

# Histogram reweighting For Rods

In this report, I examine the Rod-like particle with  $K=8, L=32, Z=14$

- Pick a specific  $\mu_2$  as to be in the coexistence region of the case of  $K=8, L=32$ , which in our case:  $Z_2 = 14$  thus that we would expect to obtain **two peaks** in our  $N$  distribution;
- Following the Wang-Laudau algorithm demonstrated on page 16 of the *Biased sampling and related free energy technique lecture notes*, M.S.Shell 2009, we did a simulation for generating our weighting function  $\eta_{\mu 2}(N; \mu_1=0)$ , or equivalently,  $\eta_{z2}(N; Z_1=1)$  since in our simulation we define

$$Z_i = e^{\beta \mu_i} \dots (1)$$

In addition, this time, since we now have 2 species, we fix  $Z_2 = 14$  and we then set  $Z_1 = 1$  as to generate the weighting function  $\eta_{z2}(N; Z_1=1)$ . In other words, our acceptance probabilities would now looks like:

**For Addition :**  $P_{acc1} = \min[1, (Z_1 * V / ((N_1 + 1) * K) * \exp(\eta_{z2}[N_1 + 1] - \eta_{z2}[N_1]))];$

$$P_{acc2} = \min[1, (Z_2 * V / ((N_2 + 1) * K)];$$

**For Deletion :**  $P_{del1} = \min[1, (N_1 * K / (Z_1 * V)) * \exp(\eta_{z2}[N_1 - 1] - \eta_{z2}[N_1]);]$

$$P_{del2} = \min[1, N_2 * K / (Z_2 * V)];$$

- In RLC Vink's notation:

$$g_{\beta, z2}(N_1; Z_1=1) = e^{-\eta_{z2}(N_1; Z_1=1)} \dots (2).$$

Thus that we directly obtained our "integrated" DOS:  $g_{\beta, z2}(N_1; Z_1=1)$ .

- Then, Instead of doing multiple long MC runs for obtaining the distribution depending on different  $Z$ s, the  $N$  distribution for example, we can easily generate the distribution/probability distribution of the number of particles  $P_i(N; Z_i)$  by just applying the histogram reweighting method onto the "integrated" DOS we just got for  $Z=1$ :  $g_{\beta}(N; Z_1=1)$ . More precisely, It's the equation.4 on Page 10, *Lecture 7 and 8 RLC Vink*:

$$P(N) \propto g_{\beta}(N) e^{\beta K N} \dots (3)$$

- Where in Vink's notation  $K$  stands for chemical potential (the  $\mu$  in M.S.Shell's notation) and therefore in our notation:  $e^{\beta K N} = e^{\beta \mu N} = Z^N$ ; Hence, combining equation (1), (2), (3) we got the final equation to do the histogram reweighting:

$$P_i(N_1; Z_1=Z_i) \propto e^{-\eta_{z2}(N_1; Z_1=1)} Z_i^N \dots (4)$$

## Histogram reweighting for Rod: $K=8, L=32, Z=14$



