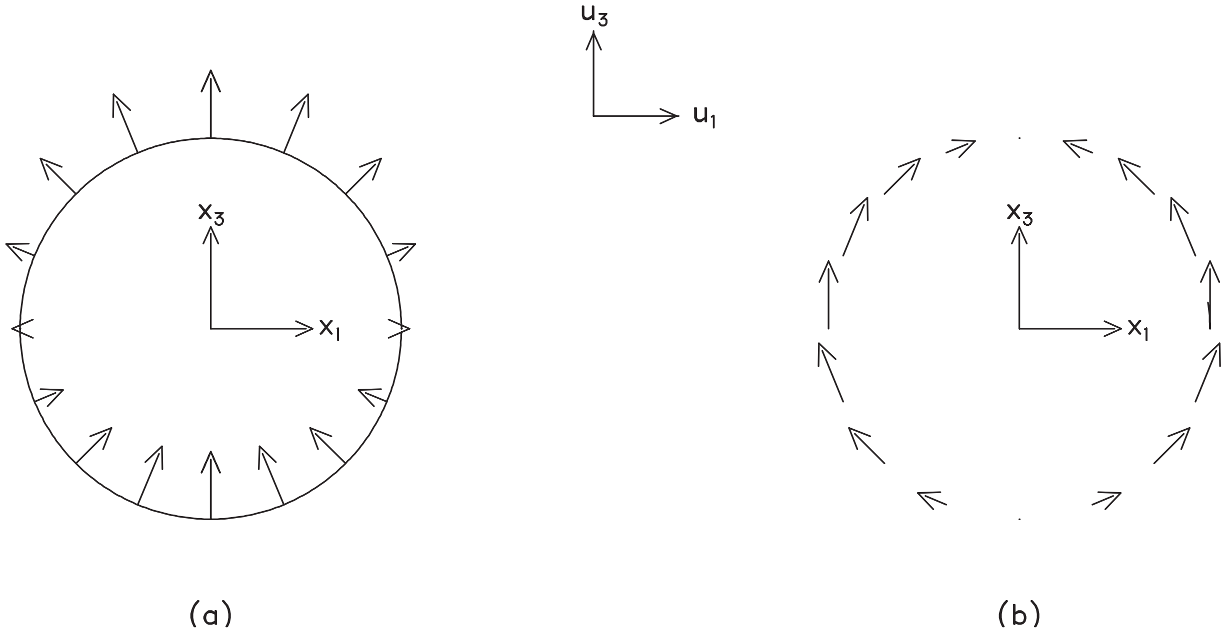

Fig. 9.6. Plot in the (x_1, x_3) plane of (a) the P and (b) S wave far-field motion generated by a point source in the x_3 direction. To draw the plots, assume that there is a pair of axes u_1 and u_3 on each point of the circle and plot (9.5.31) and (9.5.32) for (a), and (9.5.33) and (9.5.34) for (b). (After Pujol and Herrmann, 1990.)

9.6 Green's function for the elastic wave equation

Green's function is obtained from (9.5.16) with $T(t)$ replaced by $\delta(t)$ and is given by

$$G_{ij}(\mathbf{x}, t; \boldsymbol{\xi}, 0) = \frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\frac{1}{r^3}\left[H\left(t - \frac{r}{\alpha}\right) - H\left(t - \frac{r}{\beta}\right)\right]t$$
$$+ \frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{r}\delta\left(t - \frac{r}{\alpha}\right) - \frac{1}{4\pi\rho\beta^2}(\gamma_i\gamma_j - \delta_{ij})\frac{1}{r}\delta\left(t - \frac{r}{\beta}\right) \tag{9.6.1}$$

(Burridge, 1976; Hudson, 1980). Note that G_{ij} is a tensor-valued function (or tensor, for short) (Problem 9.18). The arguments $\boldsymbol{\xi}$ and 0 in G_{ij} indicate the location of the source and the time it acts. The first term on the right-hand side of (9.6.1) can be obtained directly from (9.5.18) because $\delta(t)$ is the unit element with respect to the operation of convolution (see Appendix A).

From (9.5.16) and (9.6.1) and using the relation

$$T(t) * \delta(t - t_o) = T(t - t_o) \tag{9.6.2}$$

it follows that

$$u_{ij}(\mathbf{x}, t) = T(t) * G_{ij}(\mathbf{x}, t; \boldsymbol{\xi}, 0). \tag{9.6.3}$$

From the book of José Pujol, Elastic Wave Propagation and Generation in Seismology (2003).

earthquake engineering are occasionally collected in the near field. But when one takes up in more detail the question of where the near field ends and where the far field begins, it becomes apparent that far-field terms also can be big enough to cause earthquake damage to engineering structures. (See Problem 4.1.)

4.2.1 PROPERTIES OF THE FAR-FIELD P-WAVE

We introduce here the *far-field P-wave*, which for (4.23) has the displacement $\mathbf{u}^P$ given by

$$u_i^P(\mathbf{x}, t) = \frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{r}X_0\left(t - \frac{r}{\alpha}\right). \tag{4.24}$$

As in (4.23), this is for a point force $X_0(t)$ in the x_j-direction at the origin. Along a given direction $\boldsymbol{\gamma}$ from the source, it follows from (4.24) that this wave

(i) attenuates as r^{-1};

(ii) has a waveform that depends on the time–space combination $t - r/\alpha$, and therefore propagates with speed α (recall that $\alpha^2 = (\lambda + 2\mu)/\rho$);

(iii) has a displacement waveform that is proportional to the applied force at retarded time; and

(iv) has a direction of displacement at $\mathbf{x}$ that is parallel to the direction $\boldsymbol{\gamma}$ from the source. This follows from the property $u_i^P \propto \gamma_i$ (see (4.24)). The far-field P-wave is therefore *longitudinal* (sometimes called *radial*) in that its direction of particle motion is the same as the direction of propagation. If $t = 0$ is chosen as the time at which $X_0(t)$ first becomes nonzero, then r/α is the *arrival time* of the P-wave at r.

4.2.2 PROPERTIES OF THE FAR-FIELD S-WAVE

The *far-field S-wave* in (4.23) has displacement $\mathbf{u}^S$ given by

$$u_i^S(x, t) = \frac{1}{4\pi\rho\beta^2}(\delta_{ij} - \gamma_i\gamma_j)X_0\left(t - \frac{r}{\beta}\right). \tag{4.25}$$

As in (4.23), this is for a point force $X_0(t)$ in the x_j-direction at the origin. Recall that $\boldsymbol{\gamma}$ is the unit vector directed from the source to the receiver. Along a given direction $\boldsymbol{\gamma}$, this wave

(i) attenuates as r^{-1};

(ii) propagates with speed β and has arrival time r/β at $\mathbf{x}$;

(iii) has a displacement waveform that is proportional to the applied force at retarded time; and

(iv) has a direction of displacement $\mathbf{u}^S$ at $\mathbf{x}$ that is perpendicular to the direction $\boldsymbol{\gamma}$ from the source. (From (4.25) it is easy to show that $\mathbf{u}^S \cdot \boldsymbol{\gamma} = 0$.) The far-field S-wave is therefore a *transverse* wave, because its direction of particle motion is normal to the direction of propagation.

Radiation patterns for $\mathbf{u}^P$ and $\mathbf{u}^S$ are given in Figure 4.2.

The same formulas from the book of Aki and Richards (1980). Note that both editions of the book have a typo in eq (4.25), a 1/r factor is missing.

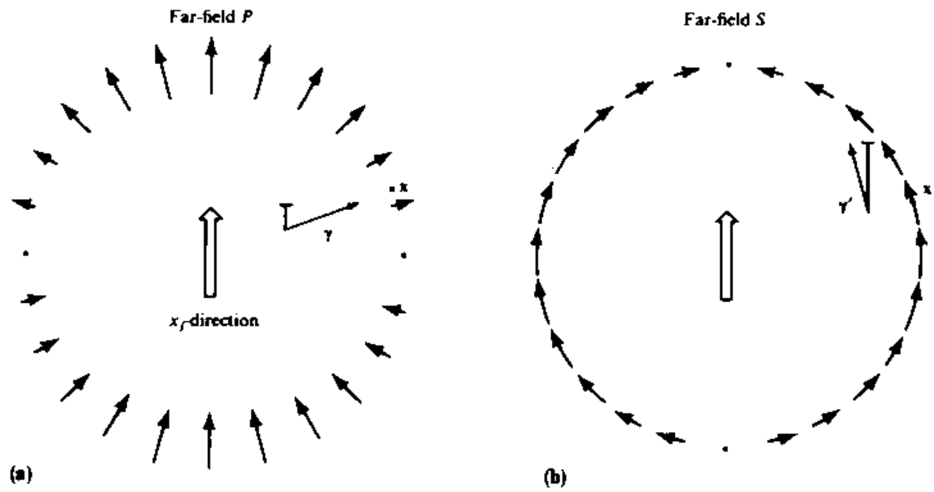

FIGURE 4.2
Radiation patterns showing the amplitude and direction of P and S motions in the far field, for a point force ⇧ in the x_j-direction within an infinite homogeneous isotropic medium. Directions for P and S are given by properties (iv) for each wave (see text), the particular choice of transverse direction $\boldsymbol{\gamma}'$ for S being determined by requiring axial symmetry. (a) The magnitude of $\mathbf{u}^P$ is given by $\mathbf{u}^P \cdot \boldsymbol{\gamma} \propto \gamma_j$, where γ_j is the cosine of the angle between the force direction and the direction of $\mathbf{u}^P$. (b) The magnitude of $\mathbf{u}^S$ is given by $\mathbf{u}^S \cdot \boldsymbol{\gamma}' \propto \gamma'_j$, where γ'_j is the cosine of the angle between the force direction and the direction of $\mathbf{u}^S$.

4.2.3 PROPERTIES OF THE NEAR-FIELD TERM

We define the near-field displacement $\mathbf{u}^N$ in (4.23) by

$$u_i^N(\mathbf{x}, t) = \frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\frac{1}{r^3}\int_{r/\alpha}^{r/\beta} \tau X_0(t-\tau)\,d\tau. \tag{4.26}$$

As in (4.23), this is for a point force $X_0(t)$ in the x_j-direction at the origin.

In our derivation (see above) of this near-field component, we see that there are contributions both from the gradient of the P-wave potential (ϕ) and from the curl of the S-wave potential ($\boldsymbol{\psi}$). In this sense, $\mathbf{u}^N$ is composed of both P-wave and S-wave motions. It is neither irrotational (i.e., having zero curl), nor solenoidal (i.e., having zero divergence), and this indicates that it is not always fruitful to decompose an elastic displacement field into its P-wave and S-wave components. Furthermore, $\mathbf{u}^N$ has both longitudinal and transverse motions, since the longitudinal component is

$$\mathbf{u}^N \cdot \boldsymbol{\gamma} = \gamma_j \frac{1}{2\pi\rho r^3}\int_{r/\alpha}^{r/\beta} \tau X_0(t-\tau)\,d\tau$$

and the transverse component is

$$\mathbf{u}^N \cdot \boldsymbol{\gamma}' = -\gamma'_j \frac{1}{4\pi\rho r^3}\int_{r/\alpha}^{r/\beta} \tau X_0(t-\tau)\,d\tau$$

(see Fig. 4.2 for definitions of $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}'$).