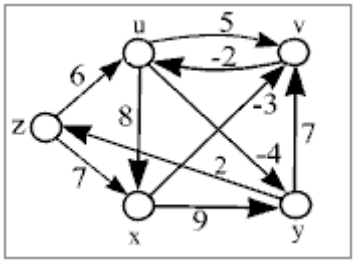
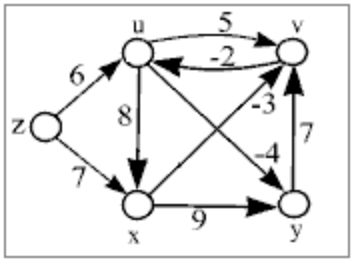
CSCI 270: Algorithms
Spring 2019 Homework 2
Due Date: 17 April 2019.

1. Modify the depth first search so that it gives the connected component number for each vertex. Here, two vertices belonging to the same connected component will get the same number.
2. Design a $\theta(V+E)$ algorithm that finds whether an undirected graph G=(V,E) contains a cycle.
3. How does the strongly connected component of a directed graph change after inserting an edge to this graph? Explain your answer.
4. A directed graph is uni-connected, if for all $u, v \in V$, there is a path from u to v and from v to u. Given an directed graph G=(V,E), design an algorithm that finds whether it is uni-directed in $O(V+E)$. Discuss its correctness and running time.
5. Given a DAG and two of its vertices *s* and *t*, find an efficient algorithm that finds the total number of different paths from *s* to *t*. Analyze the running time of your algorithm and discuss its correctness.
6. Let e=(u,v) be the minimum weight edge of an undirected graph. Prove that e=(u,v) must be in the MST or simply give a counter example in which e=(u,v) does not belong to the MST. (Assume that the graph does not have any other edge with the same weight as e=(u,v)).
7. Assume that all edge weights of a graph are integers between 1 and c, where c is a constant. Can we make Kruskal's MST algorithm faster? Explain.

   Use the following graph for questions 8-9.



8. Run the Bellman-Ford algorithm on the graph, starting from vertex y. After each iteration, show how the current shortest path distances change.
9. Run the Floyd-Warshall algoritm. Show the steps.
10. Consider the following graph. Run DFS starting from u to find the edge types. While running DFS, if there is a decision between multiple neighbor nodes, choose the letter closest to the beginning of the alphabet first.



Rules:

1. The homework must be completed individually. In the event that academic misconduct such as plagiarism or cheating is discovered, the student will receive no credit for the work, and the event reported to the Dean of your school. Please consult the Academic Integrity Statement given in the syllabus for more details about academic honestly.
2. Bring the hard copy of your homework to the beginning of the class on the due date.
3. You will lose 25 points for each day your homework is submitted late.
4. If your homework is typed (i.e., not hand-written) you will receive 5 additional points.