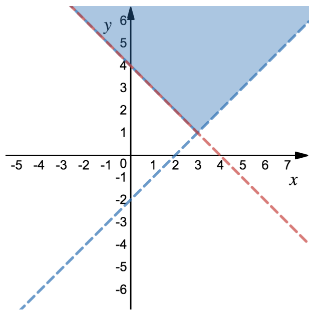

Figure 3.6: The graph of $x - y < 2$ and $x + y > 4$.

Note 3.1. A system of inequalities has no solution if there are no overlapping area. The solution set is $\oslash$.

Example 3.4. Graph the solution set of the systems:

$$\begin{cases} x + y < 4 \\ -2 \leq x < 1 \\ y > -3 \end{cases}$$

The boundary lines of each equation are

$$\begin{cases} x + y = 4 \\ x = -2 \\ x = 1 \\ y = -3 \end{cases}$$

To graph the first inequality, draw $x + y = 4$ in dashed. Because the test point $(0, 0)$ satisfies the inequality, we shade the half-planes contains this point.

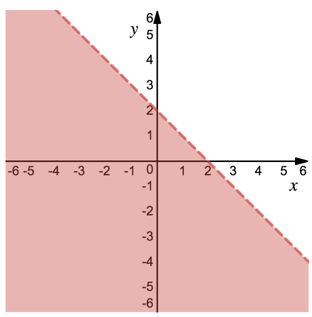

Figure 3.7: The graph of $x + y < 2$.

Let's consider the second given inequality. Its boundary lines are $x = -2$ and $x = 1$. The line of $x = 1$ is not included. Because x is between these two vertical lines, we shade the region between them. We must intersect this region with the red region in Figure 3.7. The resulting region is shown in purple.

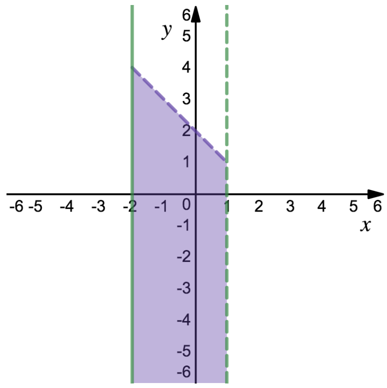

Figure 3.8: The graph of $x + y < 2$ and $-2 \le x < 1$.

Finally, let's consider the last given inequality, $y > -3$. Its boundary line is $y = -3$, which graphs as a horizontal line. Because of the greater than symbol in $y > -3$, the graph consists of the half-plane above the line $y = -3$. We must intersect this half-plane with the region in Figure 3.8. The resulting is shown in blue shading in Figure 3.9.

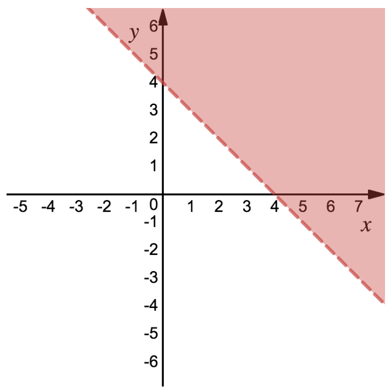

Figure 3.9: The graph of $x + y < 2$, $-2 \le x < 1$ and $y > -3$.

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