Pratical work: EM algorithm

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Simulation

We first generate:

1. A sample of $n = 100$ observations with a Poisson law using $\lambda = 3$.
2. A sample of $n = 200$ observations with a Poisson law using $\lambda = 15$.
3. A vector of 300 coordinates, which the 100 first are 1 and the others are 2.

```
sample3 <- rpois(100, 3)
print(sample3)
```

```
##   [1] 3 3 1 4 2 2 3 2 3 1 5 1 3 3 1 2 2 6 4 2 2 2 4 3 0 1 2 4 4 2 2 3 4 5 3 4 2
##  [38] 2 2 1 2 0 7 4 1 1 3 2 3 4 4 4 2 6 4 4 1 3 4 4 4 0 2 3 5 6 4 4 6 0 3 2 2 1
##  [75] 5 1 4 3 3 2 2 1 5 3 5 4 3 2 6 0 3 0 2 3 4 8 2 5 3 2
```

```
sample15 <- rpois(200, 15)
print(sample15)
```

```
##   [1] 11 12 20 15 24 25 13 17  9 14 18 12 14 11 12 11 16  7 13 17 12  9 16 21 17
##  [26] 10 18 13 17 13 10 17 19 13 20 14 11 15 13 15 11 16 16 20 17 12  4 13 23 13
##  [51] 16 16 19 17 20 14 16 16 14 21 14 18 15 15 10 11 11 20  4 16 11 23 13 14 16
##  [76] 14 23 17 14 12 16 17 21 11 17 17 14 19 16 18 14  9 11 13 10 17 14 16 17 16
## [101] 11 12 17 13 19 15 14 17 13 14 14 12 15 11 15 15 13 17 18 11 21 18 20 28 16
## [126] 15 16 16 15 12 17 19 23 13 16 14 13 22 16 11 18 20 14 17 13 15 17 18 18 17
## [151] 15 17 23 15 11 18 16 16 22 17 12 15 22 15 17 13  9 13 19 19 19 13 16 17 17
## [176]  8 19 19 16 16 16 26 13 17 10 12 17 17 16 14 13 16 11 13 12 20 19 13 14 13
```

```
v <- c()
for (i in 1:100)
{
  v <- c(v, 1, recursive = TRUE)
}
for (i in 1:200)
{
  v <- c(v, 2, recursive = TRUE)
}
```

Now, we are going to generate a Poisson law using two components:

```
#Settings constants
pi1 <- 0.4
pi2 <- 0.6
lambda1 <- 3
```

```r
lambda2 <- 15
sample1 <- rpois(300, lambda1)
sample2 <- rpois(300,lambda2)

mixed_sample = pi1 * sample1 + pi2 * sample2
print(mixed_sample)
```

```
##   [1]  7.2 10.8  4.2  8.4  8.2 10.6 12.6 12.8 11.8 13.6  9.2 15.8 11.0  8.4 10.6
##  [16] 10.0 10.4 10.2 12.4  8.8  6.0  9.8  9.2 10.8  9.6 15.0 12.2 11.0  9.4 12.8
##  [31] 10.2 12.8 10.0 10.6  9.8  4.8 12.0 10.8 11.0 11.0  9.2  7.6  8.2 11.2 12.8
##  [46] 12.4 13.6 12.0 10.2  9.6 12.8  9.2 15.6 13.8 10.6 11.6 10.2 11.2 14.8 12.8
##  [61] 11.2 10.6 12.4  9.6 12.6 13.8  7.0 10.8 11.0  8.6  9.6 12.0 13.0  7.2  9.8
##  [76] 10.4 10.6 10.0 10.2 11.8  8.8 12.2  6.8 11.6 13.6  7.8 10.6  8.6 12.4 11.8
##  [91]  6.8  8.2 12.0  9.8  8.2 10.0 12.0  8.6 10.8 12.2 13.6  7.8 11.0 12.8 11.4
## [106]  9.4 10.8  5.4  8.8  7.4  7.8  7.6 10.2 13.0 10.4  9.6 12.6  5.8 11.6  7.0
## [121] 12.6  9.2  8.2 13.8 13.4 12.2 11.4  7.8  7.0 13.6 15.8  9.8 13.0  9.0 12.0
## [136] 10.6  8.6  8.6 11.4  9.2 10.0 15.4  8.8  9.0 11.8 10.6 12.0  8.2  8.0 11.0
## [151]  7.8 11.6  6.4  8.8  8.6  6.8 10.2 14.6 10.6 12.6  9.0  8.2  6.8  9.2 12.0
## [166] 10.8  9.4 12.8  5.4  7.8  8.2  8.2  8.6  8.0 13.4  5.6 12.8  9.8 11.4 12.0
## [181]  7.6  7.0  8.8  9.2  8.2 15.4 11.8 11.6 11.4 12.4 10.0 11.6  7.6  8.6 13.0
## [196]  7.4 14.6  9.6  9.0 10.2 12.2  6.0  9.4  7.2  9.2 15.2 15.2 14.0 11.4 11.4
## [211]  9.8  9.2  8.0  8.8  9.6 13.0  9.6 11.4  8.8 10.2 14.2 11.0 10.6  9.8 11.6
## [226] 13.4 10.4  7.4 11.2  9.4  7.6  6.8  9.6 12.0  9.8  8.0  6.8 11.2  9.8 11.0
## [241] 14.2 10.0 11.2  9.4 12.2 13.6 14.6  8.0  8.8  8.0 11.4  9.6  8.2 10.2  5.6
## [256]  9.8 11.2 14.8  6.2 10.0 11.4  8.8 10.2 10.2 10.2  8.8 10.0 10.0 11.0 11.6
## [271]  6.4 11.6  7.6 10.0  5.6  9.2  4.0  9.4 10.6  7.0 13.8 13.0  7.4 10.8  9.8
## [286] 11.4 13.4 12.6  7.6  7.8  7.2  9.0 13.0 11.2  7.8  6.4 12.0 10.4  5.2 13.2
```

EM-algorithm

We first want the initialization of the EM-algorithm: