哈工大 2008 年 秋 季 学期

# 形 式 语 言　试题 A

| 学号 | |
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| 姓名 | |

| 题号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 总分 |
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注意行为规范

遵守考场纪律

1. [15 points] True or false(√ for true and × for false)

(1) If $L$ is a regular language and $M$ is a context free language, then $L\cap M$ is a context free language.

(2) The language $\{ a^i b^j a^i b^j \mid i,j \geq 0 \}$ is context free.

(3) The union of a context free language and a regular language must be context free.

(4) A PDA with two stacks can recognize any recursively enumerable language.

(5) Every regular language without ε has a context-free grammar in the Chomsky normal form.

2. [10 points] Let $L=\{ 0^n 1^n \mid n \geq 0 \}$

(1) Is $L$ a regular language?　Justify your answer.

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(2) Is $L$ a context free language? Justify your answer.

3. Consider the Turing machine M given by

$$M = (\{q_0, q_1, q_2\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined in it, as described below.

$$\delta(q_0,0) = (q_0, B, \rightarrow) \qquad \delta(q_0,1) = (q_1, B, \rightarrow)$$

$$\delta(q_1,1) = (q_1, B, \rightarrow) \qquad \delta(q_1, B) = (q_2, B, \rightarrow)$$

(1)[4 points] Specify execution trace of $M$ on the two input strings 011 and 11, i,e., provide the complete sequence of ID's for both inputs.

(2)[3 points] Provide a regular expression for the language of this Turing machine.

(3)[3 points] Suppose we added the following transition to the above machine.

$$\delta(q_1, 0) = (q_0, B, \rightarrow)$$

Provide a regular expression for the language of the resulting Turing machine.

4.(1)[5 points] Suppose $L$ is a regular language. Then there exist a constant $n$(which depends on $L$) such that for every string $w$ in $L$ such that $|w| \geq n$, we can break $w$ into three strings, $w=xyz$, such that (1) $y \neq \varepsilon$, (2) $|xy| \leq n$, (3) for all $k \geq 0$, $xy^k z \in L$.

This is called pumping lemma for regular language. Show why it is true.

(2)[7 points ] Consider the following operation on languages:

$$\text{INIT}(L)=\{\ x \mid \text{for some } y, \text{ the string } xy \text{ is in } L\ \}$$

Prove that context free languages are closed under the INIT operation.

5. [10 points] Let $M$ be a DFA such that $M=(\ Q, \Sigma, \delta, q_0, F\ )$ with $F=\{\ q_f\ \}$ and

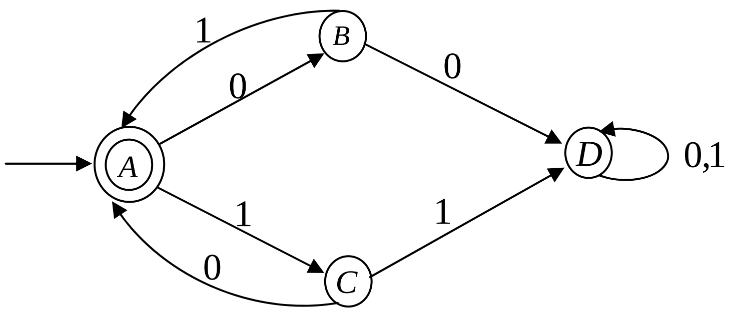
$$\forall a \in \Sigma, \delta(q_f, a) = \delta(q_0, a)$$

(1) Show that for all non-empty strings $w \in \Sigma^*$, it must be the case that $\hat{\delta}(q_f, w) = \hat{\delta}(q_0, w)$.

(2) Let $w \in L(M)$ be any string in the language of $M$. Prove by induction that for all $k \geq 0$, the string $w^k \in L(M)$.

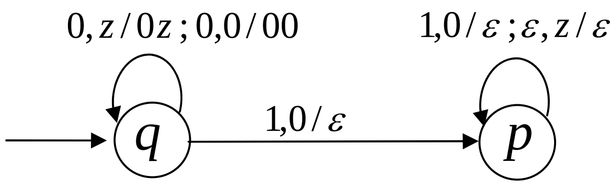
6. [10 points] Define a simple DFA(henceforth, called SFA) as a DFA in which the start state, once left, cannot be re-entered. In other words, in an SFA there is no transition going into the initial state.

(1) The following diagram presents a DFA for the language of the regular expression $(10+01)^*$. Give an SFA for the same language.



(2) Suppose you are given a DFA $M$=( $Q$, $\Sigma$, $\delta$, $q_0$, $F$ ). Describe how you will construct an SFA $M'$ such that $L(M') = L(M)$. (While you do not have to give a formal proof that your construction is correct, you must give a couple of lines of informal justification.)

7. A PDA is as following.




(1) [7 points] Convert this PDA into CFG(taking [qxp] as variable)

(2) [3 points] Simplify the CFG you have just got in part (1).

8. [10 points] Provide grammars for following languages.

(1) $L$={ $w$ | $w \in \{0,1\}^*$ and not contain 010 as substring }

(2) $L$={ $w$ | $w \in \{0,1\}^*$ and contains twice as many 0's as 1's }

9. [15 points]

(1) Design a PDA to recognize the following language.

$L=\{ w \mid w\in\{0,1\}^* \text{ and } n_1(w) \le n_0(w) \le 2n_1(w) \}$

(2) Design a DPDA to recognize the following language.

$L=\{ 0^n1^m \mid n \ge m \}$

(3) Give a Turing machine to compute the function $f(n,m)=n-m$.