CSE 20 - Fall 2022 - Exam 2

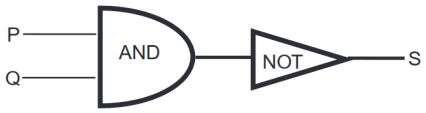
INSTRUCTIONS — READ THIS NOW

- Please limit your time working on the exam to **1 hour and 30 minutes** (include checking your work). If you cannot upload your paper to Gradescope before the time expires, then you will automatically receive a zero on the exam.
- Remember to write your name and PID at the top of **every page** of your submission.
- Write your answer for each problem on a separate page. Items that belong to the same exercise may be put together on one page. Make sure to tag your work correctly on Gradescope.
- **Always show all your work!** To receive full credit, your answers must be neatly written, logically organized, and justified. Any results outside our textbook and lecture notes must be cited appropriately.
- This exam has **six (6)** problems. The total point possible is **50**.

Note: For the first two problems, you may refer to Table 6 from Section 1.3 of Rosen's textbook (Page 27). This table was also provided on the annotated notes of Day05. Remember to show all steps of your work and indicate which rule you are using.

1. (6 points) Show that the logic circuit below implements the same operation as

$$(P \oplus Q) \vee (\neg P \wedge \neg Q).$$



2. (6 points) Use logical equivalence properties to prove the following statement. Do not use the truth table.

$$p \to (q \vee r) \equiv (p \wedge \neg q) \to r$$

3. (7 points) Translate the following statements into compound propositions and decide whether the system is consistent or not.

    1. If you pass CSE 20 then you will take CSE 21
    2. If you pass CSE 20 then you will take MATH 15B
    3. If you take CSE 21 then you will not take MATH 15B
    4. You pass CSE 20

4. (6 points) Provide two logical propositions (one in CNF and the other in DNF) that represent the output of the truth table below. Also explain how you obtain your answer.

p	q	r	output
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

(CNF: Conjunctive Normal Form; DNF: Disjunctive Normal Form)

5. (9 points) Express the **negations** of each of the following statements so that all negation symbols immediately precede the predicates.

   (a) $\forall x \exists y P(x, y) \vee \exists x \forall y \neg Q(x, y)$

   (b) $\exists x \forall y \left(\neg P(x, y) \rightarrow Q(x, y)\right)$

   (c) $\exists x \exists y \left(Q(x, y) \leftrightarrow Q(y, x)\right)$

6. (16 points) Let $\mathbb{R}^{\leq 0}$ be the set of all non-positive real numbers. On the domain $\mathbb{R}^{\leq 0} \times \mathbb{R}^{\leq 0}$, define the predicate $P(x, y)$ to be "$x * y = 7$." Decide whether the following quantified statements are **True or False** and **explain** your answer using an appropriate proof strategy.

   Note: $x * y$ (also known as $x \cdot y$) is the product of x and y

   (a) $\forall x \forall y \; P(x, y)$

   (b) $\exists x \exists y \; P(x, y)$

Domain: $\mathbb{R}^{\leq 0}$, the set of all non-positive real numbers. $P(x, y)$ means "$x * y = 7$."

(c) $\forall x \exists y \; P(x, y)$

(d) $\exists x \forall y \; P(x, y)$