Induction Exercise 5

Prove that
$$\sum_{i=1}^{n}(2i-1) = 1 + 3 + 5 + ... + (2n-1) = n^2.$$

Proof. If $n = 1$, then
$$\sum_{i=1}^{1}(2i-1) = 1 = 1^2.$$
Thus the statement is true for $n = 1$.

Now assuming that
$$\sum_{i=1}^{k}(2i-1) = k^2$$
we find that
$$\begin{aligned}\sum_{i=1}^{k+1}(2i-1) &= \left(\sum_{i=1}^{k}(2i-1)\right) + (2(k+1)-1)\\ &= k^2 + 2(k+1) - 1\\ &= k^2 + 2k + 1\\ &= (k+1)(k+1)\\ &= (k+1)^2.\end{aligned}$$

Hence by the priniciple of mathematical induction we have established that the statement is true for all $n \in \mathbb{N}$. □