

# Overview: File and Indexing

## File Storage:

1. Physical storage hierarchy
2. Secondary storage
3. Storage access
4. Redundant arrays of independent disks (RAID)
5. Organize records in a file

## Reading materials:

- Chapter 12, Database System Concepts, 7th Edition
- Chapter 16-17, Fundamentals of Database Systems

## Organize Records in a File

An operating system (OS) manages a *file system*, where a file is stored on a number of disk blocks.

- A database is stored as a collection of files.
- A file contains a sequence of records.
- A record contains a sequence of fields.

student ID	student name	gpa	...
1	Tom	3.7	...
2	David	3.6	...
3	Jack	3.9	...
4	Eva	4.1	...
5	Tony	2.7	...
6	Adam	3.8	...
...	...	...	...

# Organize Records in a File

- Blocking factor: the average number of records per block in a file.

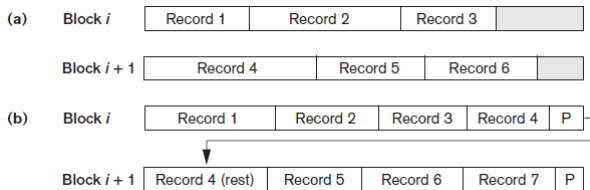


Figure: Types of record organization (a) Unspanned (b) Spanned

We use unspanned organization by default in the future discussions.

## Example: Blocking Factor

Blocking factor: the average number ( $bfr$ ) of records per block in a file.

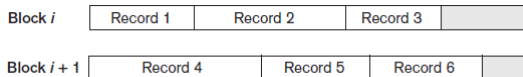


Figure: Unspanned Record Storage

Suppose that we have

- A file with  $r = 300$  records,
- Block size  $B = 4096$  bytes,
- Each record is of size  $R = 100$  bytes.

- $bfr = \lfloor \frac{B}{R} \rfloor = \lfloor \frac{4096}{100} \rfloor = 40.$

4    ■ # of blocks =  $\lceil \frac{r}{bfr} \rceil = \lceil \frac{300}{40} \rceil = 8.$

# Organize Records in a File

Given a field  $X$ , records can be organized, in a file:

- Ordered (sequential) on field  $X$  ( $X$  is called the ordering field)
- Unordered (heap) file

Possible operations on a file containing records:

- Search
- Insertion
- Deletion
- Update

# Organize records in a file



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- Records and blocking
- Sequential files
- Unordered files

# Sequential File

Sequential file of EMPLOYEE with the *ordering field* Name:

	Name	Sex	Birth_date	Job	Salary	Sex
Block 1	Aaron, Ed					
	Abbott, Diane					
	Acosta, Marc					
Block 2	Adams, John					
	Adams, Robin					
	Akers, Jan					
Block 3	Alexander, Ed					
	Alfred, Bob					
	Allen, Sam					
Block 4	Allen, Troy					
	Anders, Keith					
	Anderson, Rob					
Block 5	Anderson, Zach					
	Angeli, Joe					
	Archer, Sue					
Block 6	Arnold, Mack					
	Arnold, Steven					
	Atkins, Timothy					
Block n-1	Wong, James					
	Wood, Donald					
	Woods, Manny					
Block n	Wright, Pam					
	Wyatt, Charles					
	Zimmer, Byron					

# Search

Search for a record with  $X = K$  in a file with  $b$  blocks ordered on  $X$ .

- Sequential scan.
- Binary search (the location of the  $i$ -th record can be calculated).

	Name	Sex	Birth_date	Job	Salary	Sex
Block 1	Aaron, Ed					
	Abbott, Diane					
	Acosta, Marc					
Block 2	Adams, John					
	Adams, Robin					
	Akers, Jan					
Block 3	Alexander, Ed					
	Alfred, Bob					
	Allen, Sam					
Block 4	Allen, Troy					
	Anders, Keith					
	Anderson, Rob					
Block 5	Anderson, Zach					
	Angeli, Joe					
	Archer, Sue					
Block 6	Arnold, Mack					
	Arnold, Steven					
	Atkins, Timothy					
⋮						
Block $n-1$	Wong, James					
	Wood, Donald					
	Woods, Manny					
Block $n$	Wright, Pam					
	Wyatt, Charles					



# Sequential File

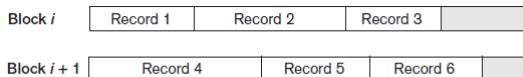
Type of Organization	Access/Search Method	Average Blocks to Access a Specific Record
Ordered	Sequential scan	$b/2$
Ordered	Binary search	$\log_2 b$

Figure: Average access times for a sequential file of  $b$  blocks

Suppose that we have

- An ordered file with  $r = 300$  records, the block size  $B = 4096$  bytes, records are of fixed size  $R = 100$  bytes.
- $bfr = 40$ , the file is stored on  $b = \lceil \frac{r}{bfr} \rceil = 8$  blocks.
- Worst-case # of I/Os in a binary search?  $\lceil \log_2 b \rceil = 3$  I/Os.

# Sequential File



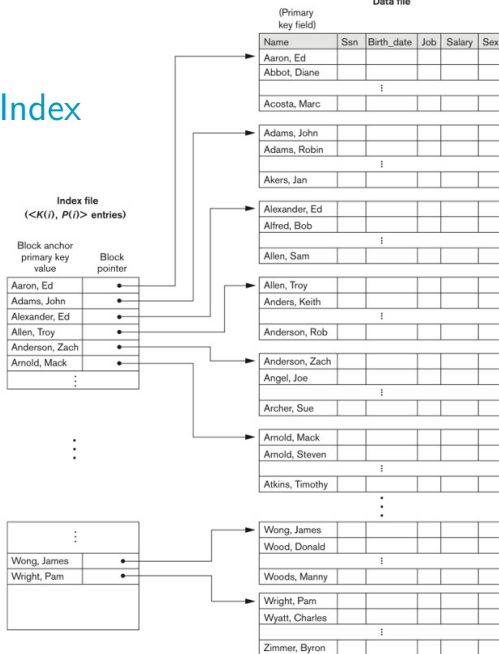
- Search: can we do better than binary search?
- How to handle insertion/deletion?

# Index on Sequential Files

- Index Entry  $\langle \text{key, block pointer} \rangle$ : represent the blocks in a file using a set of  $\langle \text{key, block pointer} \rangle$  pairs — each pair is called an *index entry*.

If the *ordering field* is key, the index is called *primary index*.

# Primary Index



In a block, the record whose key goes to the index entry is called the *anchor record* or *block anchor*.

## Primary Index - Example

Suppose that we have

- an ordered file with  $r = 300000$  records,
- the block size  $B = 4096$  bytes,
- records are of fixed size  $R = 100$  bytes,
- so the blocking factor is  $bfr = 40$ ,
- so the file has  $n_b = \lceil \frac{r}{bfr} \rceil = 7500$  blocks,
- a primary index on key  $X$ .

Answer the following questions:

- How many index entries ( $\langle \text{key}, \text{block pointer} \rangle$  pairs) does the index file have?
- If each index entry takes  $R_i = 4$  bytes, how many entries can each block hold?
- How many blocks does the index file have?

## Primary Index - Example

- How many index entries  $n_i$  ( $\langle \text{key}, \text{block pointer} \rangle$  pairs) does the index file have?  
 $n_i = n_b$ , one entry per data block.
- If each index entry takes  $R_i = 4$  bytes, how many entries can each block hold?  
 $bfr_{idx} = \lfloor \frac{B}{R_i} \rfloor = 1024$ .
- How many blocks does the index file have?  $b_{idx} = \lceil \frac{n_i}{bfr_{idx}} \rceil = \lceil \frac{7500}{1024} \rceil = 8$ .

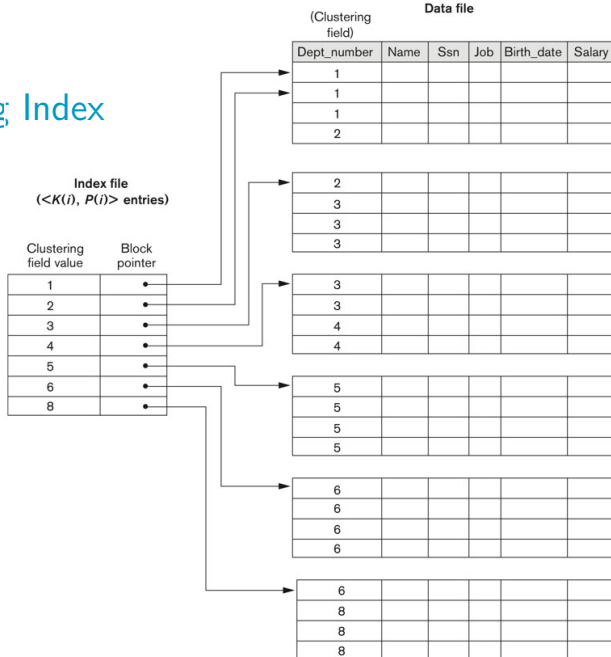
# Index on Sequential Files

- Index Entry  $\langle \text{key}, \text{block pointer} \rangle$ : represent the blocks in a file using a set of  $\langle \text{key}, \text{block pointer} \rangle$  pairs — each pair is called an *index entry*.

If the *ordering field* is a key, we build *primary index*.

If the *ordering field* is not a key, we build *clustering index*.

# Clustering Index





# Organize records in a file



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- Records and blocking
- Sequential file
- Unordered file

## Unordered file

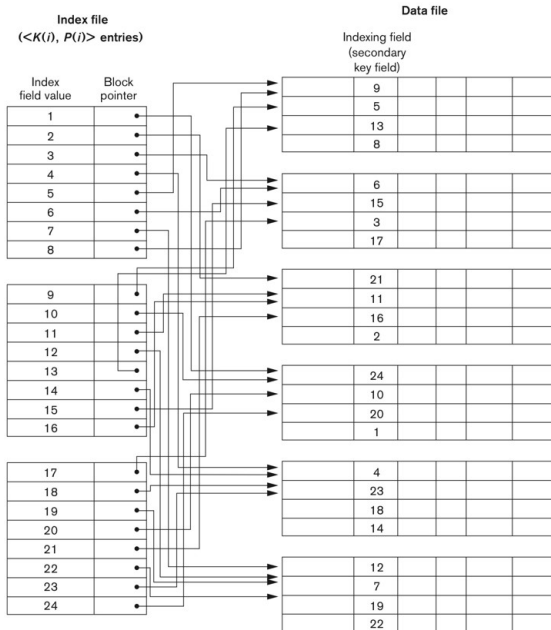
Select studentID From Student  
Where studentName = `Jack` and `GPA < 3`

student ID	student name	gpa	...
1	Tom	3.7	...
2	David	3.6	...
3	Jack	3.9	...
4	Eva	4.1	...
5	Tony	2.7	...
6	Adam	3.8	...
...	...	...	...

- A file that is unordered w.r.t. the indexing field may be ordered by other fields.
- If the indexing field is a key, we build a *secondary index* with *only* index entries.

# Unordered File

Secondary index on key



## Example: Unordered File Index on Key Values

Suppose that we have

- an unordered file with  $r = 300000$  records,
- the block size  $B = 4096$  bytes,
- a secondary index on the unordered field where the field is a key,
- each index entry:  $s_i = 4$  bytes.
- the number of distinct values in the indexing field:  $n_{dis} = 102400$ .

Answer the following questions:

- How many blocks does the index file occupy?  
 $b_{idx} = n_{dis} / (B / s_i) = 102400 / (4096 / 4) = 100$
- How to retrieve a record of key  $K$ ?  $O(\log_2 b_{idx})$ .

# Single Level Ordered Indexes

**Table 17.2** Properties of Index Types

Type of Index	Number of (First-Level) Index Entries	Dense or Nondense (Sparse)	Block Anchoring on the Data File
Primary	Number of blocks in data file	Nondense	Yes
Clustering	Number of distinct index field values	Nondense	Yes/no <sup>a</sup>
Secondary (key)	Number of records in data file	Dense	No
Secondary (nonkey)	Number of records <sup>b</sup> or number of distinct index field values <sup>c</sup>	Dense or Nondense	No

<sup>a</sup>Yes if every distinct value of the ordering field starts a new block; no otherwise.

<sup>b</sup>For option 1.

<sup>c</sup>For options 2 and 3.

- Dense index: one index entry for *each* record in the data file
- Non-dense (sparse) index: one index entry for a set of records in the data file

# Organize records in a file



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- Records and blocking
- Sequential file
- Unordered file

# Overview: File and Indexing

- Tree-based indexes
  - Multi-level indexes
  - Dynamic multi-level indexes —  $B^+$  - tree
- Hashing
  - Static hashing
  - Dynamic hashing — extendible hashing

Reading materials:

- Chapter 13, Database System Concepts, 7th Edition
- Chapter 16-17, Fundamentals of Database Systems

<http://codex.cs.yale.edu/avi/db-book/>

# Revision

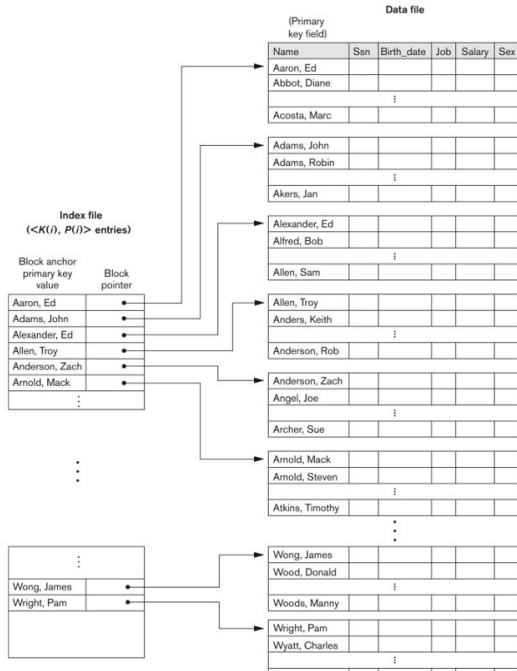
Single-Leveled Indexes:

- Primary index (key, ordered)
- Clustering index (non-key ordered)
- Secondary index (unordered)

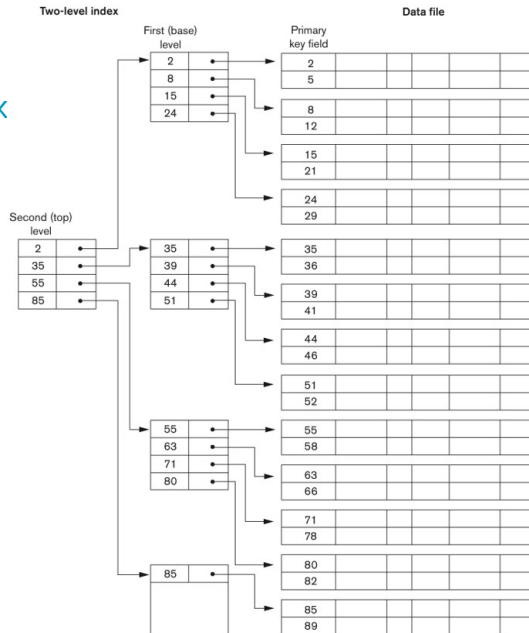
Search: binary search.



# Revision



# Multi-Level Index



# Multi-Level Indexes

- Fanout: the blocking factor of the index file, denoted as  $f$ .
- If the first level has  $r_1$  index entries,
- The second level has  $r_2 = \lceil r_1/f \rceil$  index entries,
- The third level has  $r_3 = \lceil r_2/f \rceil$  index entries,
- ...
- The top index level has one block of at most  $f$  index entries.
- There will be approximately  $t = \lceil \log_f(r_1) \rceil$  levels.

## Example

Consider a multi-level index of a data file. Assume that the fanout  $f_i = 273$ , the first level index has  $b_1 = 1099$  blocks. The secondary level blocks  $b_2 = \lceil b_1/f \rceil = 5$ . The third level blocks  $b_3 = \lceil b_2/f \rceil = 1$ . How many blocks should one access at most in order to locate a record with a given key value?

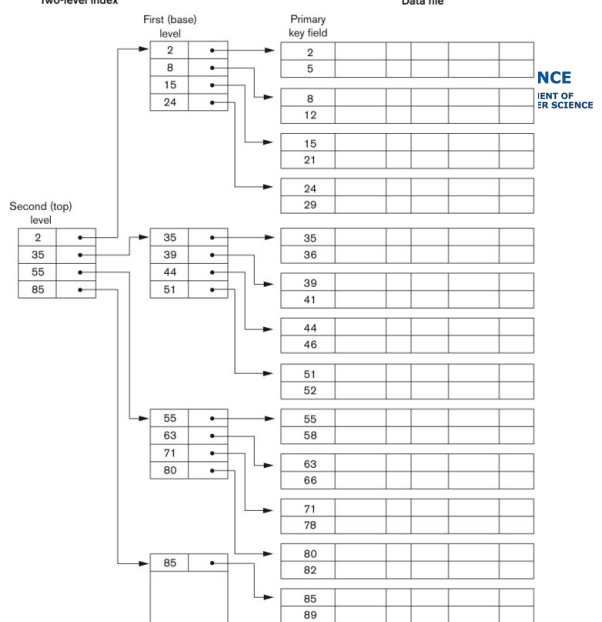
$$O(\log_f b_1) = O(\log_{273} 1099) .$$

When  $b_1 = 1099^2$ , the cost becomes  $O(2 \log_{273} 1099)$ .

The query complexity grows extremely slow with the increasing data size.

# Search on Multi-Level Indexes

Search: 46



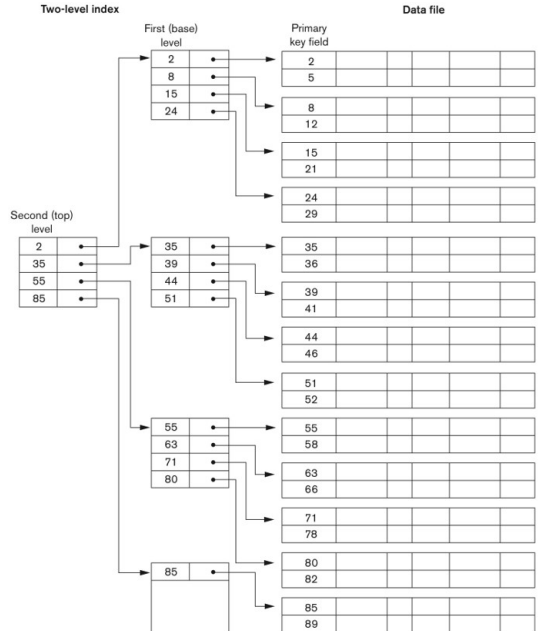
# Multi-Level Indexes

How to handle

- insertions?
- deletions?

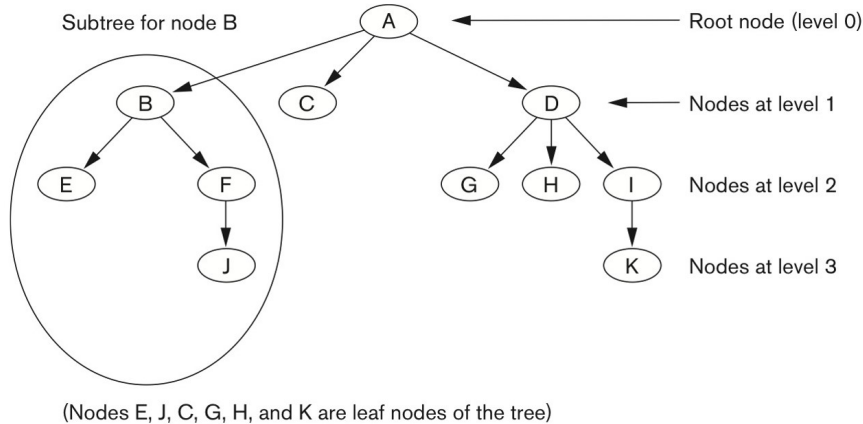
Introduce the concept of

- tree,
- search tree,
- and balanced search tree.



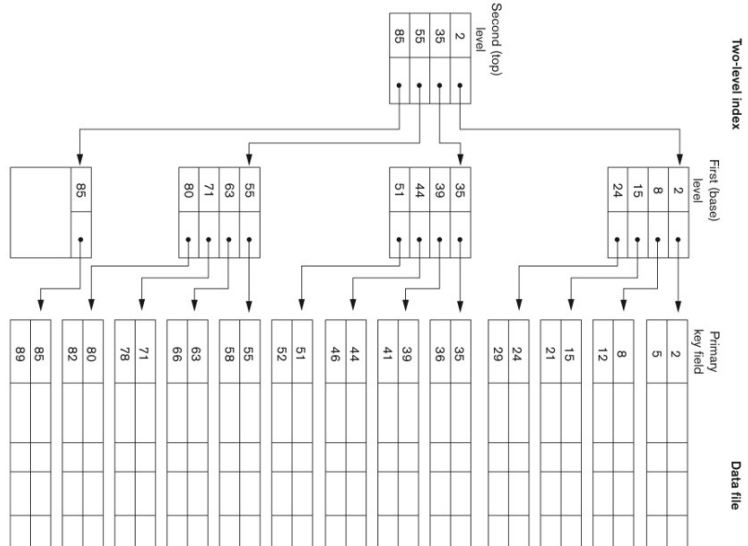
# Tree

- Nodes
- Parent/Child
- Root/Leaf
- Internal node
- Level
- Subtree
- Balanced tree:  
All leaf nodes are  
of the “same”  
level



# Multi-Level Index is a Balanced Tree

- Nodes
- Parent/Child
- Root/Leaf
- Internal node
- Level
- Subtree
- Balanced tree:  
All leaf nodes are  
at the “same”  
level



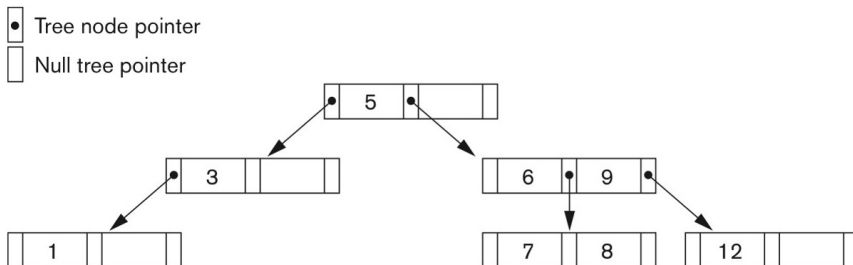


# Search Tree

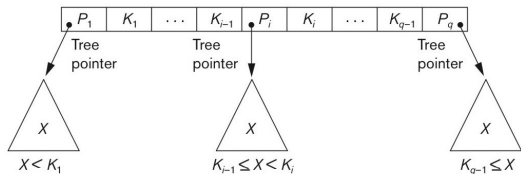
For any node  $v$  on a search tree whose value is denoted as  $K$ ,

- the values in the left subtree of  $v$  is no greater than  $K$ ;
- the values in the right subtree of  $v$  is no smaller than  $K$ .

Example of a Search Tree:

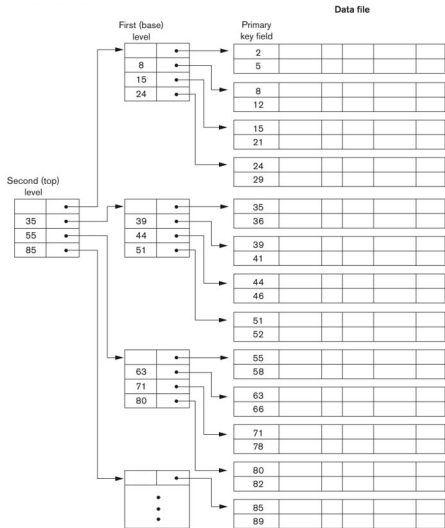


# Multi-Level Index: a Balanced Search Tree



- Search  $[30, +\infty)$
- Insert
- Delete

7, ranges  $[30, 50]$ ,



## $B^+$ -Tree: a Dynamic Multi-Level Index

Generally,  $B^+$ -Tree requires that all leaves are of the same level, each node (except for the root node) is at least *half full*. An insertion on a full node incurs a split while a deletion from a half-full node causes a redistribution/merge over the current node and its neighbors. The split and merge operation can be triggered recursively.

$B^+$ -Tree is I/O aware, and support search exact and range queries:

- Make 1 node = 1 physical page
- Balanced, height adjusted tree
- Make leaves into a linked list (for range queries)

```
SELECT cname FROM Company WHERE price = 25;
```

```
SELECT cname FROM Company WHERE 20 <= price AND price <= 30
```

# B<sup>+</sup>-Tree

B<sup>+</sup>-tree with two parameters

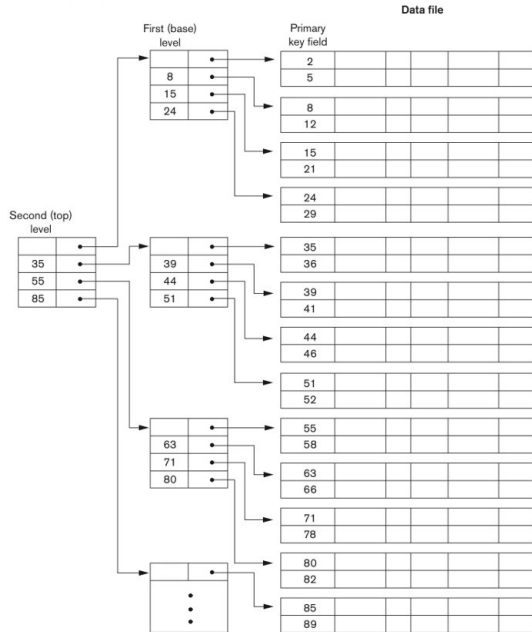
- Blocking factor of the index files (index fanout)  $f_i$  and
- Blocking factor of the data files  $f_l$ .

Each leaf node contains

- at most  $f_l$  data records
- at least  $\lceil f_l/2 \rceil$  data records

Each internal node (except the root) contains

- $f$  **tree pointers** and  $f - 1$  keys (not records!!!) where
- $f$  is at most  $f_i$  and
- $f$  is at least  $\lceil f_i/2 \rceil$



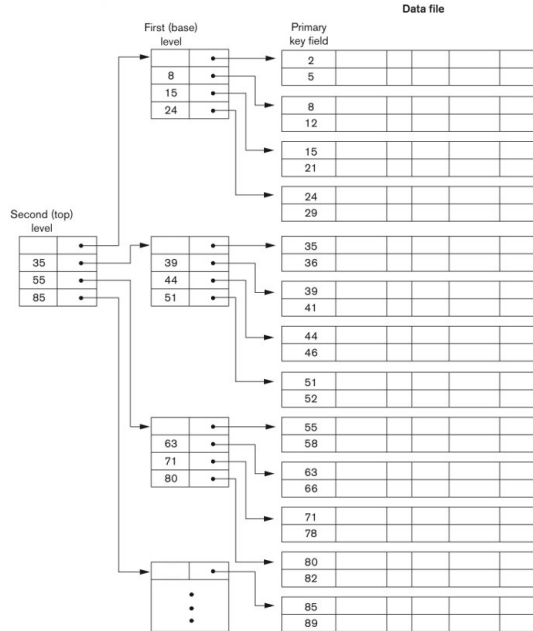
# $B^+$ -Tree

$B^+$ -tree with two parameters

- Blocking factor of the index files (index fanout)  $f_i$  and
- Blocking factor of the data files  $f_l$ .

Facts that you should know:

- How many search values per page?
- How many levels in tree?
- What is the I/O cost of an equal search?
- What is the I/O cost of a range search?
- What is the I/O cost of an update?



## $B^+$ -Tree: Cost Analysis

Let:

- $f$  = fanout, which is in  $[f_i/2, f_i]$
- $N$  = the total number of pages we need to index
- $F$  = fill-factor (usually  $= 2/3$ )

What height ( $h$ ) does our  $B^+$  Tree need to be?

$h = 2$  : Just the root node - room to index  $f$  pages

$h = 3$  :  $f$  level-1 nodes - room to index  $f^2$  pages

$h = 4$  :  $f^2$  level-1 nodes - room to index  $f^3$  pages

...

$h$  :  $f^{h-2}$  level-1 nodes - room to index  $f^{h-1}$  pages!

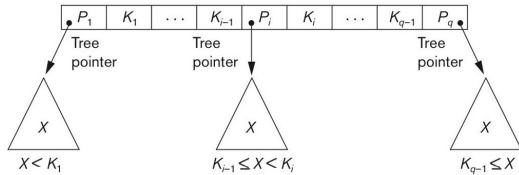
$h = O(\log_f N)$ .

## $B^+$ -Tree: Cost Analysis

Example:

- Key size = 4 bytes, Pointer size = 8 bytes  
Note: record size not relevant for these calcs (could be MBs)
- We want each node to fit in a single block/page (4 KBs)  
 $2f \times 4 + (2f + 1) \times 8 \leq 4k \Rightarrow f = 170$
- What is the height of a  $B^+$  Tree that indexes  $10^8$  pages?  $h = 5$ .

# B<sup>+</sup>-Tree: Insertion

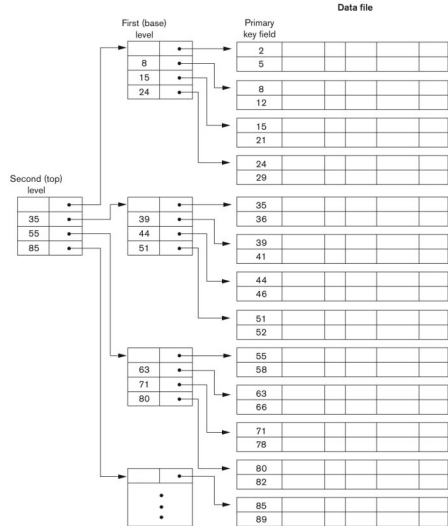


■ Search  
[30, +∞)

■ Insert

■ Delete

7, ranges [30, 50],





# $B^+$ -Tree: Insertion

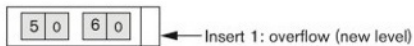
1. Search K, then
2. Insert K

Consequences:

- A leaf node is **overflow** if it has more than  $f_l$  data records
- An internal node is **overflow** if it has more than  $f_i$  pointers

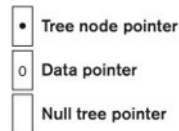
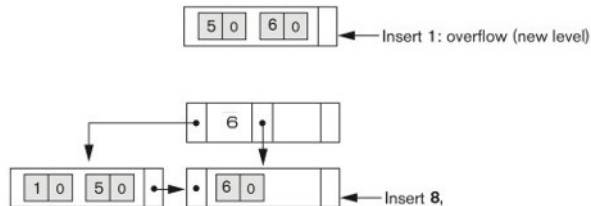
Example:  $f_i = 3, f_l = 2$

Insert 6, 5, 1



Example:  $f_i = 3, f_l = 2$

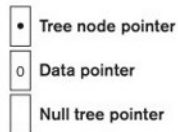
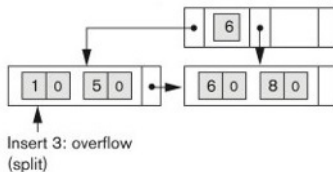
Insert 6, 5, 1, 8



Evenly split; the size of first node splitted is no smaller than the second node splitted!  
A leaf registers its first key to the index entry in the parent.

Example:  $f_i = 3, f_j = 2$

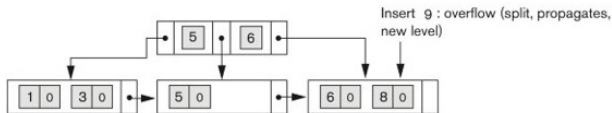
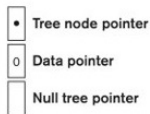
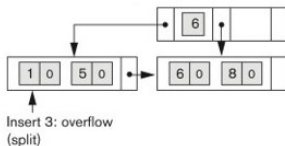
Insert 6, 5, 1, 8, 3



Example:  $f_i = 3, f_l = 2$

Insert 6, 5, 1, 8, 3:

- insertion (overflow)  $[1, 5] \Rightarrow [1, 3, 5]$
- evenly split  $b = [1, 3, 5] \Rightarrow [1, 3]$  and  $[5]$
- register  $[1, 3], 5, [5]$  to the parent of  $b$ , 5 is the key of the anchor record of node  $[5]$



## Example: $f_i = 3, f_l = 2$ - Recursive Split

Insert 6, 5, 1, 8, 3, 9  
Steps:

1. insert 9 to leaf 

6	8
---	---
2. partition the overflowed leaf  

6	8	9
---	---	---

 $\Rightarrow$ 

6	8
---	---

 & 

9
---
3. insert 

6	8
---	---

 9 

9
---

 to the parent 

5	6
---	---
4. partition the overflowed root  

5	6	9
---	---	---

 $\Rightarrow$ 

5
---

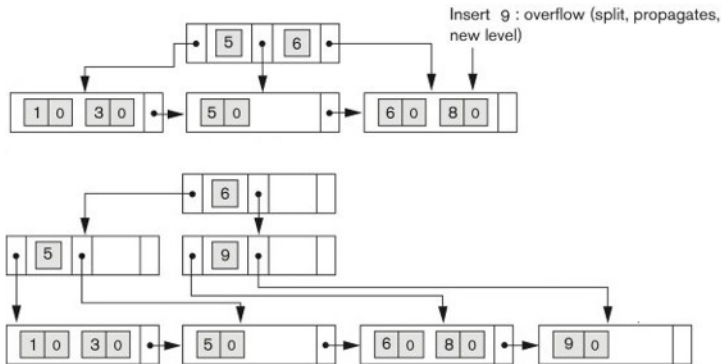
 6 

9
---
5. put 6 as the single key in the new root with left child 

5
---

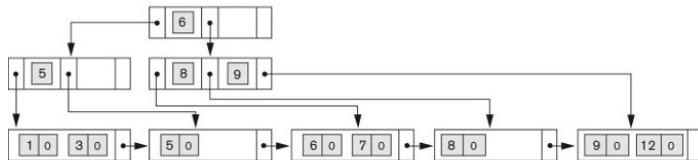
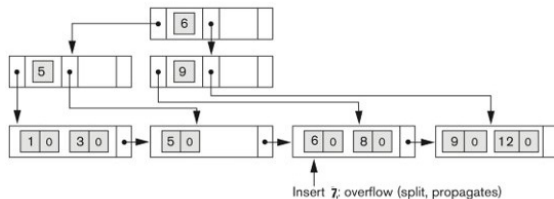
 and right child 

9
---



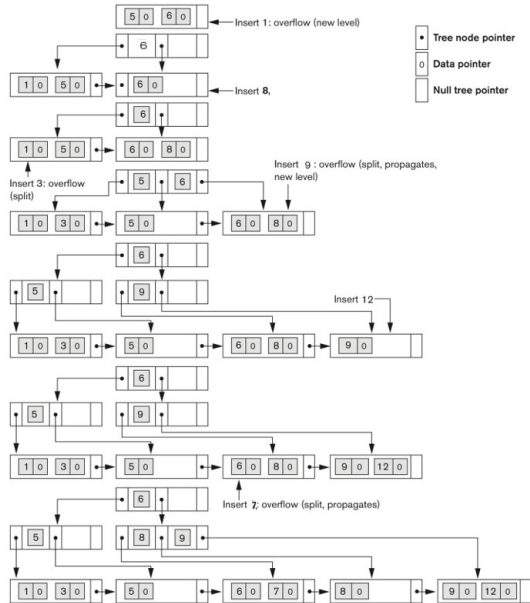
Example:  $f_i = 3, f_l = 2$

Insert 6, 5, 1, 8, 3, 9, 12, 7



Example:  $f_i = 3, f_l = 2$

Insert 6, 5, 1, 8, 3, 9, 12, 7





## $B^+$ -Tree: Insertion

Insert a data record with key  $K$ :

1.  $p$ : find the address of the data block of  $K$ ;
2.  $b$ : load  $p$  into the main memory;
3. insert the record to  $b$ ;
4. if  $b$  overflows,
  - split  $b$  into two new blocks with addresses  $c_1$  and  $c_2$ :
    - $c_1$  contains the first (smallest)  $\lceil f_l + 1 \rceil$  records
    - $c_2$  contains the last (largest)  $\lfloor f_l + 1 \rfloor$  records
  - let  $s$  be the anchor record of  $c_2$ ;
  - if  $p$  was the root before the insertion, let the new root be  $\boxed{c_1, s, c_2}$ ; otherwise,
    - let  $anc$  be the parent of  $p$ ,
    - call  $register(anc, p, s, c_1, c_2)$  — a procedure that registers the split of  $p$  to its parent
5. otherwise, write  $b$  back to address  $p$ .

## $B^+$ -Tree: Insertion

register( $anc, p, s, p_1, p_2$ ):

1. Let  $K_1, P_1, K_2, P_2, \dots, K_{n-1}, P_n$  be the content of the block of  $anc$ ;

2. Let  $i$  be the integer such that  $P_i = p$ ;

3. Register the split by letting  $b = \boxed{K'_1, P'_1, K'_2, P'_2, \dots, K'_n, P'_{n+1}}$  be

$\boxed{K_1, P_1, \dots, K_i, p_1, s, p_2, K_{i+1}, \dots, K_{n-1}, P_n}$ .

4. if  $b$  overflows,

■ split  $b$  into two new blocks with addresses  $c_1$  and  $c_2$ :

■ let  $m = \lfloor (n+1)/2 \rfloor$

■  $c_1$  contains  $\boxed{P'_1, K'_1, \dots, K'_{m-1}, P'_m}$ ;  $c_2$  contains  $\boxed{P'_{m+1}, K'_{m+2}, \dots, K'_n, P'_{n+1}}$

■ let  $s$  be the anchor record of  $K_{m+1}$ ;

■ if  $anc$  is the root, then let the new root be  $c_1, s, c_2$  otherwise,

■ let  $anc'$  be the parent of  $anc$ ,

■ call register( $anc', anc, s, c_1, c_2$ )

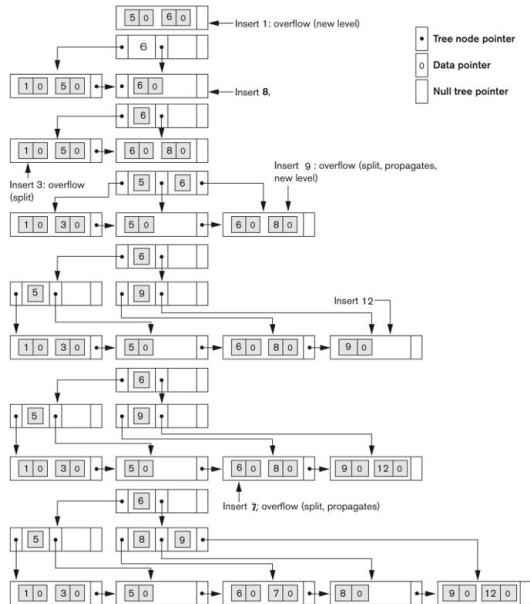
50 5. otherwise, write  $b$  back to address  $anc$ .

Example:  $f_i = 3, f_l = 2$

Insert 6, 5, 1, 8, 3, 9, 12, 7

Delete 7, 12, 9, 3, 8, 1, 5, 6

Follow the code



# Overview: Indexes

- Tree-based indexes
  - Multi-level indexes
  - Dynamic multi-level indexes —  $B^+$  - tree
- Hashing
  - Static hashing
  - Dynamic hashing — extendible hashing

Reading materials:

- Chapter 11, Database System Concepts, 7th Edition
- Chapter 16-17, Fundamentals of Database Systems

<http://codex.cs.yale.edu/avi/db-book/>