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## 1.1 $L^p$ Spaces

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# 2 Hölder Spaces

## 2.1 Hölder Continuous Functions

## 2.2 Hölder Spaces are Banach Spaces

# 3 Sobolev Spaces

# 4 Notation

(i) A multiindex is a vector $\alpha = (\alpha_1, \dots, \alpha_n)$ where each component $\alpha_i \in \mathbb{N}_0$. A multiindex has an order defined by

$$|\alpha| = \alpha_1 + \cdots + \alpha_n.$$

(ii) Using our definition of a multiindex and letting $u(x)$ be some function, we define

$$D^\alpha u(x) = \frac{\partial^{|\alpha|} u(x)}{\partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}} = \partial_{x_1}^{\alpha_1} \dots \partial_{x_n}^{\alpha_n} u(x).$$

(iii) Let $U, V \subset \mathbb{R}^n$. Then define

$$V \subset\subset U$$

to be when $V \subset \overline{V} \subset U$ and $\overline{V}$ is compact. In plain english this means $V$ is *compactly conatined* in $U$.

(iv) Let $f$ and $g$ be functions. Then define $*$ to be the Convolution operator where

$$(f * g)(x) = \int_{-\infty}^{\infty} f(\tau) g(x - \tau) d\tau = \int_{-\infty}^{\infty} f(x - \tau) g(\tau) d\tau$$

is the Convolution of the functions $f$ and $g$ which results in a third function that expresses how one of the functions modifies the other. Note that I am assuming $f$ and $g$ are both supported on an infinite interval, which may not always be the case.