

# Morning



# *Formal Language And Automata*

- Languages
- Automata
- Computation



# Language

What is a language ?

This is a sentence.

This is also a sentence.

...

So we have

{ sentence 1, sentence 2, sentence 3, .... }

the set of sentences  $\Leftrightarrow$  Language

# Sentence and Alphabet

- sentence/string = sequence of symbols chosen  
from the alphabet  $\Sigma$

example : Mouse love rice.

- alphabet = set of symbols

example : ASCII, 中文国标,  $\Sigma = \{0,1\}$

symbols  $\Rightarrow$  sentences  $\Rightarrow$  language

# Rules/Grammar

- rules = by which sentence is generated

example : rules for English

<sentence> → <noun-phrase><predicate>

<noun-phrase> → <article><noun>

<predicate> → <verb>

<article> → a|an|the

<noun> → wolf|sheep

<verb> → love|eat

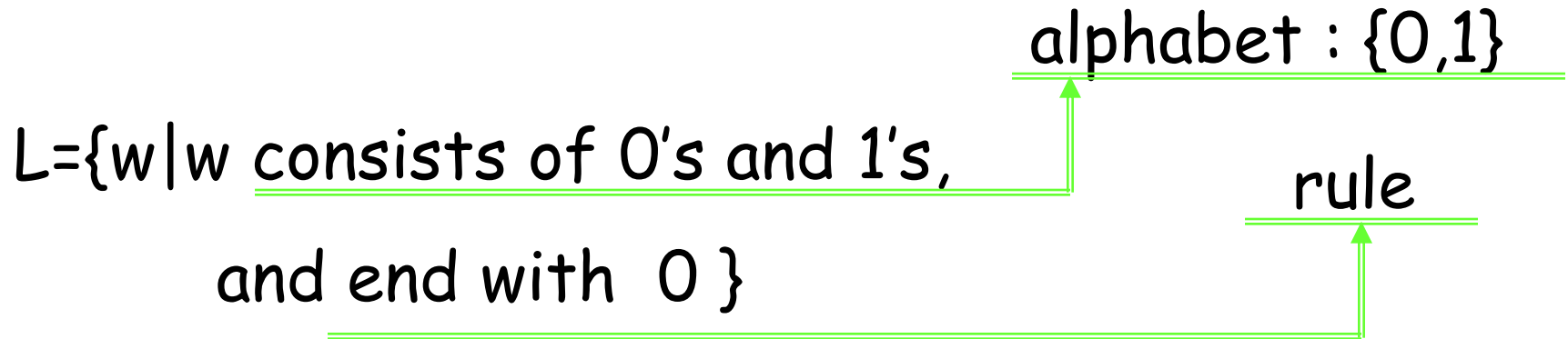
# Examples of language

## example 1.1

$L = \{w \mid \underline{w \text{ consists of 0's and 1's,}}$   
 $\quad \underline{\text{and end with 0}}\}$

alphabet : {0,1}

rule



$L = \{ 0, 00, 10, 000, 010, 100, 110, 0000, \dots \}$

$11111100 \in L, 1 \notin L, 0001 \notin L, 20 \notin L$

# Examples of language

example 1.2



rule



rule

$$L = \{ 0^n 1^n \mid n \geq 0 \}$$

alphabet = {0,1}

$$L = \{ \varepsilon, 01, 0011, 000111, 00001111, 0000011111, \dots \}$$

## example 1.3 The empty language

$$L = \{ \} = \phi$$

$L$  : contains no string



## example 1.4

$L = \{ w \mid w \text{ is a sentence in English} \}$

Bolt won two gold medals in Daegu. ✓

The sheep eats grass. ✓

The grass eats sheep. ?

♣ Formal language focus on form, not meaning

# String operations

$$w = a_1 a_2 \dots a_n$$

abba

$$v = b_1 b_2 \dots b_n$$

bbbbaaa

## Concatenation

$$wv = a_1 a_2 \dots a_n b_1 b_2 \dots b_n$$

abbabbbbaaa

## Reverse

$$w^R = a_n a_{n-1} \dots a_1$$

## Another operation

$$w^n = \underbrace{w \ w \ \dots \ w}_n$$

➤  $w = abb \Rightarrow w^2 = abbabb$ ,  $w^3 = abbabbabb$

➤ definition :  $w^0 = \varepsilon$

# Operations on languages

## ➤ The usual set operations

$$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$$

$$L_1 \cap L_2 = \{ w \mid w \in L_1 \text{ and } w \in L_2 \}$$

$$L_1 - L_2 = \{ w \mid w \in L_1 \text{ and } w \notin L_2 \}$$

## ➤ Reverse

$$L^R = \{ w^R \mid w \in L \}$$

**Example**  $L = \{ a^n b^n \mid n \geq 1 \} \Rightarrow L^R = \{ b^n a^n \mid n \geq 1 \}$

# Operations on languages

## ➤ Concatenation

$$L_1 L_2 = \{ wv \mid w \in L_1 \text{ and } v \in L_2 \}$$

Example  $L = \{ a^n b^n \mid n \geq 1 \}$ ,  $K = \{ b^n a^n \mid n \geq 1 \}$

$$LK = \{ a^n b^n b^n a^n \mid n \geq 1 \} \quad \times$$

$$LK = \{ a^n b^n b^m a^m \mid n \geq 1, m \geq 1 \}$$

$$L^2 = ?$$

## example 1.5

$$L = \{ w \in \{0, 1\}^* \mid \text{all 0's precede all 1's in } w \}$$

$$L = \{ \varepsilon, 0, 1, 00, 01, 11, 000, 001, 011, 111, \dots \}$$

## example 1.6

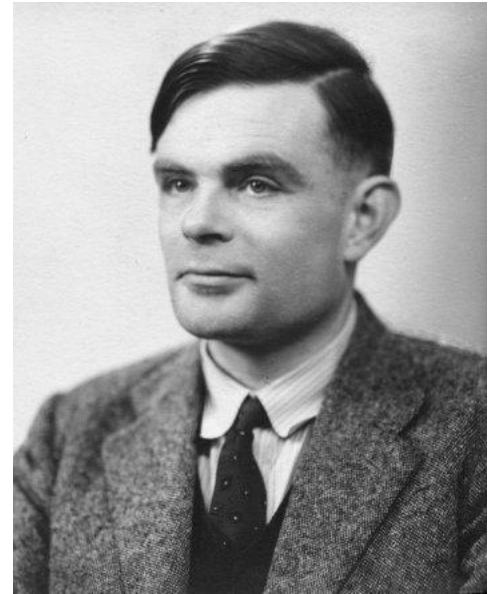
$$\begin{aligned} L_1 &= \{w \in \{0,1\}^* \mid \text{no prefix of } w \text{ contains } 1\} \\ &= \{\varepsilon, 0, 00, 000, 0000, 00000, 000000, \dots\} \end{aligned}$$

$$\begin{aligned} L_2 &= \{w \in \{0,1\}^* \mid \text{no prefix of } w \text{ starts with } 1\} \\ &= \{\varepsilon, 0, 00, 01, 000, 001, 010, 011, 0000, 0001, \dots\} \\ &= \{w \in \{0,1\}^* \mid \text{the first character of } w \text{ is } 0\} \cup \{\varepsilon\} \end{aligned}$$

$$\begin{aligned} L_3 &= \{w \in \{0,1\}^* \mid \text{every prefix of } w \text{ starts with } 1\} \\ &= \phi \end{aligned}$$

# Automata

- ◆ Alan Marthison Turing
  - On Computable Numbers  
With an Application to  
the Entscheidungs Problem
  - Turing Machine





# Automata

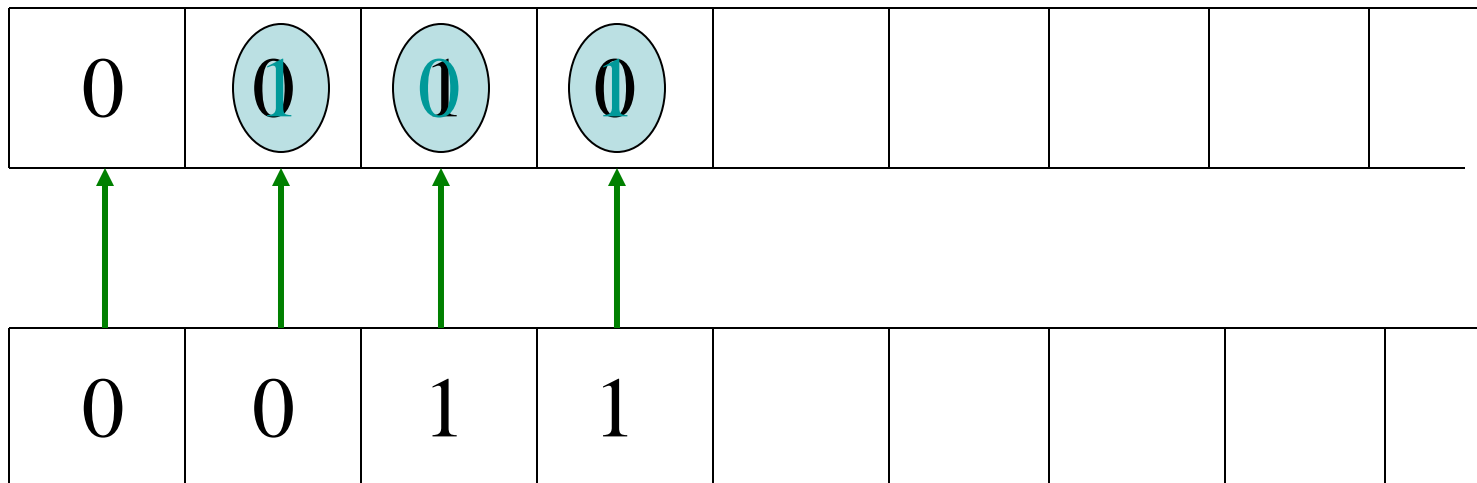
- ◆ Finite Automata
  - Deterministic Finite Automata
  - Non-deterministic Finite Automata
- ◆ Push Down Automata
- ◆ Turing Machine



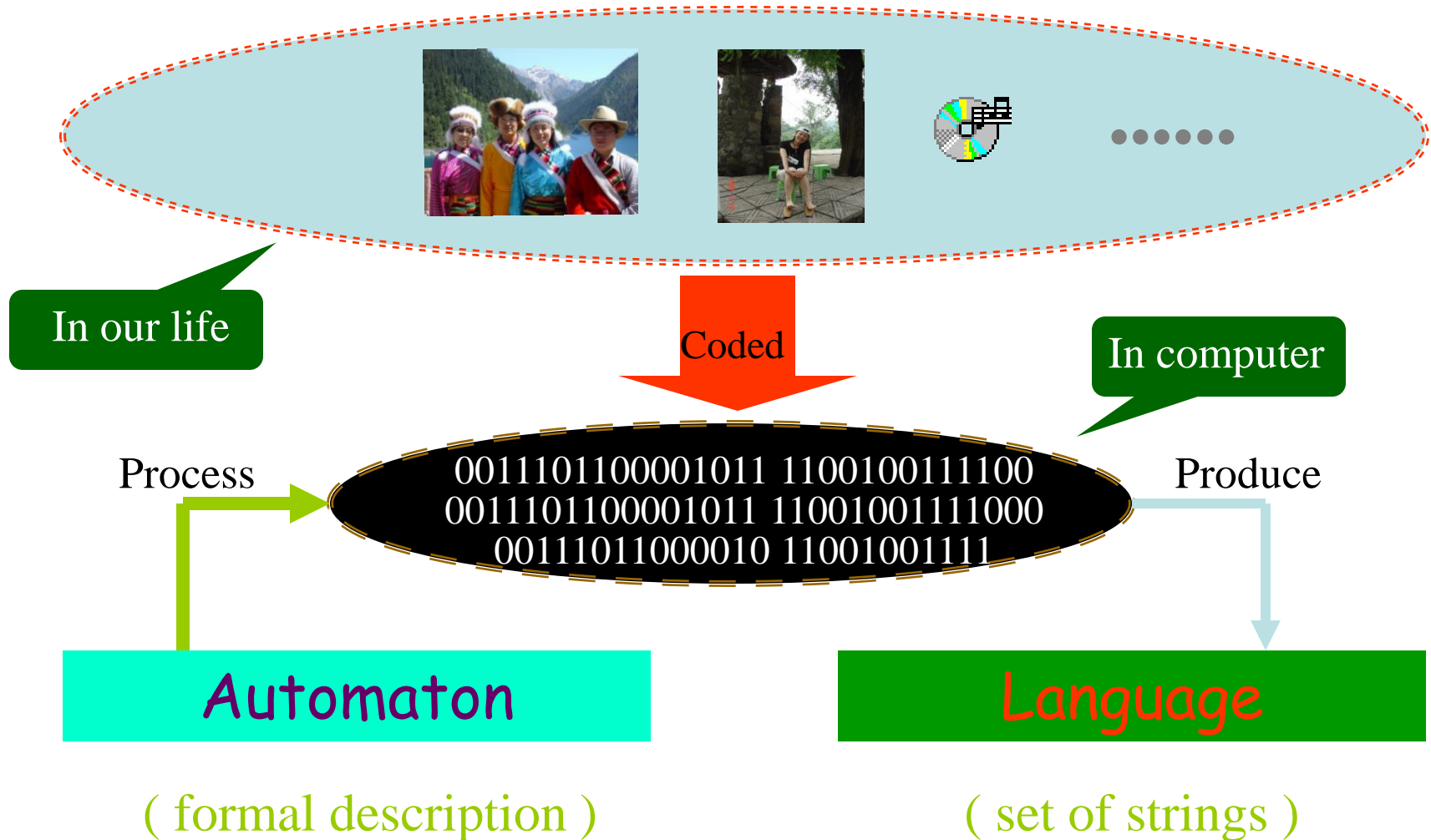
# Computation

$$\begin{array}{r} 2 \\ + 3 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 0010 \\ + 0011 \\ \hline 0101 \end{array}$$



# Computation for computer



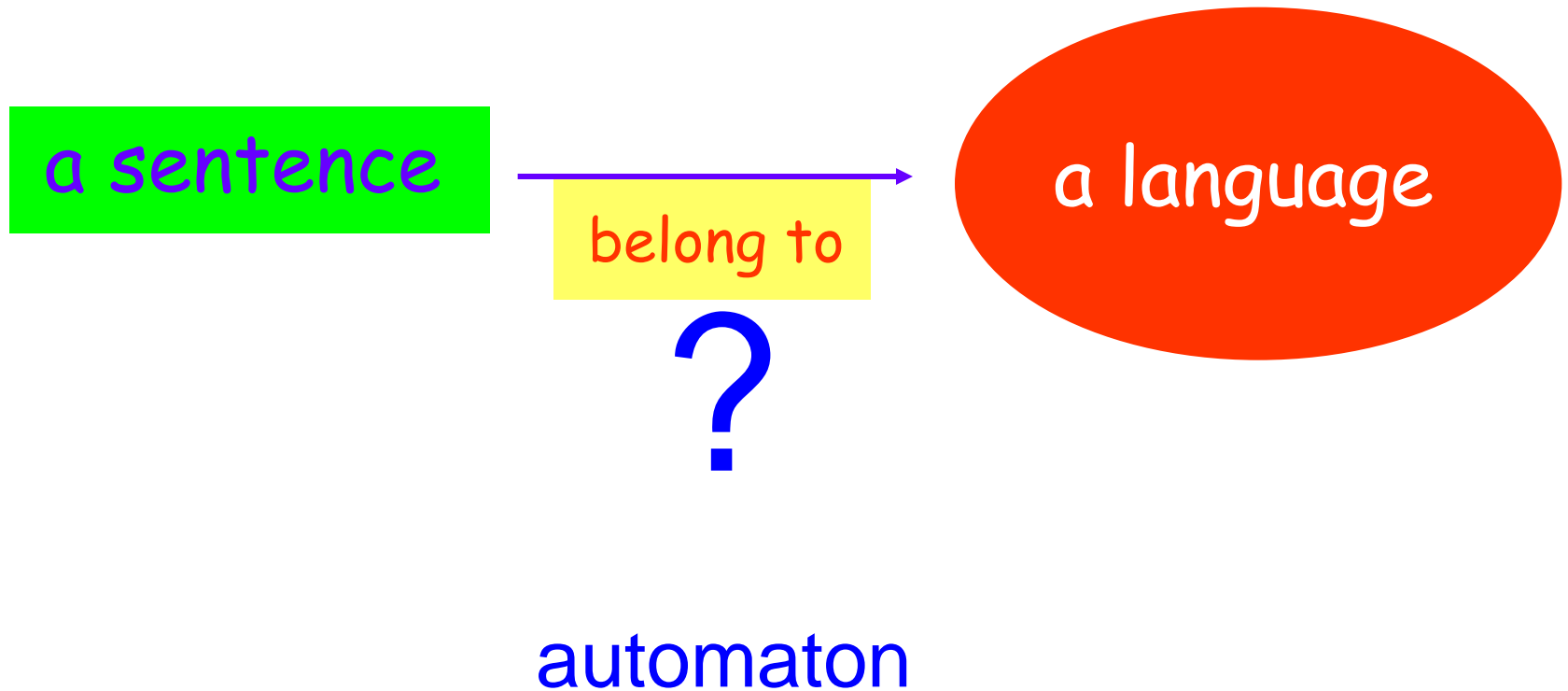
# Computation

- ◆ Computable Problems
  - write a program to solve
- ◆ Intractable Problems
  - find someway to work around

# Undecidable Problem

```
main ( )
{
    Int n, total, x, y, z ;
    scanf("%d", &n);
    total=3;
    while(1){
        for(x=1;x<=total-2;x++)
            for(y=1;y<=total-x-1;y++){
                z=total-x-y;
                if(exp(x,n)+exp(y,n)==exp(z,n))
                    printf("hello,world\n");
            }
        total++;
    }
}
```

# Undecidable Problem



# Content

Automata

Languages

Grammars

Construction

Properties

Design

Finite  
Automaton

Regular  
Language

Regular  
Expression

Recognize

Generate

Push Down  
Automaton

Context Free  
Language

Context Free  
Grammar

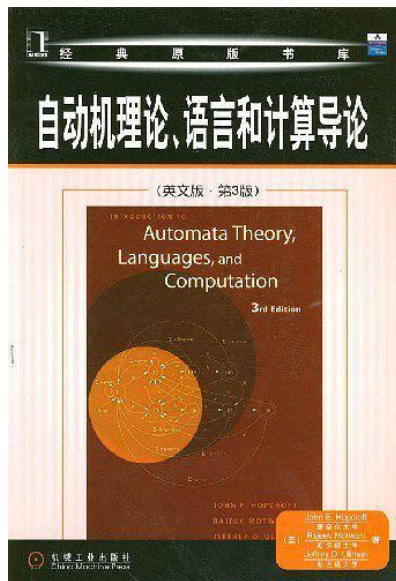
Turing  
Machine

Recursively  
Enumerable

( Phrase  
Grammar)

# Text book

## 1. Introduction to Automata Theory , Languages , and Computation (Third Edition )



—— John E. Hopcroft  
Rajeev Motwani  
Jeffrey D. Ullman



# Text book

2. An Introduction to Formal Languages  
and Automata (Third Edition )

—— Peter Linz

# Goal

1. Understanding "theoretical" concepts  
----- method of formal description
2. Get a sense of how to reason formally
3. Improving reading ability in English

# Homework

- All exercises listed on qq-group
- Submit on random ID
- Check by free talk
- Discussions in group

# Honor and Collaboration

- Collaboration is strongly encouraged
- Solutions must be written independently
- Responsible for Understanding and explaining

# Exam

- Only final exam
- Open exam

You are allowed to refer to text-book, class handouts, and notes during the exam

- Closed exam

Nothing allowed except one pen

# Grading Policy

- Homework : 20%
- Final exam : 80%

# *Instructor*

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Good good study  
day day up!