Induction Exercise 4

Prove that

$$\sum_{i=0}^{n} i(i+1) = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Proof. If $n = 1$, then

$$\sum_{i=0}^{1} i(i+1) = 2 = \frac{1(2)(3)}{3}.$$

Thus the statement is true for $n = 1$.

Now assuming that

$$\sum_{i=0}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$$

we find that

$$\begin{aligned}
\sum_{i=0}^{k+1} i(i+1) &= \left(\sum_{i=0}^{k} i(i+1)\right) + (k+1)((k+1)+1) \\
&= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) \\
&= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3} \\
&= \frac{(k+1)(k+2)(k+3)}{3} \\
&= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}.
\end{aligned}$$

Hence by the priniciple of mathematical induction we have established that the statement is true for all $n \in \mathbb{N}$. $\square$