Contents

5 Sobolev Spaces

5.1

Suppose $k \in \{0, 1, \dots\}$, $0 < \gamma < 1$. Prove $C^{k,\gamma}(\overline{U})$ is a Banach space.

Proof. In progress... $\square$

5.3

Denote by U the open square $\{x \in \mathbb{R}^2 \,|\, |x_1| < 1, |x_2| < 1\}$. Define

$$u(x) = \begin{cases} 1 - x_1, & \text{if } x_1 > 0, |x_2| < x_1 \\ 1 + x_1, & \text{if } x_1 < 0, |x_2| < -x_1 \\ 1 - x_2, & \text{if } x_2 > 0, |x_1| < x_2 \\ 1 + x_2, & \text{if } x_2 < 0, |x_1| < -x_2. \end{cases}$$

For which $1 \le p \le \infty$ does u belong to $W^{1,p}(U)$.

Proof. In progress... $\square$

5.5

Let U, V be open sets, with $V \subset\subset U$. Show there exists a smooth function ζ such that $\zeta \equiv 1$ on V, $\zeta = 0$ near ∂U. (Hint: Take $V \subset\subset W \subset\subset U$ and mollify χ_W.)

Proof. In progress... $\square$

5.6

Assume U is bounded and $U \subset\subset \bigcup_{i=1}^N V_i$. Show there exist C^∞ functions ζ_i $(i = 1, \dots, N)$ such that

$$\begin{cases} 0 \le \zeta_i \le 0, \ \text{supp}\ \zeta_i \subset V_i \, (i = 1, \dots, N) \\ \sum_{i=1}^N \zeta_i \text{ on } U. \end{cases}$$

Proof. In progress... $\square$