

Computer Graphics and Image Processing

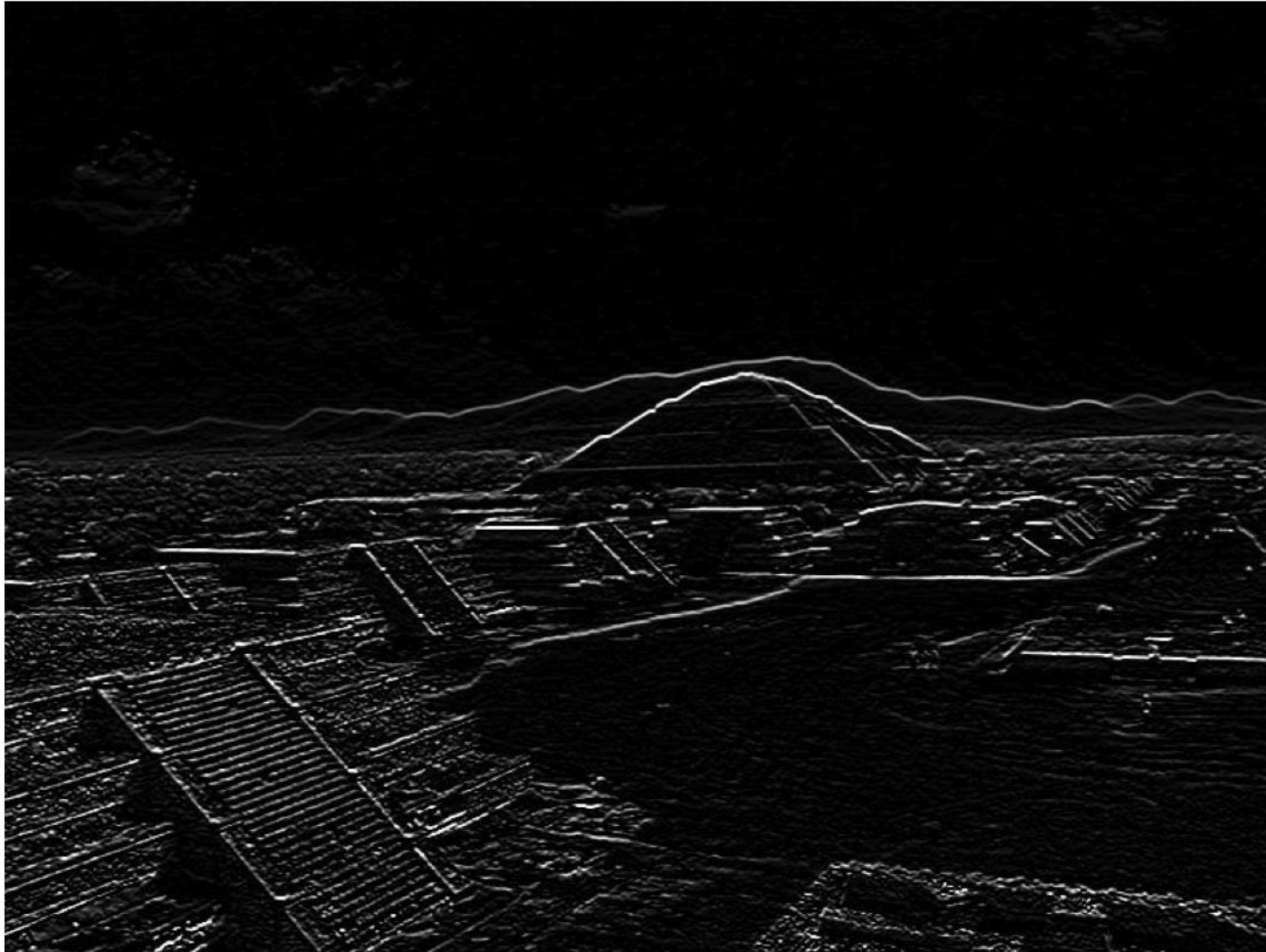
Part 3: Image Processing
5 – Edges

Martin Urschler, PhD

Edges are important image features



Edges are important image features



Semantic &
shape
information!

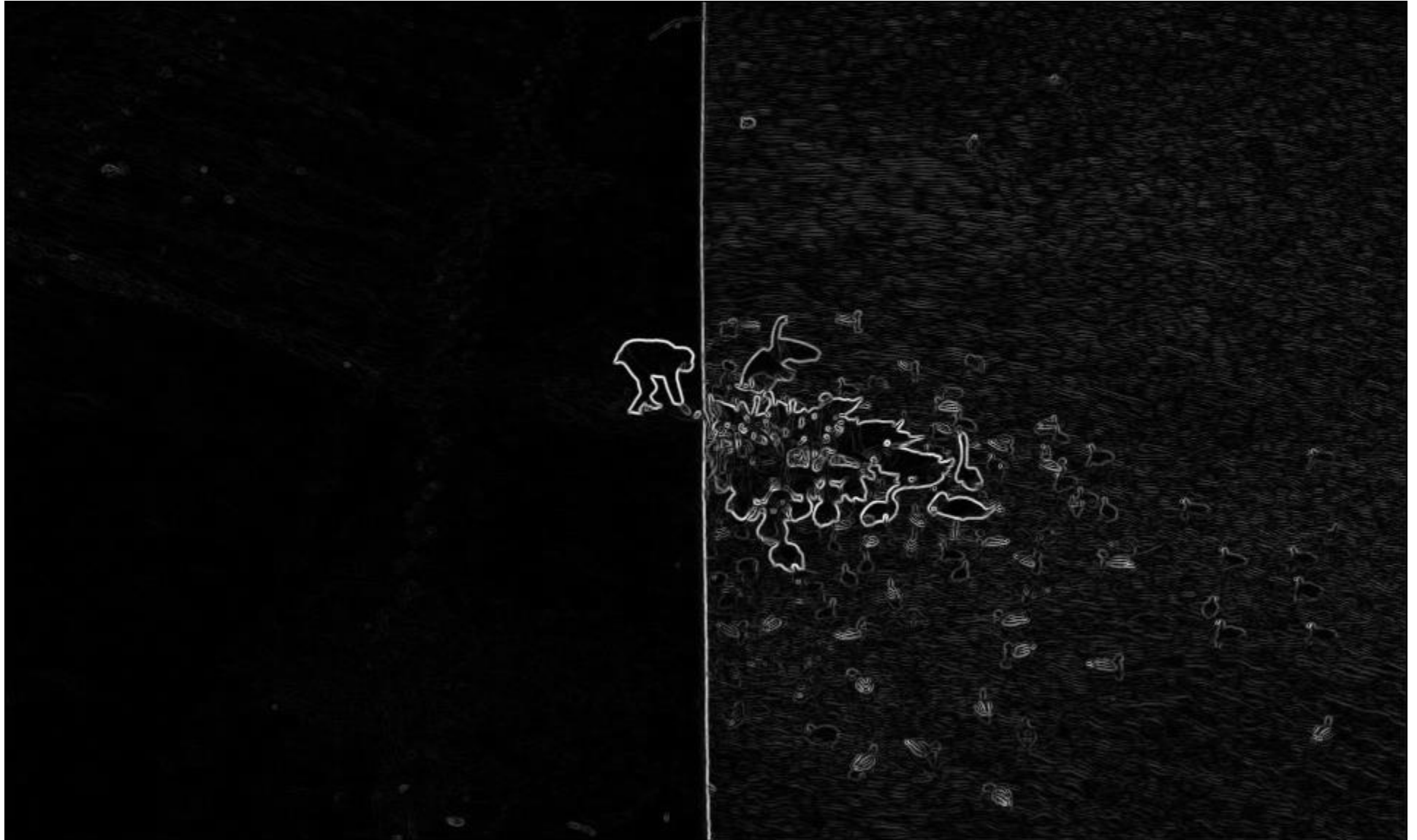
Moving window transform for edges

- Edge detection is a major application of image filtering
- Informal definition of an **edge**:
 - Locations of sudden grey level / color changes
 - Transition between objects or between object and background
 - Locations that attract visual attention (salient regions)
- Problem: Noise in the image has similar properties!
- Conventional 3-step edge detection approach:
 1. Noise reduction (preserving edges as much as possible)
 2. Edge enhancement
 3. Edge localization (single pixel wide edges)

Example image

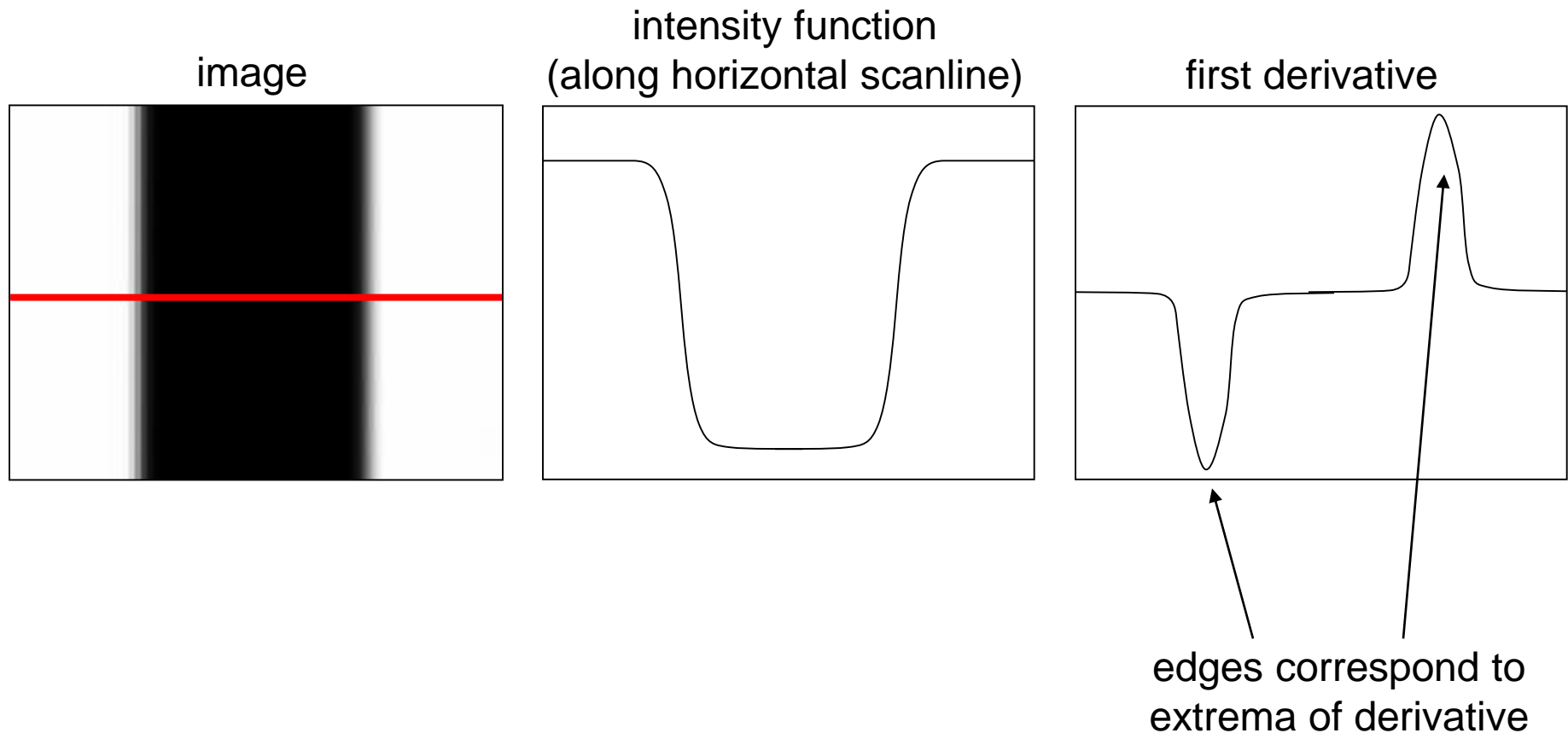


Edges



Edge detection

- An edge is a location of rapid change in the image intensity function



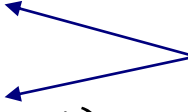
Moving window transform for edges

- Estimation of the grey level **gradient** at pixels:

$$g_x(x, y) = \frac{\partial f(x, y)}{\partial x} \approx \frac{f(x + 1, y) - f(x - 1, y)}{2}$$

$$g_y(x, y) = \frac{\partial f(x, y)}{\partial y} \approx \frac{f(x, y + 1) - f(x, y - 1)}{2}$$

Finite difference
approximations

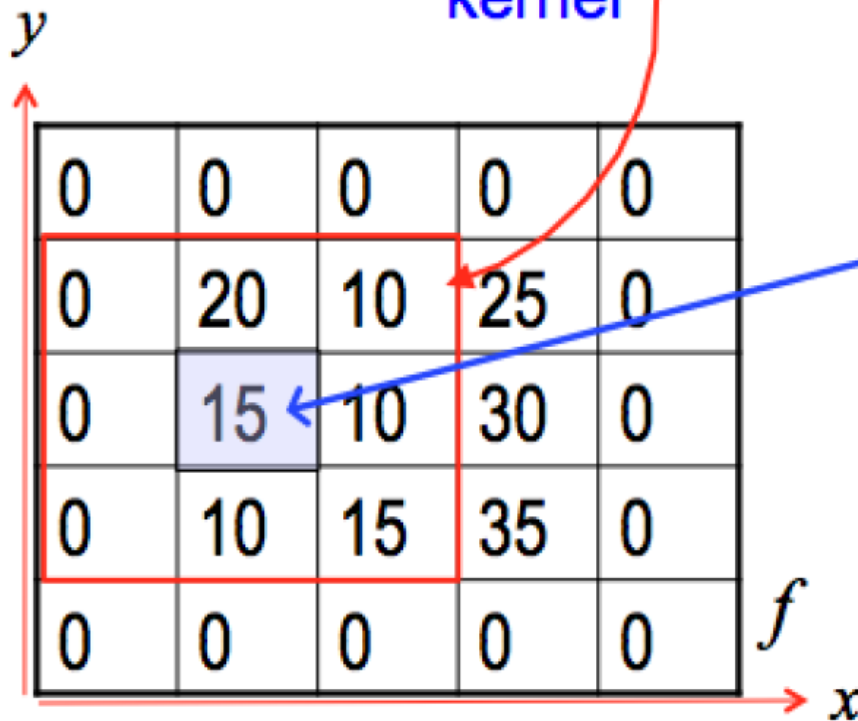


Derivative filter in x (vertical edge)

Prewitt operator

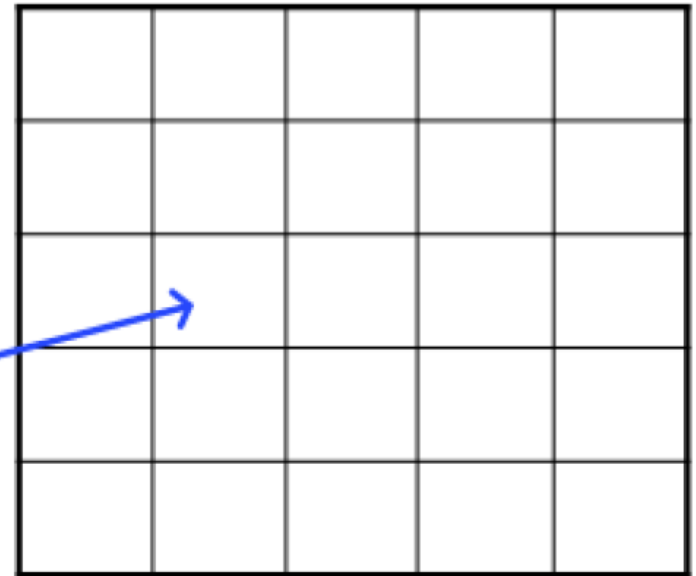
| | | |
|----|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

kernel



Prewitt combines derivative and Smoothing in a 3x3 kernel!

$$g = \text{MWT}(f) \Rightarrow$$



Finite difference approximation of the partial derivative

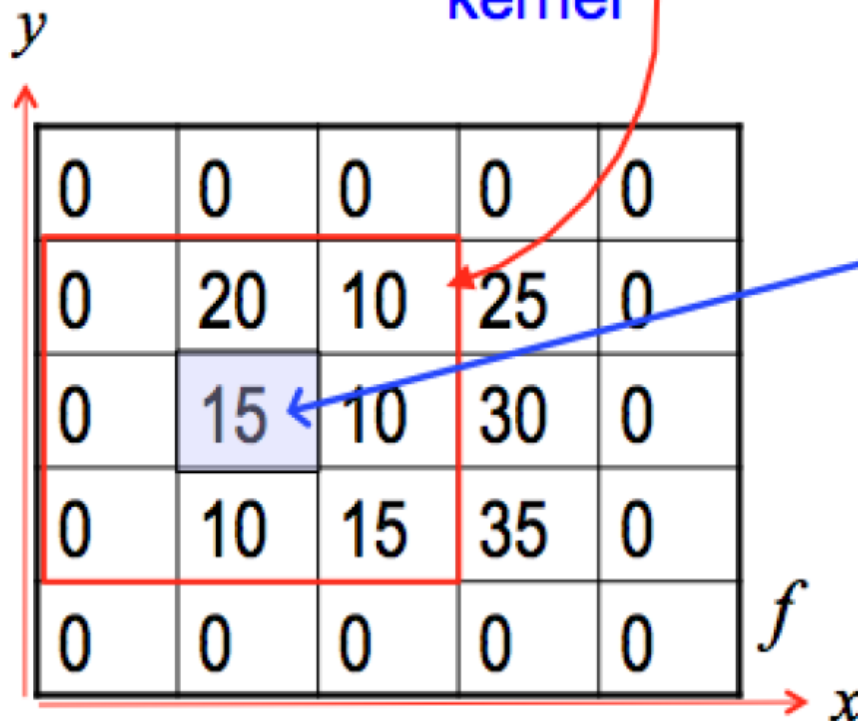
$$g(x, y) = \frac{\partial f(x, y)}{\partial x}$$

Derivative filter in x (vertical edge)

Prewitt operator

| | | |
|----|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

kernel



Prewitt combines derivative and Smoothing in a 3x3 kernel!

$$g = \text{MWT}(f) \Rightarrow$$

| | | | | |
|--|----|----|-----|--|
| | | | | |
| | 20 | 20 | -20 | |
| | 35 | 45 | -35 | |
| | 25 | 40 | -25 | |
| | | | | |

Finite difference approximation of the partial derivative

$$g(x, y) = \frac{\partial f(x, y)}{\partial x}$$

Moving window transform for edges

- Noise smoothing using a low-pass filter (mean, Gaussian, etc.)

- Separable Prewitt kernels (includes 3×3 mean):

$$\frac{1}{6} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

vertical edge

$$\frac{1}{6} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

horizontal edge

- Separable Sobel kernels (includes 3×3 weighted mean, as an approximation to Gaussian kernel):

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Moving window transform for edges

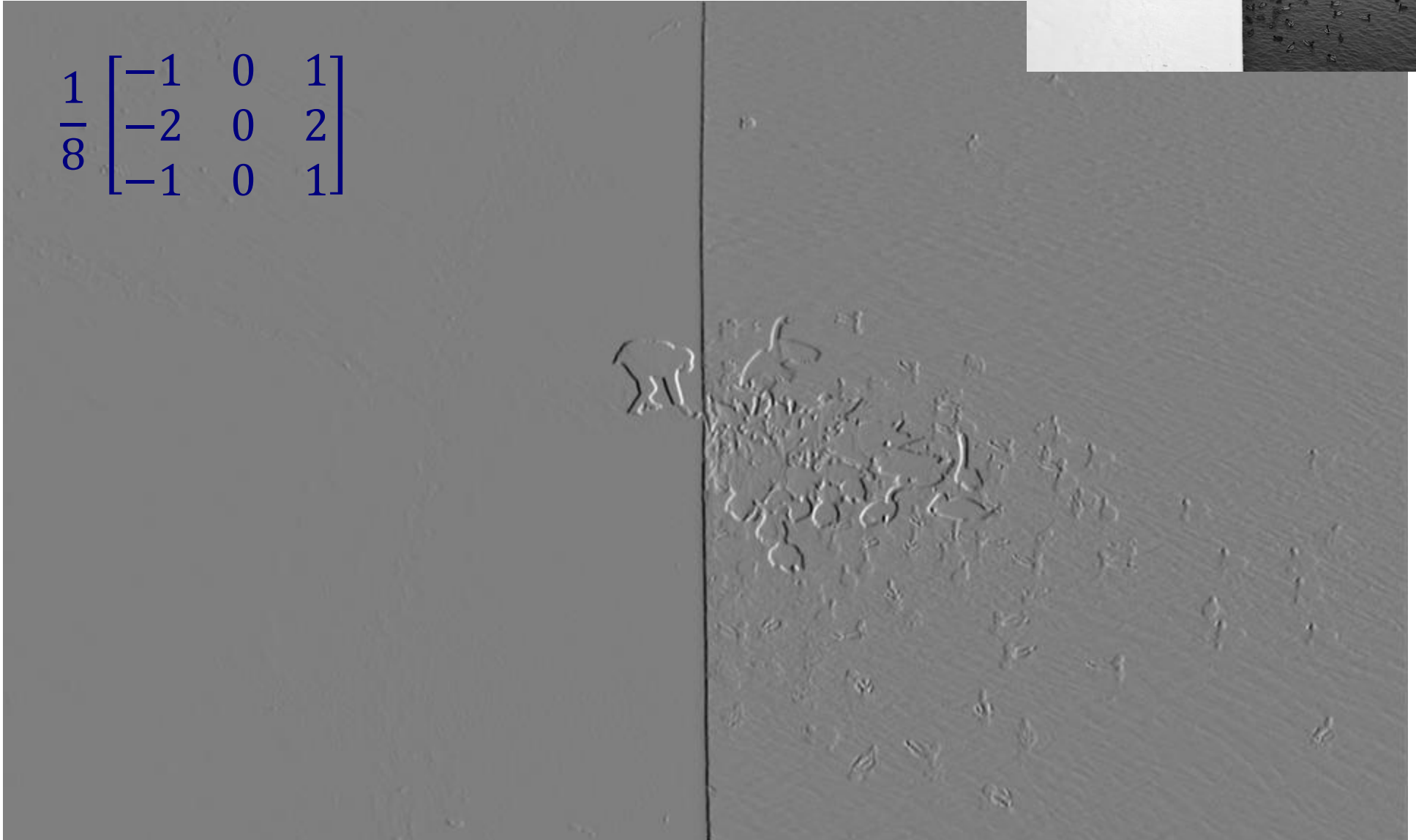
- Separable Sobel kernel by (outer) matrix product

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Gradient in x: vertical edges

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

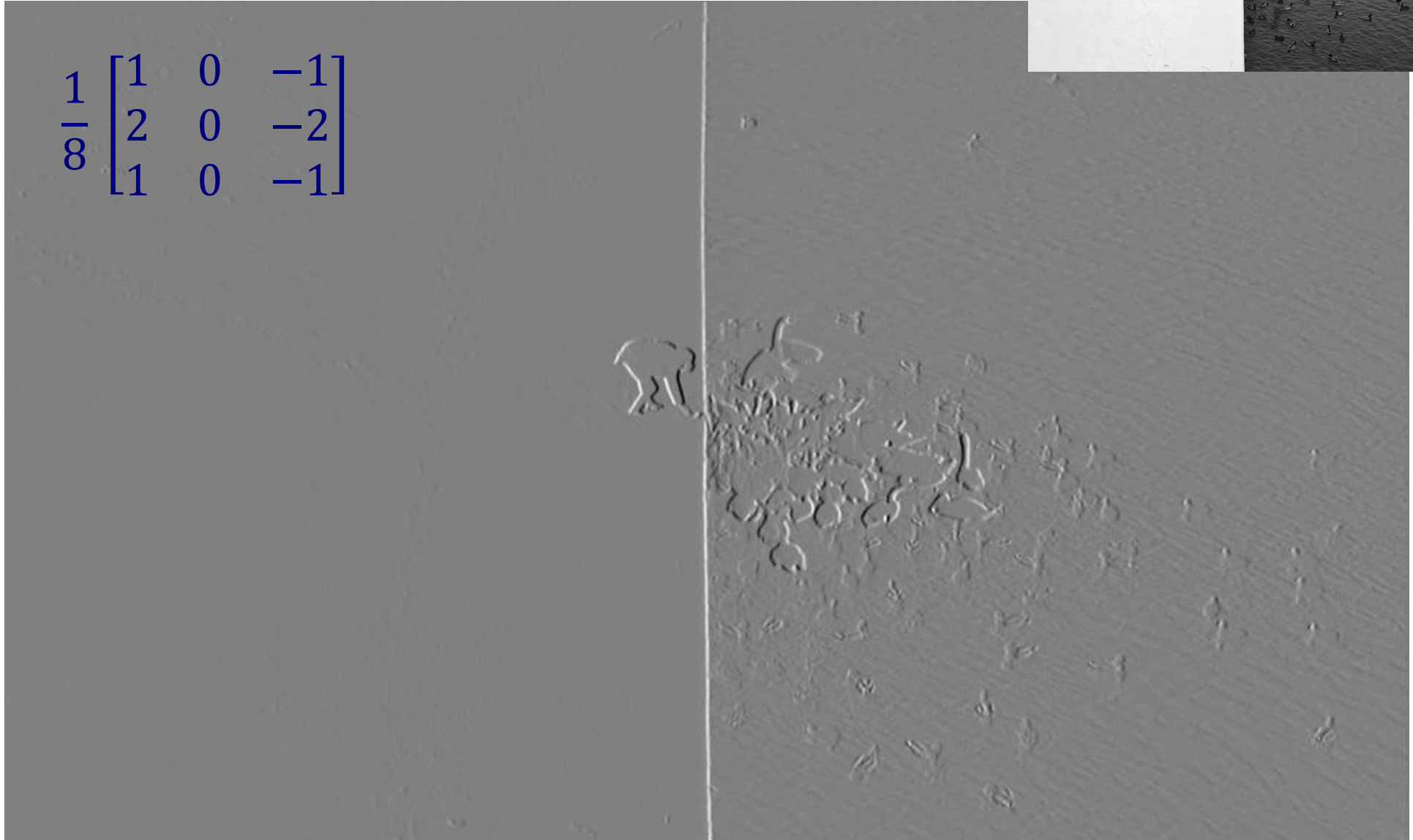


Edge detection using gradients

- Attention: Sign-alternating kernels!
 - Output images with positive and negative values
 - Display: by mapping zero gradient to mid-grey level
 - Positive gradient values appear brighter
 - Negative gradient values appear darker

Gradient in x: vertical edges

$$\frac{1}{8} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$



Gradient in y: horizontal edges



$$\frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

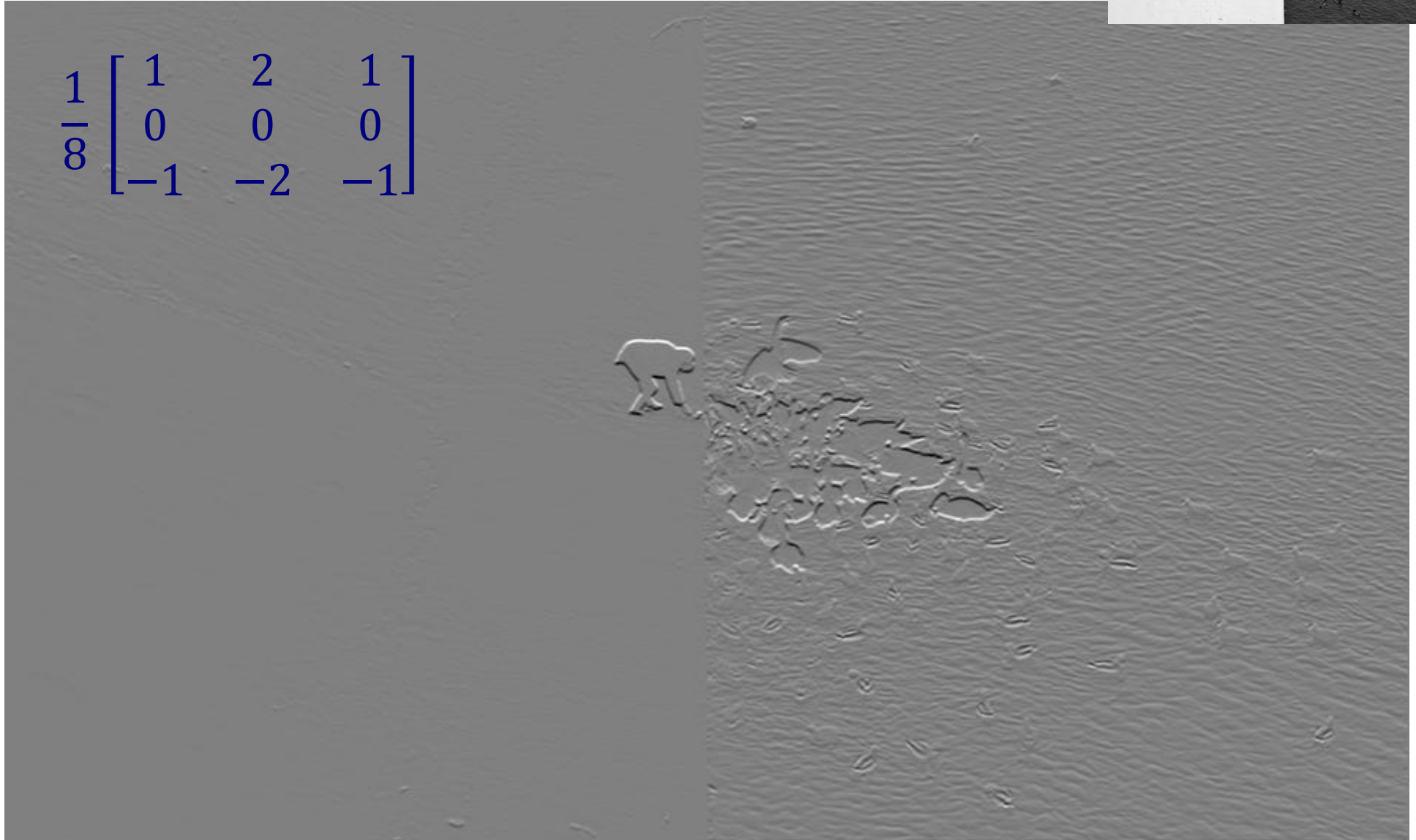
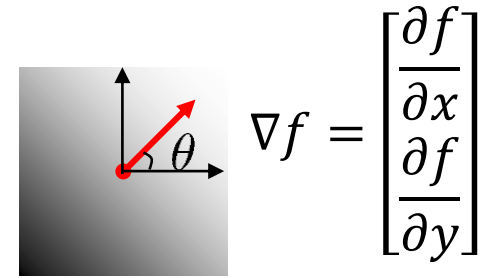
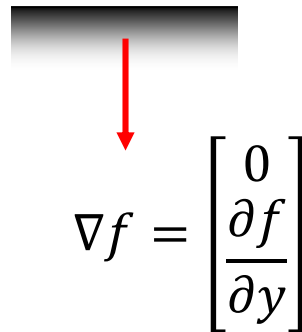
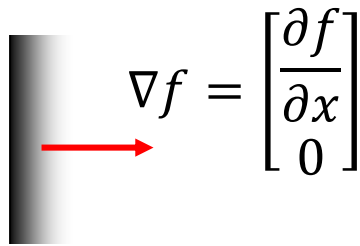


Image gradient vector

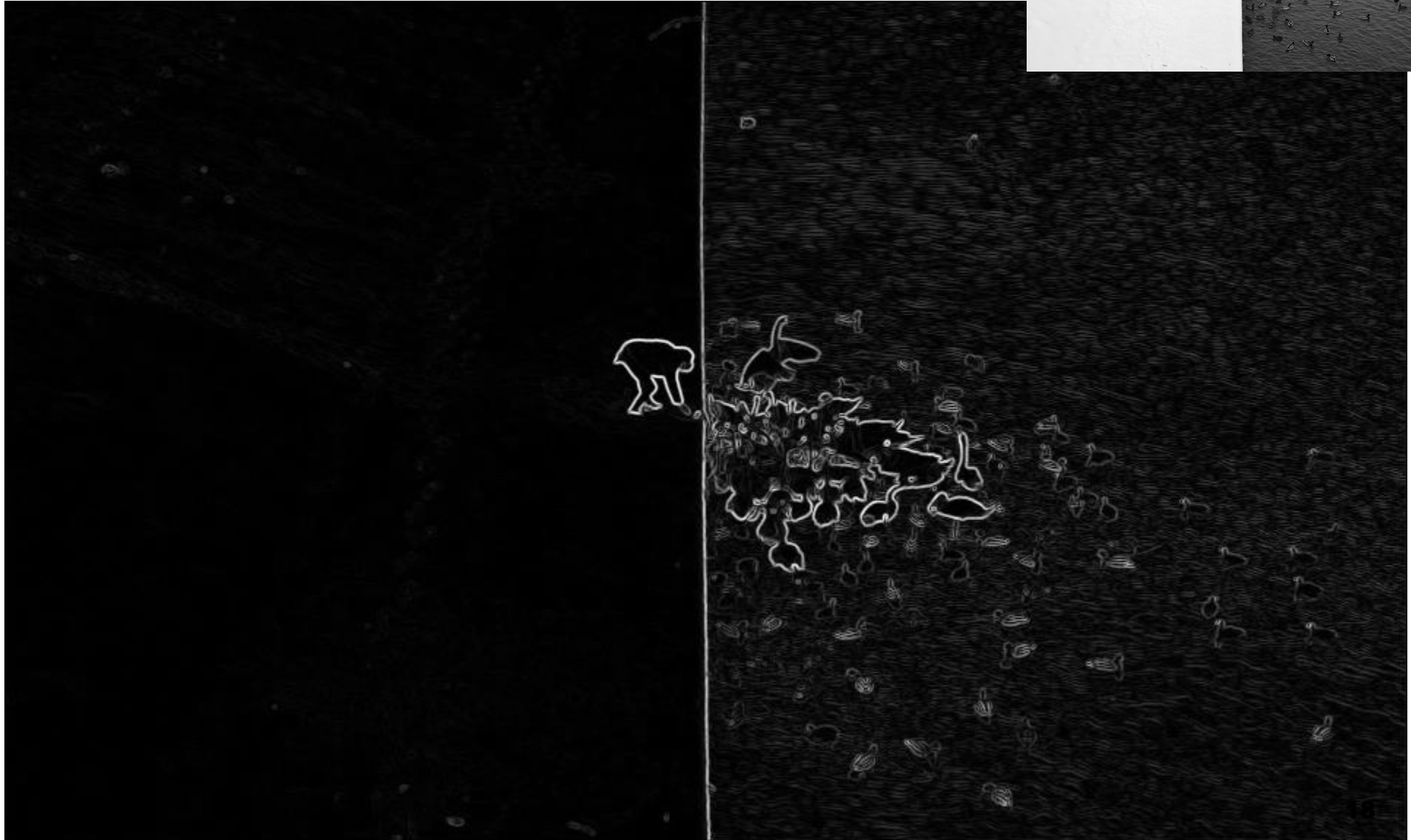
- The gradient vector of image pixels: $\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$



- The gradient vector points in the direction of most rapid increase in intensity (perpendicular to the edge)
- The gradient direction is given by $\theta(x, y) = \tan^{-1} \left(\frac{g_y(x, y)}{g_x(x, y)} \right)$
- The edge strength is given by the gradient magnitude

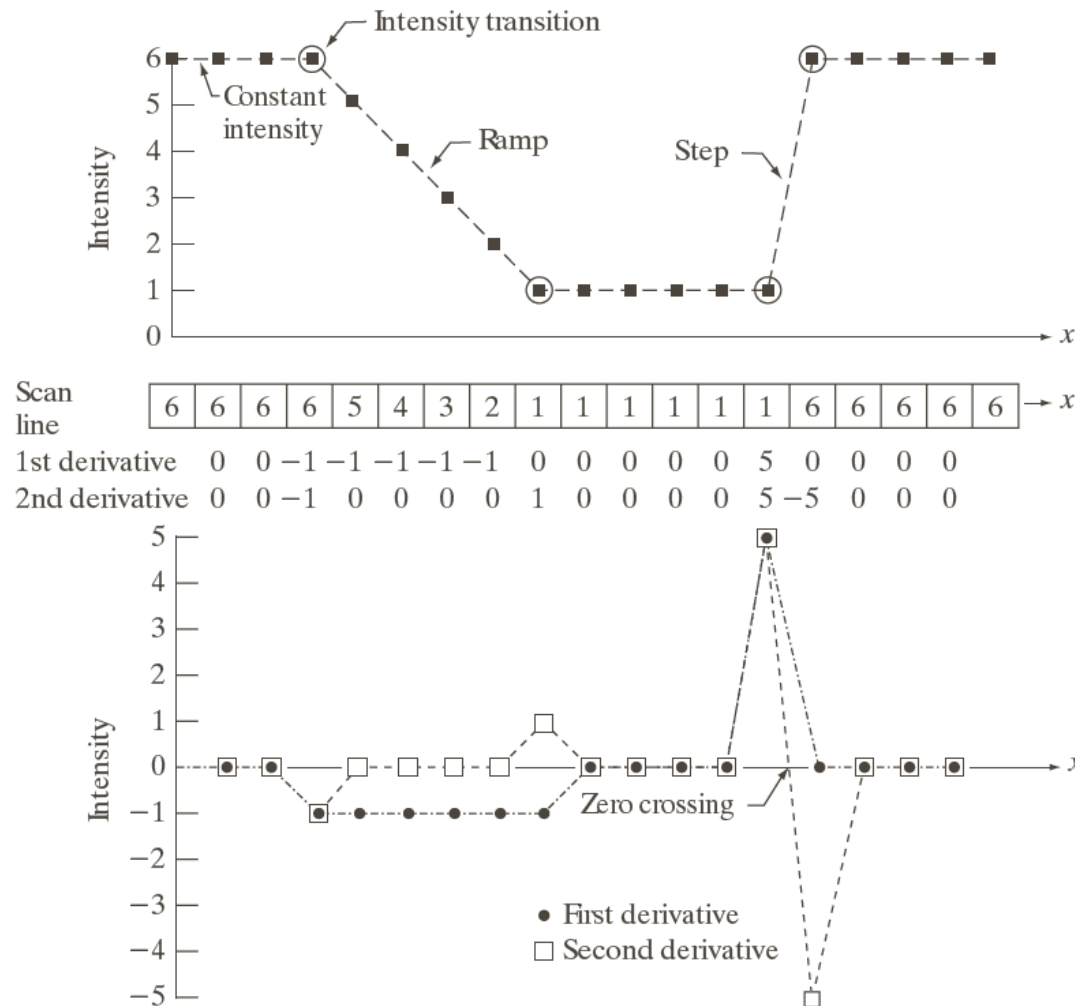
$$g_m(x, y) = \sqrt{g_x^2(x, y) + g_y^2(x, y)} \approx |g_x(x, y)| + |g_y(x, y)|$$

Gradient magnitude



More fun with derivatives...

- 2nd derivative is where the gradient changes



a
b
c

FIGURE 3.36
Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Laplacian filter

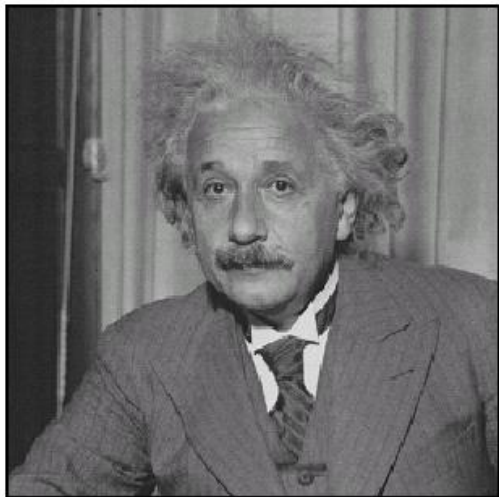
- We can approximate 2nd derivative using either of these Laplacian filter kernels
 - Correspond to sum of 2nd derivatives in x and y $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$



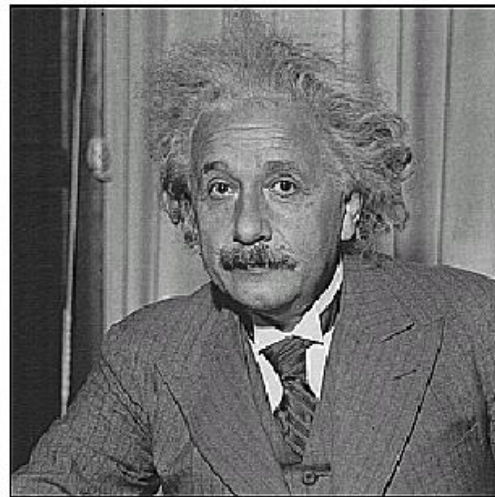
Enhances image details

Image sharpening

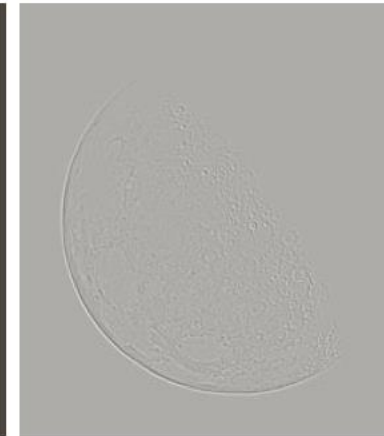
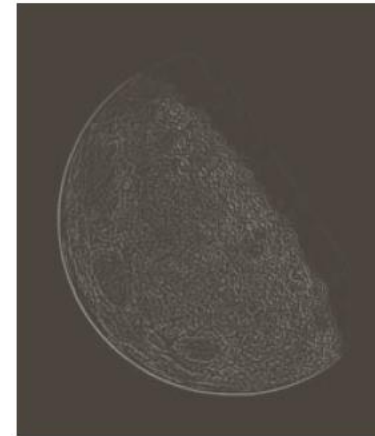
- Add the Laplacian filter result to the original image
- Note that Laplacian has positive and negative values (careful scaling!)
- Drawback: 2nd derivative also enhances noise!



before



after



a
b c
d e

FIGURE 3.38

(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Back to building an edge detector



original image



final output

Building an edge detector



gradient magnitude of the pre-smoothed image

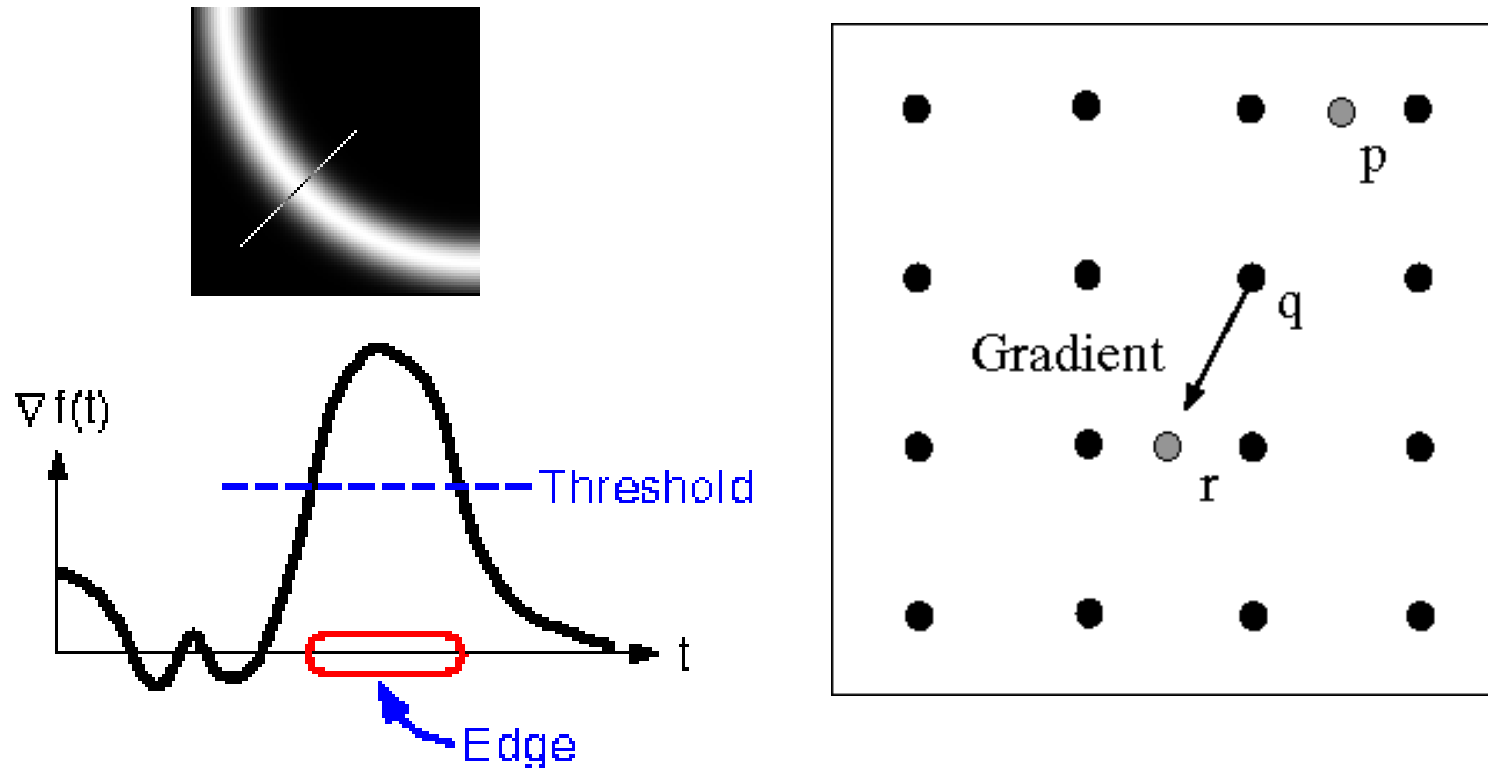
Building an edge detector



How to turn
these thick
regions of the
gradient into
curves?

Thresholded gradient magnitude

Non-maximum suppression



- For each location q above threshold, check that the gradient magnitude is **higher than at neighbors p and r along the direction of the gradient**
 - May need to interpolate to get the magnitudes at p and r

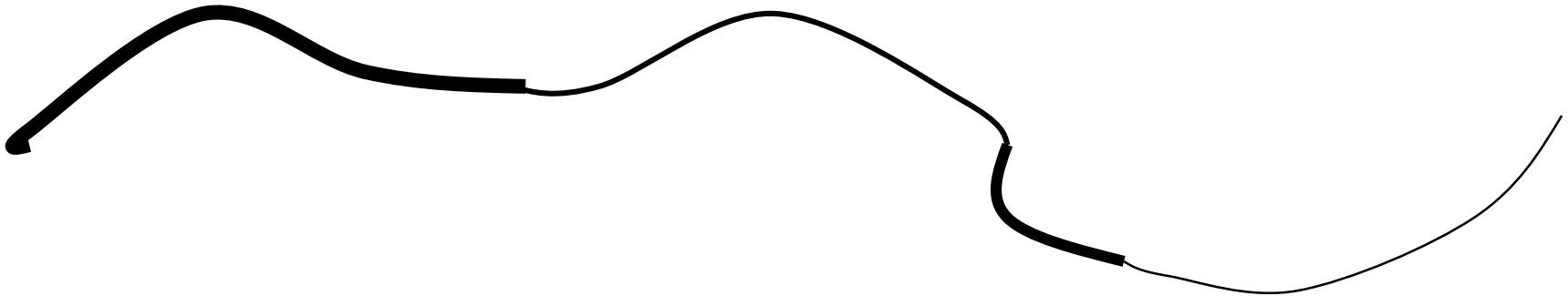
Non-maximum suppression



Another
problem:
pixels along
this edge
didn't
survive the
thresholding

Hysteresis thresholding

- Use a high threshold to start edge curves, and a low threshold to continue them



Hysteresis thresholding



original image



**high threshold
(strong edges)**



**low threshold
(weak edges)**



hysteresis threshold

Canny edge detector

- Compute x and y gradient images
- Find magnitude and orientation of gradient
- Non-maximum suppression:
 - Thin wide “ridges” down to single pixel width
- Linking and thresholding (hysteresis):
 - Define two thresholds on gradient magnitude: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

J. Canny, [A Computational Approach To Edge Detection](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Canny edge detector result



original image



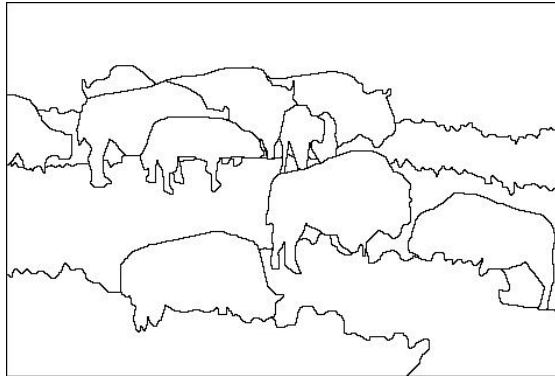
final output

What are meaningful contours?

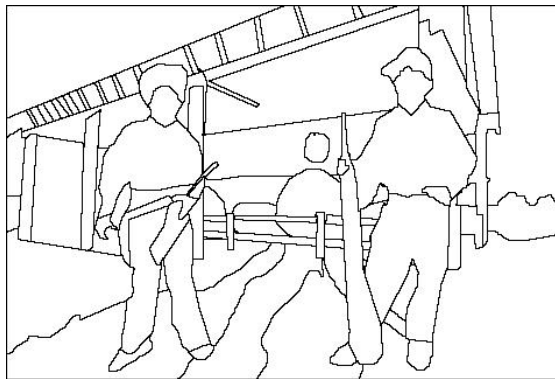
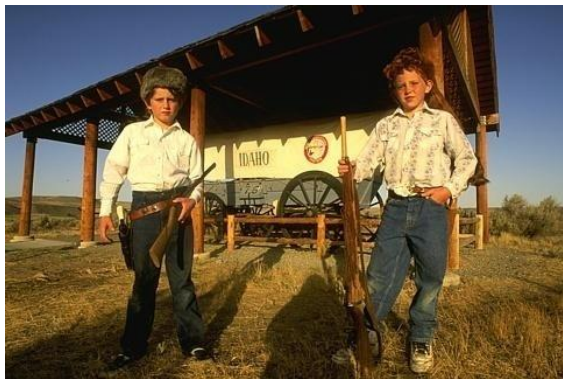
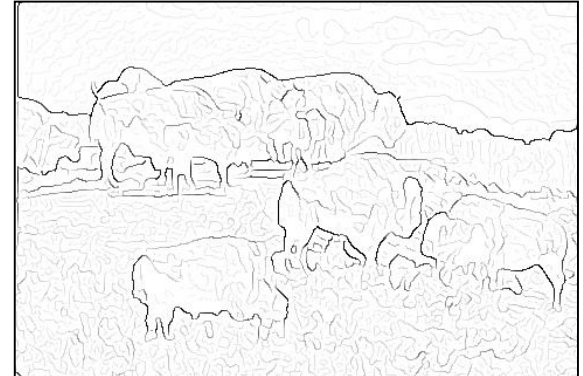
image



human segmentation



gradient magnitude



■ [Berkeley segmentation database](#)

Outlook Data-driven edge detection

