

1. Title - 10 Words Max:

- A Study of Adaptive Finite Element Methods, and their Applications.
- A Study of Function Spaces, and Adaptive Finite Element Methods.

2. Overview/Abstract - 300 Words Max:

By definition, Partial Differential Equations (PDE) are equations involving more than one independent variable, so that the derivatives occurring in the equation are partial derivatives, where a partial derivative is essentially the rate of change of some variable while holding the several other relevant variables constant. PDEs find their merit in that many physical, biological, and engineering related problems can be represented by means of PDEs. However, these equations often do not have exact solutions, so it becomes important to have Numerical Methods to approximate these solutions. Adaptive Finite Element Methods (AFEM) are fundamental Numerical Methods used to approximate PDEs, where we implement these methods by means of computers, specifically, by means of algorithms that use simple arithmetic operations that result in approximate solutions in a numerical form. With these definitions in mind, the goals of this project are to first conduct a study into Function Spaces such as Hilbert and Sobolev Spaces, which are relevant to the analysis of PDEs and the construction of their solutions. Then, we will progress into the study of Numerical Methods induced by weak formulations of PDEs, namely, Finite Element Methods. And finally, we will study state of the art AFEMs by researching and applying relevant Numerical Methods from a survey in this topic.

3. Originality, Creativity, and Significance - 300 Words Max:

The originality of this project will come from our research into these state of the art AFEMs, and how we can apply these methods to model different problems, primarily, in the form of computer programs. The creativity of this project will come from how we decide to implement these methods, as there are many different ways to implement the same program or algorithm, which directly represents the creativity and personality of the programmer. Also, creativity will come from how we decide to optimize and refine these programs or algorithms, as a programmer has to be very clever in how they design their programs, and also in how they continue to improve them in the aim to make them more efficient, stable, and well-conditioned. Finally, the significance of this project will come from the study and analysis of PDEs, and the theory behind these equations. As stated before, many physical, biological, and engineering related problems can be represented mathematically by means of PDEs. Specifically, we find applications in many important fields of study such as Schrödinger's equation in Physics, Reaction-Diffusion equations in Biology and Chemistry, the Black-Scholes equation in Economics, etc... It is easy to see that PDEs are used in basically all scientific areas, but come with the caveat that many do not have exact solutions, which is where Numerical Methods, such as AFEMs, gain their significance.

4. Potential Benefit - 250 Words Max:

A potential benefit to me, regarding the summer project, would be my continued study in Mathematical Analysis, PDEs, and Numerical Methods, as these are fields that I am very interested and having the opportunity to continue my study in these fields would be a great privilege. Also, as a Computer Scientist, another potential benefit to me would be learning how to model real world problems involving PDEs, that would in turn expose me to more mathematically oriented programming languages that I would not learn in my Computer Science degree otherwise. A final potential benefit to me would be the valuable research experience that I would gain during a summer research project involving both Mathematical and Computer Science related topics. I believe this project would help prepare me for the prospect of continuing my study in graduate school, be it in either Computer Science or Mathematics.

5. Relevant Experience - 250 Words Max:

In terms of summer research I am relatively inexperienced as I have never had the opportunity to be part of a summer research project. However, mathematically, I have studied and taken courses in Ordinary Differential Equations, Real Analysis, Undergraduate Calculus 1 to 3, Linear Algebra 1 and 2, and by the end of this semester I will have completed courses in PDEs and Numerical Methods. I believe this experience would constitute a satisfactory mathematical background, and would allow me to contribute to a summer research project mainly tackling PDEs and Numerical Methods. In terms of the programming and algorithm design aspects, I also believe I would have a satisfactory background as I am finishing the fourth year of my Computer Science degree, and in this time I have been exposed to many different programming languages and algorithm design techniques that would aid me in implementing these Numerical Methods.