Compsci 373 Tutorial 1

Geometry I and II

Office hours:

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- Email: mrog173@aucklanduni.ac.nz
- Tutor for weeks 2-3 and weeks 10-12
- Office hours for weeks I am tutoring (Please email first):
    - In Computer Science Communal Lounge
    - Wednesdays: 14:00-15:00
    - Fridays: 13:00-14:00

Coderunner

- There will be weekly quizzes and exercises on Coderunner
- Some are graded and some are not
- Sandbox quizzes are not graded
- https://coderunner2.auckland.ac.nz/

- Make sure you use coderunner2!

Overview:

- Dot Product
- Cross Product
- Matrix Multiplication
- Vector Normalisation
- 2x2 Matrix Determinant
- Inverse of a matrix
- Distance from Plane to the Origin
- Distance from Point to Plane
- Affine Transform Matrix Translation
- Affine Transform Matrix Scaling
- Affine Transform Matrix Rotation
- Affine Transform Matrix Shearing
- Orthogonal projection

Question: Dot Product

Calculate the dot product of vectors u = $\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$ and v = $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$:

Question: Dot Product

Calculate the dot product of vectors u = $\begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$ and v = $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$:

Question: Cross Product

Calculate the vector cross product of vectors u = $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ and v = $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$:

Question: Cross Product

Calculate the vector cross product of vectors u = $\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ and v = $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$:

Question: Matrix Multiplication

Given M = $\begin{pmatrix} -2 & 3 \\ 3 & 3 \end{pmatrix}$ and N = $\begin{pmatrix} 2 & -3 \\ 3 & -3 \end{pmatrix}$. Compute M x N:

Question: Matrix Multiplication

Given M = $\begin{pmatrix} 4 & 2 & 1 \\ -3 & 1 & 5 \end{pmatrix}$ and N = $\begin{pmatrix} 1 & -2 \\ 0 & -3 \\ 4 & 0 \end{pmatrix}$. Compute M x N:

Question: Vector Normalisation

Normalise the vector v = $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$:

Question: Vector Normalisation

Normalise the vector v = $\begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix}$:

Question: Determinant of a 2x2 Matrix

Given M = $\begin{pmatrix} 1 & -3 \\ 5 & 3 \end{pmatrix}$:

Question: Determinant of a 2x2 Matrix

Given M = $\begin{pmatrix} 6 & 5 \\ 5 & 3 \end{pmatrix}$:

Not examinable

Question: Determinant of a 3x3 Matrix

Given M = $\begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 4 & 0 & 1 \end{pmatrix}$, find the determinant:

Working:

$$Det(M) = a \times Det\left(\begin{pmatrix} e & f \\ h & i \end{pmatrix}\right) - b \times Det\left(\begin{pmatrix} d & f \\ g & i \end{pmatrix}\right) + c \times Det(\begin{pmatrix} d & e \\ g & h \end{pmatrix})$$
$$Det(M) = a \times (ei - fh) - b \times (di \ - fg) + c \times (dh - eg)$$
$$Det(M) = 1 \times (-1 \times 1 - 3 \times 0) - 2 \times (2 \times 1 - 3 \times 4) + 4 \times (2 \times 0 - 4 \times -1)$$
$$Det(M) = 1 \times (-1) - 2 \times (-10) + 4 \times (4) = 35$$

Answer: 35

Question: Inverse of a 2x2 Matrix

Assume a square matrix of M = $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$, calculate the inverse if it exists:

Question: Inverse of a 2x2 Matrix

Assume a square matrix of M = $\begin{pmatrix} 2 & 4 \\ -1 & 5 \end{pmatrix}$, calculate the inverse if it exists:

Question: Inverse of a 2x2 Matrix

Assume a square matrix of M = $\begin{pmatrix} 3 & 2 \\ -3 & 3 \end{pmatrix}$, calculate the inverse if it exists:

Question: Distance from Plane to the Origin

How far is the plane 3x + y - 2z = 5 from the origin (0, 0, 0)?

Formula: $\frac{d}{|\vec{n}|}$

Question: Distance from Plane to the Origin

How far is the plane 10x + 10y - z = 109 from the origin (0, 0, 0)?

Formula: $\dfrac{d}{|\vec{n}|}$

Question: Distance from Plane to the Origin

How far is the plane 52x + 429y - 832z = 0 from the origin (0, 0, 0)?

Formula: $\dfrac{d}{|\vec{n}|}$

Question: Distance from Point to Plane

Find the distance from point Q = (3, 4, 2) to the plane defined by the equation:

3x + y - 2z = 5

Formula: $D = \left|\frac{ax_0+by_0+cz_0+d}{\sqrt{a^2+b^2+c^2}}\right|$

Question: Distance from Point to Plane

Find the distance from point P = (1, 1, 1) to the plane defined by the equation:

x + y + z = 1

Formula: $D = \left|\frac{ax_0+by_0+cz_0+d}{\sqrt{a^2+b^2+c^2}}\right|$

Affine Transformation Matrix (Translation)

Matrix T represents a translation vector $= \begin{pmatrix} x \\ y \end{pmatrix}$

$$T = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

Applying a translation $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ *to the vector* $v = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \times 3 + 0 \times 4 + 2 \times 1 \\ 0 \times 3 + 1 \times 4 + 3 \times 1 \\ 0 \times 3 + 0 \times 4 + 1 \times 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 1 \end{pmatrix}$$

Affine Transformation Matrix (Scaling)

Matrix S represents a scaling with parameters = $\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$

$$S = \begin{bmatrix}\alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1\end{bmatrix}$$

Example:

Applying scaling $\begin{pmatrix}2 \\ 1\end{pmatrix}$ *to the vector* $v = \begin{pmatrix}2 \\ 2 \\ 1\end{pmatrix}$

$$\begin{bmatrix}2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{pmatrix}2 \\ 2 \\ 1\end{pmatrix} = \begin{pmatrix}2 \times 2 + 0 \times 2 + 0 \times 1 \\ 0 \times 2 + 1 \times 2 + 0 \times 1 \\ 0 \times 2 + 0 \times 2 + 1 \times 1\end{pmatrix} = \begin{pmatrix}4 \\ 2 \\ 1\end{pmatrix}$$

Affine Transformation Matrix (Rotation)

- Rotation matrix in 2D
- Matrix R represents a rotation of x degrees anti-clockwise
- 2D rotation matrix in homogeneous system is 3 x 3

x	-90	0	90	180
$\sin(x)$	-1	0	1	0
$\cos(x)$	0	1	0	-1

$$R = \begin{bmatrix} \cos(x) & -\sin(x) & 0 \\ \sin(x) & \cos(x) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

Applying a 90° rotation to the vector $v = \begin{pmatrix} 7 \\ 8 \\ 1 \end{pmatrix}$

$$\begin{bmatrix} \cos(90°) & -\sin(90°) & 0 \\ \sin(90°) & \cos(90°) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 7 \\ 8 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \times 7 + (-1) \times 8 + 0 \times 1 \\ 1 \times 7 + 0 \times 8 + 0 \times 1 \\ 0 \times 7 + 0 \times 8 + 1 \times 1 \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \\ 1 \end{pmatrix}$$

Affine Transformation Matrix (Shearing)

- Matrix H represents a shearing with parameters = $\begin{pmatrix} S_x \\ S_y \end{pmatrix}$
- S_x represents horizontal shearing parameter ($\mathrm{x}' = 1 \times x + \mathrm{y} \times S_x$)
- S_y represents vertical shearing parameter ($\mathrm{y}' = 1 \times \mathrm{y} + x \times S_y$)

$$H = \begin{bmatrix} 1 & S_x & 0 \\ S_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

Applying $S_x = 3$, $S_y = -2$ *to the vector* $v = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$$\begin{bmatrix} 1 & 3 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \times 2 + 3 \times 1 + 0 \times 1 \\ -2 \times 2 + 1 \times 1 + 0 \times 1 \\ 0 \times 2 + 0 \times 1 + 1 \times 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$$

Question: Affine Transformation Matrix

Consider the 2D Cartesian Coordinates of the point P = $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Which statement about P', the transformed point P after performing first a translation by t = $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, then a rotation by -90 degrees and finally a scaling by 2 in both x and y direction is true.

Question: Magnitude of Orthogonal Projection

Consider the vectors b = $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and a = $\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$. What is the magnitude of orthogonal projection, b_a of b onto a?

Formula: $\mathrm{b_a} = \frac{a \cdot b}{|\vec{a}|}$