Simplified Physics for Spring-Connected Repellant Points

Lars Rönnbäck

June 23, 2010

1 Formulas

Let the force with which points repel each other be

$$\vec{F}_r = \frac{\mathcal{K}_r}{d(p_1, p_2)}$$

where $\mathcal{K}_r$ is a constant and $d(p_1, p_2)$ is the distance between the points p_1 and p_2. Let the force with which spring-connected points attract each other be

$$\vec{F}_a = \mathcal{K}_a d(p_1, p_2)^2$$

where $\mathcal{K}_a$ is a constant and $d(p_1, p_2)$ is the distance between the points p_1 and p_2. The acceleration of a point caused by an applied force can be derived from

$$\vec{F} = m\vec{a}$$

and furthermore, the point will gain a velocity of

$$\vec{v} = \vec{a}t$$

over the time period t. Combined these yield

$$\vec{v} = \frac{\vec{F}t}{m}$$

for the velocity. If masses and time resolution can be chosen arbitrarily the formula will be in its simplest form for every point having $m = 1$ using the resolution $t = 1$. In this case the formula becomes:

$$\vec{v} = \vec{F}$$

If the distance between two points $p_1(x_1, y_1)$ and $p_2(x_2, y_2)$ is is expressed using the Euclidean metric

$$d(p_1, p_2) = d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

this gives the x and y components of $\vec{F}_r$ as

$$F_r^x = \frac{\mathcal{K}_r(x_2 - x_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$F_r^y = \frac{\mathcal{K}_r(y_2 - y_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If instead the distance is expressed using the taxi cab metric

$$d(p_1, p_2) = d((x_1, y_1), (x_2, y_2)) = |x_2 - x_1| + |y_2 - y_1|$$

the components of $\vec{F}$ becomes

$$F_r^x = \frac{\mathcal{K}_r(x_2 - x_1)}{(|x_2 - x_1| + |y_2 - y_1|)^2}$$

$$F_r^y = \frac{\mathcal{K}_r(y_2 - y_1)}{(|x_2 - x_1| + |y_2 - y_1|)^2}$$

On the $n\pi/4, n = 0, 2, 4, 6$ radians this distance coincides with the Euclidean distance, but for $n = 1, 3, 5, 7$ we will measure a distance that is $\sqrt{2}$ times longer. On the former repelling forces will have an advantage and on the latter the attracting ones. This non-uniformity will slightly affect the layout, but the advantage of having simpler calculations makes up for this.

1.1 Algorithms

The following is a pseudo-code description of a verlet algorithm.

1. An array of the current locations and x and y component velocities of points are kept.

2. All distances are calculated and stored, resulting in $\binom{n}{2}$ calculations, where n is the number of points.

3. For every point determine which other points lie within the contribution radius and calculate the x and y components of the repelling forces.

4. For every point calculate the x and y components of the attracting forces caused by spring-connected points.

5. Adjust the x and y component velocities.

6. Calculate the new position of the point, given that the time step is 1 second.