CS 146 PS4

1.

a. $J(\beta_t) = (e^{\beta_t} - e^{-\beta_t})\epsilon_t + e^{-\beta_t}$

$\frac{dJ(\beta_t)}{d\beta_t} = (e^{\beta_t} + e^{-\beta_t})\epsilon_t - e^{-\beta_t}$

Setting equal to zero:

$(e^{\beta_t} + e^{-\beta_t})\epsilon_t - e^{-\beta_t} = 0$

$e^{\beta_t} + e^{-\beta_t} = \frac{e^{-\beta_t}}{\epsilon_t}$

$e^{2\beta_t} + 1 = \frac{1}{\epsilon_t}$

$\beta_t = \frac{1}{2}[log(\frac{1}{\epsilon_t} - 1)]$

b. $\beta_1 = \frac{1}{2}[log(\frac{1}{\epsilon_1} - 1)]$, where ϵ_1 is 0

$\rightarrow \beta_1 = inf$

2.

a. Just do {1,2}, {5},{7}

$c_1 = 1.5, c_2 = 5,\ c_3 = 7.\ Obj = .5^2 + .5^2 + 0 + 0 = 0.5$

b. Consider $c_1 = 1$, $c_2=2$, $c_3=6$ (center of 5&7)

$obj = 0 + 0 + 1^2 + 1^2 = 2$ (which is greater than 0.5 and thus suboptimal)

It will not be improved as it will always converge to the local minimum and not the global min.

3.

a. $\frac{\partial l}{\partial \mu_j} = \frac{\partial}{\partial \mu_j}[\sum_k \sum_n \gamma_{nk} log\omega_k + \sum_k \sum_n \gamma_{nk} logN(x_n|\mu_k, \Sigma_k)]$

$= \frac{\partial}{\partial \mu_j}[\sum_k \sum_n \gamma_{nk} logN(x_n|\mu_k, \Sigma_k)]$

$= \frac{\partial}{\partial \mu_j}[\sum_k \sum_n \gamma_{nk} log[2\pi^{-k/2} det(\Sigma_k)^{-1/2} exp(\frac{-1}{2}(x - \mu_k)^T \Sigma_k^{-1}\ (x - \mu_k))]]$

$= \frac{\partial}{\partial \mu_j}[\sum_k \sum_n \gamma_{nk}[log(2\pi^{-k/2} det(\Sigma_k)^{-1/2}) + (\frac{-1}{2}(x - \mu_k)^T \Sigma_k^{-1}\ (x - \mu_k))]]$

$= \frac{\partial}{\partial \mu_j}\sum_k \sum_n \gamma_{nk}(\frac{-1}{2}(x - \mu_k)^T \Sigma_k^{-1}\ (x - \mu_k))$

$= \sum_n \gamma_{nj}(\sum_j x_n - \sum_j \mu_j\ )$

b. $\sum_n \gamma_{nj}(\sum_j x_n - \sum_j \mu_j) = 0$

$$\sum_n \gamma_{nj} \sum_j x_n - \sum_n \gamma_{nj} \sum_j \mu_j = 0$$

$$\sum_j \mu_j = \frac{1}{\sum_n \gamma_{nj}} \sum_n \gamma_{nj} \sum_j x_n$$

$$\mu_j = \frac{1}{\Sigma_n \gamma_{nj}} \sum_n \gamma_{nj} x_n$$

c. $\omega_1 = \frac{.2+.2+.8+.9+.9}{5} = 0.6$

$$\omega_2 = \frac{.8+.8+.2+.1+.1}{5} = 0.4$$

$$\mu_1 =. \frac{.2(5)+.2(15)+.8(25)+.9(30)+.9(40)}{.2+.2+.8+.9+.9} = 29$$

$$\mu_2 = \frac{.8(5)+.8(15)+.2(25)+.1(30)+.1(40)}{.8+.8+.2+.1+.1} = 14$$