SoftEng306
Software Engineering Design 2

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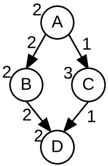
Department of Electrical, Computer, and Software Engineering
University of Auckland

Task scheduling with communication delays

Scheduling task graphs with communication delays on homogeneous processors



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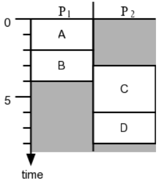

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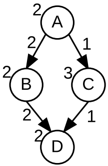

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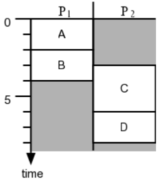

$P|prec, c_{ij}|C_{max}$

- Traditional and general problem
- Strong NP-hard

$\Rightarrow$ Heuristics, most popular is list scheduling

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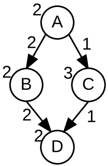

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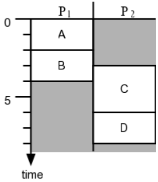

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- Traditional and general problem
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$\Rightarrow$ Heuristics, most popular is list scheduling

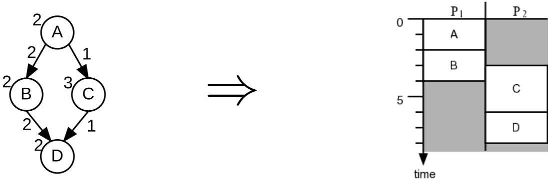
But here,

$\Rightarrow$ Optimal solver, based on state space search

Content

Scheduling problem

Finding start time and processor allocation for every task

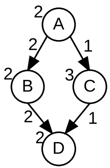


- $t_s(n)$: start time of task n
- $proc(n)$: processor of task n

Given by task graph $G = (V, E, w, c)$

- $w(n)$: execution time of task n
  - weight of node
- $c(e_{ij})$: remote communication cost between tasks n_i and n_j
  - weight of edge

Constraints



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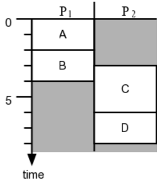

Processor constraint

$$proc(n_i) = proc(n_j) \Rightarrow \begin{cases} & t_s(n_i) + w(n_i) \leq t_s(n_j) \\ \text{or} & t_s(n_j) + w(n_j) \leq t_s(n_i) \end{cases}$$

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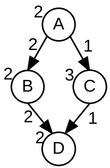

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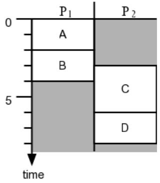

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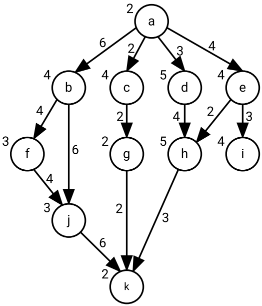
Precedence constraint

For each edge e_{ij} of E

$$t_s(n_j) \geq t_s(n_i) + w(n_i) + \begin{cases} 0 & \text{if } proc(n_i) = proc(n_j) \\ c(e_{ij}) & \text{otherwise} \end{cases}$$

Critical path and bottom level

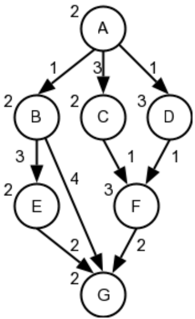
- Path length (here): sum of task weights on path
- Critical path: longest path through graph
  - Here: a, d, h, k and a, b, f, j, k, length 14
- Bottom level: longest path to exist task starting with node
  - E.g.: $bl_w(a) = 14$, $bl_w(b) = 12$, $bl_w(h) = 7$

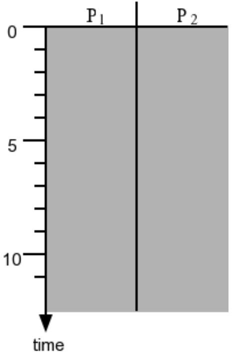


List-Scheduling

1. Order nodes of DAG according to a priority, while respecting their dependences
2. Iterate over node list from 1.) and schedule every node to the processor that allows its earliest start time.

Here: node order: A,C,D,F,B,E,G





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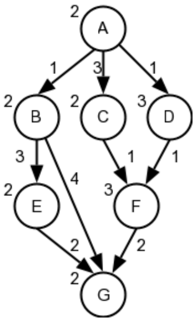

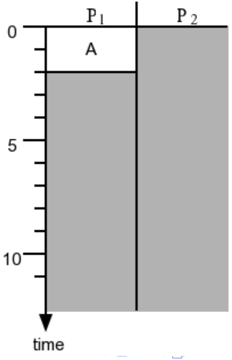

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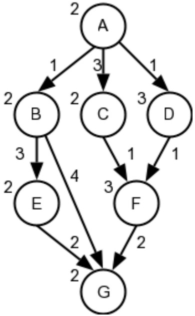

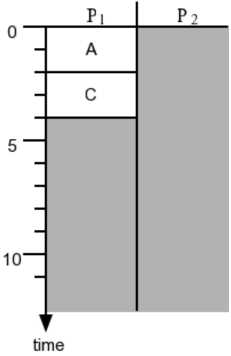

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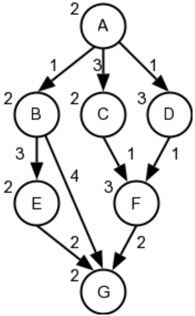

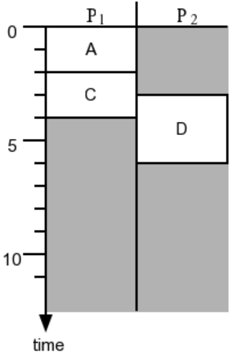

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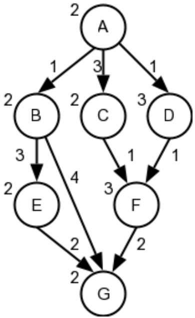

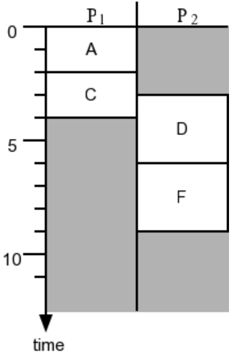

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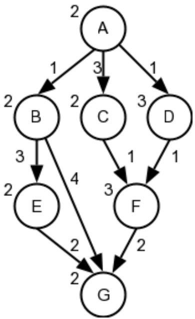

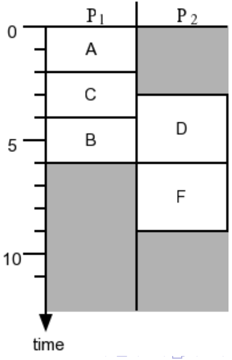

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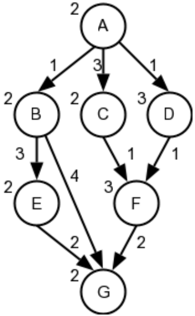

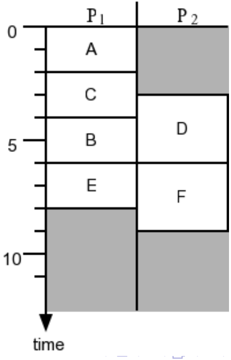

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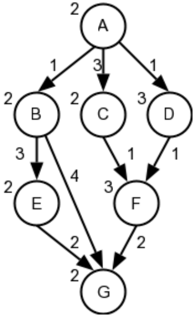

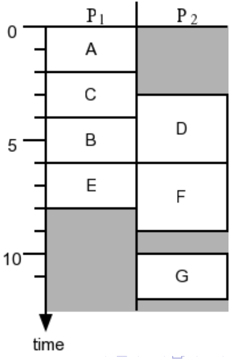

Exhaustive solution search

- State Space Search
  - Exhaustive search through all possible solutions
  - Every state (node) s represents partial solution
  - Combinatorial problems $\Rightarrow$ search tree
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  - Deeper nodes are more complete solutions
- Search techniques
  - Branch and Bound – easy, limited memory search techniques
  - A* – great performance, but memory problem !

Solution space for scheduling problem

One possibility: like list scheduling, trying out all task orders and all processor allocations

- State: partial schedule
- Initial state: empty schedule
- Cost function $f(s)$: underestimate of makespan for complete schedule based on s

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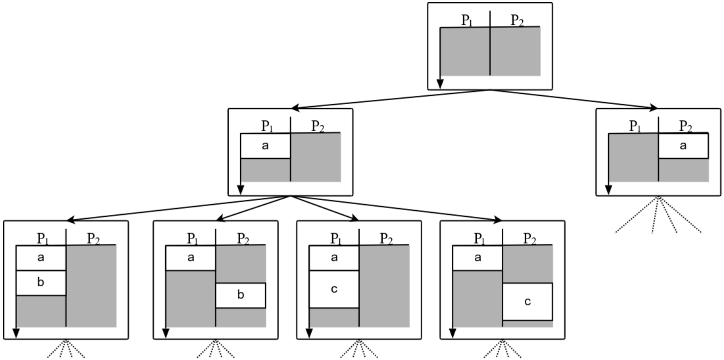
Expansion

- Given state s, let $\mathbf{free}(s)$ be free tasks

for all $i \in \mathbf{free}(s)$ **do**
    for all $P \in \mathbf{P}$ **do**
        Create new state: i scheduled on P as early as possible

Solution tree

- Task graph on two processors



Lower bounds on (partial) schedules

- Perfect load balance plus current idle time

$$\frac{\sum_{i \in \mathbf{V}} w(n_i) + idle(s)}{|\mathbf{P}|}$$

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- Max (start time of scheduled tasks plus their bottom level)

$$\max_{n_i \in s}\{t_s(n_i) + bl_w(n_i)\}$$

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DFS on state space (depth until $f(s) \geq B$):
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- Memory required is $O(|V|P)$
- Benefits from tight upper bounds for initial B

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  - Expand most promising state first (best $f(s)$) $\Rightarrow$ Head of *OPEN*
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while $OPEN \neq \emptyset$ **do**
  $s \leftarrow PopHead(OPEN)$
  if s is complete solution **then**
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  Expand state s into children and compute $f(s_{child})$ for each
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- Very, very memory hungry (Breadth First Search)
- With given $f(s)$ function, A* explores least number of states!