SDPB 2.5.1

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Contents

1 Introduction

SDPB is an arbitrary-precision semidefinite program solver, specialized for "polynomial matrix programs" (defined below). This document describes SDPB's usage and input/output. Much more detail about its design is given in [1]. The reader is encouraged to look there for a better understanding of SDPB's parameters and internal operation.

1.1   Installation and Requirements

See `Install.md` for up-to-date instructions on getting pre-made binaries or building from source.

2   Polynomial Matrix Programs

`SDPB` solves the following type of problem, which we call a *polynomial matrix program* (PMP). Consider a collection of symmetric polynomial matrices

$$M_j^n(x) \;=\; \begin{pmatrix} P_{j,11}^n(x) & \cdots & P_{j,1m_j}^n(x) \\ \vdots & \ddots & \vdots \\ P_{j,m_j1}^n(x) & \cdots & P_{j,m_jm_j}^n(x) \end{pmatrix} \tag{2.1}$$

labeled by $0 \leq n \leq N$ and $1 \leq j \leq J$, where each element $P_{j,rs}^n(x)$ is a polynomial in x. Given $b \in \mathbb{R}^N$, we would like to

$$\begin{aligned} \text{maximize} \quad & b_0 + b \cdot y \quad \text{over} \quad y \in \mathbb{R}^N, \\ \text{such that} \quad & M_j^0(x) + \textstyle\sum_{n=1}^N y_n M_j^n(x) \succeq 0 \quad \text{for all } x \geq 0 \text{ and } 1 \leq j \leq J. \end{aligned} \tag{2.2}$$

The notation $M \succeq 0$ means "M is positive semidefinite."

3   Input to `SDPB`

`SDPB` takes the following input:

- for each $j = 1, \ldots, J$:
  - polynomial matrices $M_j^0(x), \ldots, M_j^N(x)$ of maximum degree d_j,
  - bilinear bases $q_m^{(j)}(x)$ $(m = 0, \ldots, \lfloor d_j/2 \rfloor)$,
  - sample points $x_k^{(j)}$ $(k = 0, \ldots, d_j)$,
  - sample scalings $s_k^{(j)}$ $(k = 0, \ldots, d_j)$,
- an objective function $b_0 \in \mathbb{R}$ and $b \in \mathbb{R}^N$.

A bilinear basis is a collection of polynomials $q_m^{(j)}(x)$ such that $\deg q_m^{(j)} = m$, for example monomials $q_m^{(j)}(x) = x^m$. (A better choice for numerical stability is usually orthogonal polynomials on the positive real line.) The sample points and sample scalings determine how the PMP is represented internally as an SDP. In principle, they do not affect the solution of the PMP, but in practice they can affect numerical stability. The constant b_0 is completely irrelevant to the solution algorithm, but is included for convenience. See [1] for details.

3.1 Input Format

SDPB reads the data above in the following XML format.

Listing 1: XML input format for SDPB

```
input to SDPB ≡
  <sdp>
    ⟨xml for objective⟩
    ⟨xml for polynomial vector matrices⟩
  </sdp>

xml for objective ≡
  <objective>
    <elt>b_0</elt>

    ...

    <elt>b_N</elt>
  </objective>

xml for polynomial vector matrices ≡
  <polynomialVectorMatrices>
    ⟨xml for polynomial vector matrix M_1^n(x)⟩

    ...

    ⟨xml for polynomial vector matrix M_J^n(x)⟩
  </polynomialVectorMatrices>

xml for polynomial vector matrix M_j^n(x) ≡
  <polynomialVectorMatrix>
    <rows>m_j</rows>
    <cols>m_j</cols>
    <elements>
      ⟨xml for polynomial vector P_{j,11}^n(x)⟩

      ...

      ⟨xml for polynomial vector P_{j,m_j 1}^n(x)⟩

      ...

      ⟨xml for polynomial vector P_{j,1m_j}^n(x)⟩

      ...

      ⟨xml for polynomial vector P_{j,m_j m_j}^n(x)⟩
    </elements>
    <samplePoints>
      <elt>x_0^{(j)}</elt>

      ...

      <elt>x_{d_j}^{(j)}</elt>
    </samplePoints>
    <sampleScalings>
      <elt>s_0^{(j)}</elt>

      ...

      <elt>s_{d_j}^{(j)}</elt>
```

```
    </sampleScalings>
    <bilinearBasis>
      ⟨xml for polynomial $q_0^{(j)}(x)$⟩
      ...
      ⟨xml for polynomial $q_{\lfloor d_j/2 \rfloor}^{(j)}(x)$⟩
    </bilinearBasis>
  </polynomialVectorMatrix>

xml for polynomial vector $P_{j,rs}^n(x)$ ≡
  <polynomialVector>
    ⟨xml for polynomial $P_{j,rs}^0(x)$⟩
    ...
    ⟨xml for polynomial $P_{j,rs}^N(x)$⟩
  </polynomialVector>

xml for polynomial $a_0 + a_1 x + \ldots a_d x^d$ ≡
  <polynomial>
    <coeff>$a_0$</coeff>
    ...
    <coeff>$a_d$</coeff>
  </polynomial>
```

The data can be spread over multiple files. In such cases the different input files should all follow the above format except that ⟨*xml for objective*⟩ should be omitted from all but one file. The parser will combine the constraints from all the polynomial vector matrices specified in the different files.

Several aspects of the input format are inefficient. Because the matrices are symmetric, `rows` and `cols` are redundant, and most elements are listed twice. Also, XML is extremely verbose.

To improve performance for large inputs, the XML files must first be preprocessed into a more efficient format. Instructions on how to preprocess the input and run SDPB are in `docs/Usage.md`.

3.2 `JSON` and `Mathematica` Interface

The program `sdp2input` generates preprocessed input files from `JSON` or `Mathematica` data. It automatically makes sensible choices for the bilinear bases $q_m^{(j)}(x)$, the sample points $x_k^{(j)}$ and the sample scalings $s_k^{(j)}$. If you wish to experiment with these choices, there is also an included `Mathematica` notebook `SDPB.m`.

The `Mathematica` definition of a PMP is slightly different but trivially equivalent to

([2.2](#)). It is:

$$\text{maximize} \quad a \cdot z \quad \text{over} \quad z \in \mathbb{R}^{N+1},$$
$$\text{such that} \quad \sum_{n=0}^{N} z_n W_j^n(x) \succeq 0 \quad \text{for all } x \geq 0 \text{ and } 1 \leq j \leq J, \tag{3.1}$$
$$n \cdot z = 1.$$

where $W_j^n(x)$ are matrix polynomials. The normalization condition $n \cdot z = 1$ can be used to solve for one of the components of z in terms of the others. Calling the remaining components $y \in \mathbb{R}^N$, we arrive at ([2.2](#)), where $M_j^n(x)$ are linear combinations of $W_j^n(x)$ and b_0, b_n are linear combinations of the a_n. This difference in convention is for convenient use in the conformal bootstrap.

`SDPB.m` defines a function `WriteBootstrapSDP[file, sdp]`, where `file` is the XML file to be written to, and `sdp` has the following form, where the polynomials $Q_{j,rs}^n(x)$ are the elements of $W_j^n(x)$.

Listing 2: Usage of `WriteBootstrapSDP` in `SDPB.m`

```
function call  ≡  WriteBootstrapSDP[file, ⟨sdp⟩]

sdp  ≡  SDP[⟨objective⟩, ⟨normalization⟩, ⟨positive matrices with prefactors⟩]

objective  ≡  {a₀, ..., a_N}

normalization  ≡  {n₀, ..., n_N}

positive matrices with prefactors  ≡  {
    ⟨positive matrix with prefactor 1⟩,
    ...
    ⟨positive matrix with prefactor J⟩,
}

positive matrix with prefactor j  ≡
  PositiveMatrixWithPrefactor[⟨prefactor⟩,
    {
      {
        {Q⁰_{j,11}(x), ..., Qᴺ_{j,11}(x)}, ..., {Q⁰_{j,m_j1}(x), ..., Qᴺ_{j,m_j1}(x)}
      },
      ...
      {
        {Q⁰_{j,1m_j}(x), ..., Qᴺ_{j,1m_j}(x)}, ..., {Q⁰_{j,m_jm_j}(x), ..., Qᴺ_{j,m_jm_j}(x)}
      },
    }
  ]

prefactor  ≡
    DampedRational[c, {p₁,...,p_k}, b, x]
  or
    const
```

5

The prefactor in `PositiveMatrixWithPrefactor` is used for constructing bilinear bases and sample scalings. Specifically, if the prefactor is $\chi(x)$, the bilinear basis is a set of orthogonal polynomials with respect to measure $\chi(x)dx$ on the positive real line, and sample scalings are $\chi(x_k)$, where the x_k are sample points. The notebook `SDPB.m` only deals with damped-rational prefactors because these are relevant to the conformal bootstrap. These stand for

$$\texttt{DampedRational}[c, \ \{p_1, \ldots, p_k\}, \ b, \ x] \ \rightarrow \ c\frac{b^x}{\prod_{i=1}^{k}(x - p_i)}. \tag{3.2}$$

We do not use an exponential-times-rational `Mathematica` function directly because the `DampedRational` data structure makes it easier to extract information needed to construct a bilinear basis. The notebook `SDPB.m` makes a choice of sample points that are reasonable for conformal bootstrap applications.

As an example bootstrap application, the included notebook `Bootstrap2dExample.m` computes a single-correlator dimension bound for 2d CFTs with a $\mathbb{Z}_2$ symmetry, as in [2].

The `JSON` format is described by the schema in `docs/sdp2input_schema.json`.

3.3   An Example

Let's look at an example. Consider the following problem: maximize $-y$ such that

$$1 + x^4 + y\left(\frac{x^4}{12} + x^2\right) \ \geq \ 0 \qquad \text{for all } x \geq 0 \tag{3.3}$$

This is an PMP with 1×1 positive-semidefiniteness constraints. We will arbitrarily choose a prefactor of $e^{-x} = \texttt{DampedRational[1,\{\}, 1/E,x]}$, so that the bilinear basis consists of Laguerre polynomials. The `Mathematica` code for this example is

Listing 3: Mathematica input for the example (3.3)

```
Module[
  {
    pols = {
      PositiveMatrixWithPrefactor[
        DampedRational[1,{}, 1/E,x],
        {{{1 + x^4, x^4/12 + x^2}}}
      ]
    },
    norm = {1, 0},
    obj = {0, -1}
  },
  WriteBootstrapSDP["test.xml", SDP[obj, norm, pols]];
];
```

It produces the following XML file

Listing 4: XML file `test.xml` produced by listing 3. Decimals are truncated at 12 digits.

```xml
<sdp>
  <objective><elt>0</elt><elt>-1</elt></objective>
  <polynomialVectorMatrices>
    <polynomialVectorMatrix>
      <rows>1</rows>
      <cols>1</cols>
      <elements>
        <polynomialVector>
          <polynomial>
            <coeff>1</coeff><coeff>0</coeff><coeff>0</coeff>
            <coeff>0</coeff><coeff>1</coeff>
          </polynomial>
          <polynomial>
            <coeff>0</coeff><coeff>0</coeff><coeff>1</coeff>
            <coeff>0</coeff><coeff>0.0833333333333</coeff>
          </polynomial>
        </polynomialVector>
      </elements>
      <samplePoints>
        <elt>0.017496844815</elt><elt>0.157471603340</elt><elt>0.857345395967</elt>
        <elt>2.117118222694</elt><elt>3.936790083523</elt>
      </samplePoints>
      <sampleScalings>
        <elt>0.982655336118</elt><elt>0.854301072560</elt><elt>0.424286902403</elt>
        <elt>0.120378031823</elt><elt>0.019510742190</elt>
      </sampleScalings>
      <bilinearBasis>
        <polynomial><coeff>1</coeff></polynomial>
        <polynomial><coeff>-1</coeff><coeff>1</coeff></polynomial>
        <polynomial><coeff>1</coeff><coeff>-2</coeff><coeff>0.5</coeff></polynomial>
      </bilinearBasis>
    </polynomialVectorMatrix>
  </polynomialVectorMatrices>
</sdp>
```

4 Internal SDP

To understand the output of `SDPB`, we need a rough understanding of its internal representation of the above PMP as a semidefinite program (SDP). Much more detail is given in [1]. The PMP (2.2) is translated into a dual pair of SDPs of the following form:

$$
\begin{aligned}
\mathcal{D}: \quad &\text{maximize} \quad \mathrm{Tr}(CY) + b_0 + b \cdot y \quad \text{over} \quad y \in \mathbb{R}^N,\ Y \in \mathcal{S}^K, \\
&\text{such that} \quad \mathrm{Tr}(A_* Y) + By = c, \ \text{and} \\
&Y \succeq 0.
\end{aligned}
\tag{4.1}
$$

$$\mathcal{P}: \quad \text{minimize} \quad b_0 + c \cdot x \quad \text{over} \quad x \in \mathbb{R}^P, \ X \in \mathcal{S}^K,$$
$$\text{such that} \quad X = \sum_{p=1}^{P} A_p x_p - C,$$
$$B^T x = b,$$
$$X \succeq 0, \tag{4.2}$$

where "$\succeq 0$" means "is positive-semidefinite" and

$$\begin{aligned}
c &\in \mathbb{R}^P, \\
B &\in \mathbb{R}^{P \times N}, \\
A_1, \dots, A_P, C &\in \mathcal{S}^K.
\end{aligned} \tag{4.3}$$

Here, $\mathcal{S}^K$ is the space of $K \times K$ symmetric real matrices, and $\mathrm{Tr}(A_*Y)$ denotes the vector $(\mathrm{Tr}(A_1 Y), \dots, \mathrm{Tr}(A_P Y)) \in \mathbb{R}^P$. An optimal solution to (4.1) and (4.2) is characterized by $XY = 0$ and also equality of the primal and dual objective functions $\mathrm{Tr}(CY) + b_0 + b \cdot y = b_0 + c \cdot x$.

The residues

$$\begin{aligned}
P &\equiv \sum_i A_i x_i - X - C, \\
p &\equiv b - B^T x, \\
d &\equiv c - \mathrm{Tr}(A_*Y) - By,
\end{aligned} \tag{4.4}$$

measure the failure of x, X, y, Y to satisfy their constraints. We say a point $q = (x, X, y, Y)$ is "primal feasible" or "dual feasible" if the residues are sufficiently small,

$$\begin{aligned}
\text{primal feasible:} \quad & \texttt{primalError} \equiv \max_{i,j}\{|p_i|, |P_{ij}|\} < \texttt{primalErrorThreshold}; \\
\text{dual feasible:} \quad & \texttt{dualError} \equiv \max_i\{|d_i|\} < \texttt{dualErrorThreshold},
\end{aligned}$$

where $\texttt{primalErrorThreshold} \ll 1$ and $\texttt{dualErrorThreshold} \ll 1$ are parameters chosen by the user.

An optimal point should be both primal and dual feasible, and have (nearly) equal primal and dual objective values. Specifically, let us define $\texttt{dualityGap}$ as the normalized difference between the primal and dual objective functions

$$\begin{aligned}
\texttt{dualityGap} &\equiv \frac{|\texttt{primalObjective} - \texttt{dualObjective}|}{\max\{1, |\texttt{primalObjective} + \texttt{dualObjective}|\}}, \\
\texttt{primalObjective} &\equiv b_0 + c \cdot x, \\
\texttt{dualObjective} &\equiv \mathrm{Tr}(CY) + b_0 + b \cdot y.
\end{aligned} \tag{4.5}$$

A point is considered "optimal" if

$$\texttt{dualityGap} < \texttt{dualityGapThreshold}, \tag{4.6}$$

where $\texttt{dualityGapThreshold} \ll 1$ is chosen by the user.

5 Output of SDPB

5.1 Terminal Output

The output from running SDPB on the example problem in section 3.3 is in listing 5. The input, output, and checkpoint files are listed first, followed by various parameters. After each iteration, SDPB prints the following:

time: The current solver runtime in seconds.

mu: The value of the complementarity $\mathrm{Tr}(XY)/K$.

P-obj: The primal objective value $b_0 + c \cdot x$.

D-obj: The dual objective value $\mathrm{Tr}(CY) + b_0 + b \cdot y$.

gap: The value of dualityGap.

P-err: The primal error $\max_{i,j}\{|P_{ij}|\}$.

p-err: The primal error $\max_i\{|p_i||\}$.

D-err: The dual error $\max_i\{|d_i|\}$.

P-step: The primal step length $\alpha_{\mathcal{P}}$ described in [1].

D-step: The dual step length $\alpha_{\mathcal{D}}$ described in [1].

beta: The corrector centering parameter β_c described in [1].

If an optimal solution exists, the primal and dual error will decrease until the problem becomes primal and dual feasible. Then the primal and dual objective functions start to converge, and the complementarity μ decreases until the duality gap becomes smaller than dualityGapThreshold.

The terminal output ends with the final values of the primal/dual objectives, primal/dual errors and duality gap.

Listing 5: Output of SDPB for the input file in listing 4

```
$ ./build/sdpb -s test/test --noFinalCheckpoint --procsPerNode=1
SDPB started at 2020-Jan-16 12:09:59
SDP directory    : "test/test"
out directory    : "test/test_out"
checkpoint in    : "test/test.ck"
checkpoint out   : "test/test.ck"

Parameters:
maxIterations              = 500
maxRuntime                 = 9223372036854775807
checkpointInterval         = 3600
noFinalCheckpoint          = true
writeSolution              = x,y
findPrimalFeasible         = false
```

```
findDualFeasible          = false
detectPrimalFeasibleJump  = false
detectDualFeasibleJump    = false
precision(actual)         = 400(448)
dualityGapThreshold       = 1e-30
primalErrorThreshold      = 1e-30
dualErrorThreshold        = 1e-30
initialMatrixScalePrimal  = 1e+20
initialMatrixScaleDual    = 1e+20
feasibleCenteringParameter   = 0.1
infeasibleCenteringParameter = 0.3
stepLengthReduction       = 0.7
maxComplementarity        = 1e+100
procsPerNode              = 1
procGranularity           = 1
verbosity                 = 1

Block Grid Mapping
Node    Num Procs     Cost          Block List
=================================================
0       1             25        {(0,5)}
```

	time	mu	P-obj	D-obj	gap	P-err	p-err	D-err	P-step	D-step	beta
1	0	1.0e+40	+0.00	+0.00	0.00	+1.00e+20	+1.00	+2.88e+20	0.631	0.647	0.300
2	0	5.0e+39	+9.49e+19	-1.64e+20	1.00	+3.69e+19	+0.369	+1.02e+20	0.653	0.639	0.300
3	0	2.5e+39	+1.04e+20	-2.92e+20	1.00	+1.28e+19	+0.128	+3.68e+19	0.660	0.639	0.300
4	0	1.2e+39	+1.43e+20	-4.30e+20	1.00	+4.35e+18	+0.0435	+1.33e+19	0.652	0.638	0.300
5	0	5.8e+38	+1.91e+20	-6.13e+20	1.00	+1.51e+18	+0.0151	+4.80e+18	0.645	0.636	0.300
6	0	2.8e+38	+2.52e+20	-8.65e+20	1.00	+5.38e+17	+0.00538	+1.75e+18	0.640	0.634	0.300
7	0	1.4e+38	+3.36e+20	-1.21e+21	1.00	+1.94e+17	+0.00194	+6.38e+17	0.636	0.633	0.300
8	0	7.2e+37	+4.52e+20	-1.69e+21	1.00	+7.05e+16	+0.000705	+2.34e+17	0.635	0.633	0.300
9	0	3.6e+37	+6.16e+20	-2.34e+21	1.00	+2.57e+16	+0.000257	+8.58e+16	0.634	0.633	0.300
10	0	1.8e+37	+8.43e+20	-3.23e+21	1.00	+9.43e+15	+9.43e-05	+3.15e+16	0.633	0.633	0.300
11	0	9.3e+36	+1.16e+21	-4.46e+21	1.00	+3.46e+15	+3.46e-05	+1.16e+16	0.633	0.633	0.300
12	0	4.7e+36	+1.59e+21	-6.16e+21	1.00	+1.27e+15	+1.27e-05	+4.24e+15	0.633	0.633	0.300
13	0	2.4e+36	+2.20e+21	-8.49e+21	1.00	+4.65e+14	+4.65e-06	+1.56e+15	0.633	0.633	0.300
14	0	1.2e+36	+3.03e+21	-1.17e+22	1.00	+1.71e+14	+1.71e-06	+5.71e+14	0.633	0.633	0.300
15	0	6.1e+35	+4.18e+21	-1.62e+22	1.00	+6.26e+13	+6.26e-07	+2.09e+14	0.633	0.633	0.300
16	0	3.1e+35	+5.77e+21	-2.23e+22	1.00	+2.30e+13	+2.30e-07	+7.68e+13	0.633	0.633	0.300
17	0	1.6e+35	+7.96e+21	-3.08e+22	1.00	+8.42e+12	+8.42e-08	+2.82e+13	0.633	0.633	0.300
18	0	7.9e+34	+1.10e+22	-4.25e+22	1.00	+3.09e+12	+3.09e-08	+1.03e+13	0.633	0.633	0.300
19	0	4.0e+34	+1.52e+22	-5.86e+22	1.00	+1.13e+12	+1.13e-08	+3.79e+12	0.633	0.633	0.300
20	0	2.0e+34	+2.09e+22	-8.09e+22	1.00	+4.15e+11	+4.15e-09	+1.39e+12	0.633	0.633	0.300
21	0	1.0e+34	+2.89e+22	-1.12e+23	1.00	+1.52e+11	+1.52e-09	+5.09e+11	0.633	0.633	0.300
22	0	5.2e+33	+3.98e+22	-1.54e+23	1.00	+5.58e+10	+5.58e-10	+1.87e+11	0.633	0.633	0.300
23	0	2.6e+33	+5.50e+22	-2.13e+23	1.00	+2.05e+10	+2.05e-10	+6.85e+10	0.633	0.633	0.300
24	0	1.3e+33	+7.59e+22	-2.93e+23	1.00	+7.51e+09	+7.51e-11	+2.51e+10	0.633	0.633	0.300
25	0	6.7e+32	+1.05e+23	-4.05e+23	1.00	+2.75e+09	+2.75e-11	+9.21e+09	0.633	0.633	0.300
26	0	3.4e+32	+1.44e+23	-5.59e+23	1.00	+1.01e+09	+1.01e-11	+3.38e+09	0.633	0.633	0.300
27	0	1.7e+32	+1.99e+23	-7.71e+23	1.00	+3.70e+08	+3.70e-12	+1.24e+09	0.633	0.633	0.300
28	0	8.7e+31	+2.75e+23	-1.06e+24	1.00	+1.36e+08	+1.36e-12	+4.54e+08	0.633	0.633	0.300
29	0	4.4e+31	+3.80e+23	-1.47e+24	1.00	+4.98e+07	+4.98e-13	+1.67e+08	0.633	0.633	0.300
30	0	2.2e+31	+5.24e+23	-2.03e+24	1.00	+1.83e+07	+1.83e-13	+6.11e+07	0.633	0.633	0.300
31	0	1.1e+31	+7.23e+23	-2.80e+24	1.00	+6.70e+06	+6.70e-14	+2.24e+07	0.633	0.633	0.300
32	0	5.7e+30	+9.98e+23	-3.86e+24	1.00	+2.46e+06	+2.46e-14	+8.22e+06	0.633	0.633	0.300
33	0	2.9e+30	+1.38e+24	-5.32e+24	1.00	+9.01e+05	+9.01e-15	+3.01e+06	0.633	0.633	0.300
34	0	1.5e+30	+1.90e+24	-7.35e+24	1.00	+3.30e+05	+3.30e-15	+1.11e+06	0.633	0.633	0.300
35	0	7.4e+29	+2.62e+24	-1.01e+25	1.00	+1.21e+05	+1.21e-15	+4.05e+05	0.633	0.633	0.300
36	0	3.7e+29	+3.62e+24	-1.40e+25	1.00	+4.44e+04	+4.44e-16	+1.49e+05	0.633	0.633	0.300
37	0	1.9e+29	+4.99e+24	-1.93e+25	1.00	+1.63e+04	+1.63e-16	+5.45e+04	0.633	0.633	0.300
38	0	9.6e+28	+6.89e+24	-2.66e+25	1.00	+5.98e+03	+5.98e-17	+2.00e+04	0.633	0.633	0.300
39	0	4.8e+28	+9.51e+24	-3.68e+25	1.00	+2.19e+03	+2.19e-17	+7.33e+03	0.633	0.633	0.300
40	0	2.4e+28	+1.31e+25	-5.07e+25	1.00	+803.	+8.03e-18	+2.69e+03	0.634	0.634	0.300
41	0	1.2e+28	+1.81e+25	-7.00e+25	1.00	+294.	+2.94e-18	+985.	0.634	0.634	0.300
42	0	6.3e+27	+2.50e+25	-9.64e+25	1.00	+108.	+1.08e-18	+360.	0.636	0.635	0.300

43	0	3.2e+27	+3.44e+25	−1.32e+26	1.00	+39.1	+3.91e−19	+131.	0.642	0.639	0.300
44	0	1.6e+27	+4.72e+25	−1.81e+26	1.00	+14.0	+1.40e−19	+47.5	0.657	0.649	0.300
45	0	7.8e+26	+6.42e+25	−2.41e+26	1.00	+4.80	+4.80e−20	+16.7	0.699	0.677	0.300
46	0	3.6e+26	+8.51e+25	−3.04e+26	1.00	+1.44	+1.44e−20	+5.40	0.785	0.768	0.300
47	0	1.6e+26	+1.02e+26	−3.19e+26	1.00	+0.310	+3.10e−21	+1.25	1.00	1.00	0.300
48	0	5.3e+25	+8.01e+25	−1.83e+26	1.00	+1.16e−115	+4.30e−120	+7.84e−90	0.734	0.734	0.100
49	0	1.8e+25	+2.73e+25	−6.20e+25	1.00	+7.61e−116	+2.83e−117	+1.35e−85	0.736	0.736	0.100
50	0	6.0e+24	+9.21e+24	−2.09e+25	1.00	+1.27e−115	+1.74e−116	+5.21e−86	0.736	0.736	0.100
51	0	2.0e+24	+3.10e+24	−7.08e+24	1.00	+1.30e−115	+2.40e−116	+1.44e−86	0.736	0.736	0.100
52	0	6.9e+23	+1.04e+24	−2.39e+24	1.00	+1.27e−115	+1.24e−116	+4.03e−87	0.737	0.737	0.100
53	0	2.3e+23	+3.51e+23	−8.08e+23	1.00	+1.18e−115	+5.06e−118	+1.08e−87	0.737	0.737	0.100
54	0	7.8e+22	+1.18e+23	−2.72e+23	1.00	+1.05e−115	+8.09e−118	+2.86e−88	0.738	0.738	0.100
55	0	2.6e+22	+3.95e+22	−9.14e+22	1.00	+1.02e−115	+3.81e−117	+7.51e−89	0.738	0.738	0.100
56	0	8.8e+21	+1.32e+22	−3.07e+22	1.00	+1.49e−115	+1.50e−117	+1.97e−89	0.739	0.739	0.100
57	0	2.9e+21	+4.43e+21	−1.03e+22	1.00	+1.27e−115	+1.14e−116	+5.14e−90	0.739	0.739	0.100
58	0	9.8e+20	+1.48e+21	−3.44e+21	1.00	+1.02e−115	+1.35e−116	+1.34e−90	0.739	0.739	0.100
59	0	3.3e+20	+4.95e+20	−1.15e+21	1.00	+1.02e−115	+1.40e−117	+3.50e−91	0.740	0.740	0.100
60	0	1.1e+20	+1.66e+20	−3.85e+20	1.00	+9.59e−116	+6.80e−118	+9.11e−92	0.740	0.740	0.100
61	0	3.7e+19	+5.54e+19	−1.29e+20	1.00	+1.02e−115	+2.37e−118	+2.37e−92	0.740	0.740	0.100
62	0	1.2e+19	+1.85e+19	−4.30e+19	1.00	+8.41e−116	+6.29e−119	+6.17e−93	0.740	0.740	0.100
63	0	4.1e+18	+6.18e+18	−1.44e+19	1.00	+5.21e−116	+6.30e−120	+1.61e−93	0.740	0.740	0.100
64	0	1.4e+18	+2.06e+18	−4.80e+18	1.00	+1.36e−116	+1.64e−120	+4.17e−94	0.740	0.740	0.100
65	0	4.6e+17	+6.89e+17	−1.60e+18	1.00	+3.52e−117	+4.26e−121	+1.09e−94	0.740	0.740	0.100
66	0	1.5e+17	+2.30e+17	−5.36e+17	1.00	+9.16e−118	+1.11e−121	+2.82e−95	0.740	0.740	0.100
67	0	5.1e+16	+7.68e+16	−1.79e+17	1.00	+2.38e−118	+2.88e−122	+7.33e−96	0.740	0.740	0.100
68	0	1.7e+16	+2.56e+16	−5.97e+16	1.00	+6.18e−119	+7.54e−123	+1.90e−96	0.740	0.740	0.100
69	0	5.7e+15	+8.56e+15	−1.99e+16	1.00	+1.61e−119	+2.03e−123	+4.95e−97	0.740	0.740	0.100
70	0	1.9e+15	+2.86e+15	−6.65e+15	1.00	+4.17e−120	+7.30e−124	+1.29e−97	0.740	0.740	0.100
71	0	6.3e+14	+9.53e+14	−2.22e+15	1.00	+1.08e−120	+4.46e−124	+3.34e−98	0.740	0.740	0.100
72	0	2.1e+14	+3.18e+14	−7.41e+14	1.00	+2.82e−121	+4.55e−124	+8.67e−99	0.740	0.740	0.100
73	0	7.1e+13	+1.06e+14	−2.47e+14	1.00	+7.31e−122	+1.18e−124	+2.25e−99	0.740	0.740	0.100
74	0	2.4e+13	+3.54e+13	−8.26e+13	1.00	+1.90e−122	+3.07e−125	+5.85e−100	0.740	0.740	0.100
75	0	7.9e+12	+1.18e+13	−2.76e+13	1.00	+4.93e−123	+7.98e−126	+1.52e−100	0.740	0.740	0.100
76	0	2.6e+12	+3.94e+12	−9.20e+12	1.00	+1.28e−123	+2.07e−126	+3.94e−101	0.740	0.740	0.100
77	0	8.8e+11	+1.32e+12	−3.07e+12	1.00	+3.33e−124	+5.38e−127	+1.02e−101	0.740	0.740	0.100
78	0	2.9e+11	+4.39e+11	−1.02e+12	1.00	+8.63e−125	+1.40e−127	+2.66e−102	0.740	0.740	0.100
79	0	9.8e+10	+1.47e+11	−3.42e+11	1.00	+2.24e−125	+3.63e−128	+6.90e−103	0.740	0.740	0.100
80	0	3.3e+10	+4.89e+10	−1.14e+11	1.00	+5.82e−126	+9.42e−129	+1.79e−103	0.740	0.740	0.100
81	0	1.1e+10	+1.63e+10	−3.80e+10	1.00	+1.51e−126	+2.44e−129	+4.65e−104	0.740	0.740	0.100
82	0	3.6e+09	+5.44e+09	−1.27e+10	1.00	+3.92e−127	+6.35e−130	+1.21e−104	0.740	0.740	0.100
83	0	1.2e+09	+1.82e+09	−4.24e+09	1.00	+1.02e−127	+1.65e−130	+3.14e−105	0.740	0.740	0.100
84	0	4.0e+08	+6.06e+08	−1.41e+09	1.00	+2.65e−128	+4.28e−131	+8.15e−106	0.740	0.740	0.100
85	0	1.3e+08	+2.02e+08	−4.72e+08	1.00	+6.87e−129	+1.11e−131	+2.11e−106	0.740	0.740	0.100
86	0	4.5e+07	+6.75e+07	−1.57e+08	1.00	+1.78e−129	+2.88e−132	+5.49e−107	0.740	0.740	0.100
87	0	1.5e+07	+2.25e+07	−5.25e+07	1.00	+4.63e−130	+7.48e−133	+1.43e−107	0.740	0.740	0.100
88	0	5.0e+06	+7.51e+06	−1.75e+07	1.00	+1.20e−130	+1.95e−133	+3.70e−108	0.740	0.740	0.100
89	0	1.7e+06	+2.51e+06	−5.85e+06	1.00	+3.12e−131	+5.16e−134	+9.61e−109	0.740	0.740	0.100
90	0	5.6e+05	+8.36e+05	−1.95e+06	1.00	+8.10e−132	+1.17e−134	+2.49e−109	0.740	0.740	0.100
91	0	1.9e+05	+2.79e+05	−6.51e+05	1.00	+2.10e−132	+4.64e−135	+6.48e−110	0.740	0.740	0.100
92	0	6.2e+04	+9.31e+04	−2.17e+05	1.00	+5.37e−133	+3.64e−136	+1.68e−110	0.740	0.740	0.100
93	0	2.1e+04	+3.11e+04	−7.24e+04	1.00	+1.39e−133	+3.38e−136	+4.36e−111	0.740	0.740	0.100
94	0	6.9e+03	+1.04e+04	−2.41e+04	1.00	+3.58e−134	+0.00	+1.13e−111	0.741	0.741	0.100
95	0	2.3e+03	+3.47e+03	−8.04e+03	1.00	+6.87e−135	+0.00	+2.94e−112	0.741	0.741	0.100
96	0	7.7e+02	+1.17e+03	−2.67e+03	1.00	+4.30e−135	+0.00	+7.63e−113	0.741	0.741	0.100
97	0	2.6e+02	+394.	−886.	1.00	+5.50e−135	+2.91e−135	+1.98e−113	0.741	0.741	0.100
98	0	85.	+136.	−291.	1.00	+8.25e−135	+1.38e−135	+5.13e−114	0.738	0.738	0.100
99	0	29.	+49.8	−93.6	1.00	+6.88e−135	+0.00	+1.34e−114	0.722	0.722	0.100
100	0	10.	+21.0	−29.2	1.00	+7.73e−135	+2.19e−136	+3.74e−115	0.694	0.694	0.100
101	0	3.8	+10.5	−8.33	1.00	+7.87e−135	+3.67e−136	+1.14e−115	0.707	0.707	0.100
102	0	1.4	+5.45	−1.41	1.00	+6.88e−135	+1.10e−135	+3.35e−116	0.765	0.765	0.100
103	0	0.43	+2.99	+0.854	0.555	+6.88e−135	+1.42e−135	+7.86e−117	0.768	0.768	0.100
104	0	0.13	+2.20	+1.54	0.176	+4.49e−135	+2.22e−136	+1.83e−117	0.764	0.764	0.100
105	0	0.041	+1.95	+1.75	0.0557	+3.12e−136	+0.00	+4.32e−118	0.764	0.764	0.100
106	0	0.013	+1.88	+1.81	0.0175	+7.73e−136	+0.00	+1.02e−118	0.766	0.766	0.100
107	0	0.0040	+1.85	+1.83	0.00543	+7.83e−136	+0.00	+2.38e−119	0.768	0.768	0.100
108	0	0.0012	+1.84	+1.84	0.00168	+7.53e−137	+0.00	+5.52e−120	0.770	0.770	0.100
109	0	0.00038	+1.84	+1.84	0.000514	+2.78e−136	+0.00	+1.27e−120	0.772	0.772	0.100
110	0	0.00012	+1.84	+1.84	0.000157	+2.01e−136	+0.00	+2.89e−121	0.773	0.773	0.100

```
111       0 3.5e-05 +1.84    +1.84      4.77e-05 +1.06e-135 +0.00     +6.55e-122 0.774  0.774  0.100
112       0 1.1e-05 +1.84    +1.84      1.44e-05 +1.91e-136 +0.00     +1.48e-122 0.775  0.775  0.100
113       0 3.2e-06 +1.84    +1.84      4.37e-06 +1.03e-135 +2.48e-137 +3.32e-123 0.776  0.776  0.100
114       0 9.7e-07 +1.84    +1.84      1.32e-06 +3.47e-136 +1.15e-137 +7.44e-124 0.776  0.776  0.100
115       0 2.9e-07 +1.84    +1.84      3.97e-07 +3.13e-136 +4.87e-137 +1.66e-124 0.777  0.777  0.100
116       0 8.8e-08 +1.84    +1.84      1.19e-07 +5.57e-136 +3.17e-136 +3.71e-125 0.777  0.777  0.100
117       0 2.6e-08 +1.84    +1.84      3.59e-08 +3.03e-136 +1.94e-136 +8.28e-126 0.777  0.777  0.100
118       0 7.9e-09 +1.84    +1.84      1.08e-08 +8.13e-137 +1.80e-136 +1.84e-126 0.777  0.777  0.100
119       0 2.4e-09 +1.84    +1.84      3.24e-09 +5.99e-136 +9.64e-137 +4.11e-127 0.777  0.777  0.100
120       0 7.2e-10 +1.84    +1.84      9.73e-10 +2.26e-136 +2.06e-136 +9.14e-128 0.778  0.778  0.100
121       0 2.2e-10 +1.84    +1.84      2.92e-10 +8.56e-136 +3.81e-136 +2.03e-128 0.778  0.778  0.100
122       0 6.5e-11 +1.84    +1.84      8.77e-11 +7.51e-136 +4.42e-136 +4.52e-129 0.778  0.778  0.100
123       0 1.9e-11 +1.84    +1.84      2.63e-11 +8.85e-136 +4.13e-136 +1.01e-129 0.778  0.778  0.100
124       0 5.8e-12 +1.84    +1.84      7.90e-12 +4.98e-136 +3.71e-136 +2.24e-130 0.778  0.778  0.100
125       0 1.7e-12 +1.84    +1.84      2.37e-12 +4.77e-136 +4.39e-136 +4.97e-131 0.778  0.778  0.100
126       0 5.2e-13 +1.84    +1.84      7.11e-13 +7.87e-136 +3.71e-136 +1.10e-131 0.778  0.778  0.100
127       0 1.6e-13 +1.84    +1.84      2.13e-13 +1.47e-136 +1.12e-136 +2.46e-132 0.778  0.778  0.100
128       0 4.7e-14 +1.84    +1.84      6.40e-14 +5.50e-136 +2.25e-136 +5.50e-133 0.778  0.778  0.100
129       0 1.4e-14 +1.84    +1.84      1.92e-14 +2.38e-136 +1.21e-136 +1.21e-133 0.778  0.778  0.100
130       0 4.2e-15 +1.84    +1.84      5.76e-15 +5.66e-136 +1.77e-136 +3.16e-134 0.778  0.778  0.100
131       0 1.3e-15 +1.84    +1.84      1.73e-15 +1.88e-136 +2.70e-136 +1.93e-134 0.778  0.778  0.100
132       0 3.8e-16 +1.84    +1.84      5.19e-16 +1.06e-135 +4.49e-136 +1.93e-134 0.778  0.778  0.100
133       0 1.1e-16 +1.84    +1.84      1.56e-16 +2.13e-136 +4.49e-136 +7.29e-134 0.778  0.778  0.100
134       0 3.4e-17 +1.84    +1.84      4.67e-17 +8.79e-136 +5.18e-136 +7.98e-134 0.778  0.778  0.100
135       0 1.0e-17 +1.84    +1.84      1.40e-17 +5.56e-136 +1.20e-136 +7.57e-134 0.778  0.778  0.100
136       0 3.1e-18 +1.84    +1.84      4.20e-18 +3.21e-136 +1.30e-136 +6.05e-134 0.778  0.778  0.100
137       0 9.3e-19 +1.84    +1.84      1.26e-18 +1.30e-136 +2.94e-136 +1.27e-133 0.778  0.778  0.100
138       0 2.8e-19 +1.84    +1.84      3.78e-19 +6.93e-136 +4.69e-136 +2.74e-133 0.778  0.778  0.100
139       0 8.4e-20 +1.84    +1.84      1.13e-19 +1.59e-136 +3.72e-136 +2.09e-133 0.778  0.778  0.100
140       0 2.5e-20 +1.84    +1.84      3.40e-20 +3.17e-136 +1.14e-136 +1.91e-133 0.778  0.778  0.100
141       0 7.5e-21 +1.84    +1.84      1.02e-20 +9.84e-136 +0.00     +1.06e-132 0.778  0.778  0.100
142       0 2.3e-21 +1.84    +1.84      3.06e-21 +4.28e-136 +0.00     +1.15e-132 0.778  0.778  0.100
143       0 6.8e-22 +1.84    +1.84      9.19e-22 +1.20e-136 +0.00     +4.95e-133 0.778  0.778  0.100
144       0 2.0e-22 +1.84    +1.84      2.76e-22 +3.46e-136 +3.39e-137 +1.33e-132 0.778  0.778  0.100
145       0 6.1e-23 +1.84    +1.84      8.27e-23 +8.21e-136 +1.37e-136 +1.18e-132 0.778  0.778  0.100
146       0 1.8e-23 +1.84    +1.84      2.48e-23 +4.89e-136 +1.26e-136 +1.52e-132 0.778  0.778  0.100
147       0 5.5e-24 +1.84    +1.84      7.44e-24 +4.39e-136 +9.00e-137 +1.11e-132 0.778  0.778  0.100
148       0 1.6e-24 +1.84    +1.84      2.23e-24 +6.16e-136 +8.19e-137 +5.68e-133 0.778  0.778  0.100
149       0 4.9e-25 +1.84    +1.84      6.70e-25 +9.19e-136 +4.69e-136 +3.13e-132 0.778  0.778  0.100
150       0 1.5e-25 +1.84    +1.84      2.01e-25 +6.83e-136 +4.32e-136 +1.99e-132 0.778  0.778  0.100
151       0 4.4e-26 +1.84    +1.84      6.03e-26 +2.69e-136 +1.71e-136 +2.12e-132 0.778  0.778  0.100
152       0 1.3e-26 +1.84    +1.84      1.81e-26 +8.64e-136 +2.15e-136 +4.23e-132 0.778  0.778  0.100
153       0 4.0e-27 +1.84    +1.84      5.43e-27 +2.11e-136 +2.48e-136 +8.79e-132 0.778  0.778  0.100
154       0 1.2e-27 +1.84    +1.84      1.63e-27 +5.05e-136 +2.04e-136 +5.63e-132 0.778  0.778  0.100
155       0 3.6e-28 +1.84    +1.84      4.88e-28 +6.08e-136 +3.21e-136 +7.22e-132 0.778  0.778  0.100
156       0 1.1e-28 +1.84    +1.84      1.47e-28 +3.22e-136 +3.50e-136 +1.84e-131 0.778  0.778  0.100
157       0 3.2e-29 +1.84    +1.84      4.40e-29 +2.57e-136 +1.04e-136 +1.74e-131 0.778  0.778  0.100
158       0 9.7e-30 +1.84    +1.84      1.32e-29 +7.44e-136 +2.75e-136 +1.38e-131 0.778  0.778  0.100
159       0 2.9e-30 +1.84    +1.84      3.96e-30 +2.35e-136 +8.75e-137 +3.78e-131 0.778  0.778  0.100
160       0 8.7e-31 +1.84    +1.84      1.19e-30 +1.01e-135 +4.53e-136 +1.55e-131 0.778  0.778  0.100
-----found primal-dual optimal solution-----

primalObjective = 1.8402657631320492466880401717305542005635853203028255646576190613343016672653733682604
9865612094019
02111601886294781720509282804555570379
dualObjective   = 1.8402657631320492466880401717292438808478490702030795792640645597275696782038955172911
6356865203683
72132484769504674081833414899815973672
dualityGap      = 3.5601371877563627014999905963533505072344274310916883129388560704189497462085352238569
5653183532885
8564162988744217253002402735522747108e-31
primalError     = 5.8099713533981309349705817368413610795466571964655742634757213231464580718871358515751
1697443878587
3209402480858514950312788893168634202e-136
dualError       = 1.4684137819353812633162747029681310546719677842025686726937628183154614032611825901779
694044217367
80648081630993974340543568124277287755e-131

Saving solution to       : "test/test_out"
```

For larger, parallel runs, the exact solution that SDPB computes can vary between different machines, or even different configurations on the same run. For example, the

step size is computed by adding up many other numbers. The order in which these numbers are added can change the final sum in the lowest significant bits. This does not make the solver unstable, but it does introduce a minor amount of variation. The solver still converges to the same answer up to the error thresholds and duality gap, but the actual value of the residual errors and gap will be different.

5.2   Termination

The possible termination reasons for `SDPB` are as follows

`found primal-dual optimal solution`
> Found a solution for x, X, y, Y that is simultaneously primal feasible, dual feasible, and optimal.

`found primal feasible solution`
> Found a solution for x, X that is primal feasible. `SDPB` will only terminate with this result if the option `--findPrimalFeasible` is specified.

`found dual feasible solution`
> Found a solution for y, Y that is dual feasible. `SDPB` will only terminate with this result if the option `--findDualFeasible` is specified.

`primal feasible jump detected`
> A Newton step with primal step length $\alpha_{\mathcal{P}}$ just occurred, without resulting in a primal feasible solution. (Usually this means one should increase `precision`.)

`dual feasible jump detected`
> A Newton step with dual step length $\alpha_{\mathcal{D}}$ just occurred, without resulting in a dual feasible solution. (Usually this means one should increase `precision`.)

`maxIterations exceeded`
> SDPB has run for more iterations than specified by the option `--maxIterations`.

`maxRuntime exceeded`
> SDPB has run for longer than specified by the option `--maxRuntime`.

`maxComplementarity exceeded`
> $\mu = \mathrm{Tr}(XY)/\dim(X)$ exceeded the value specified by `--maxComplementarity`. This might indicate that the problem is unbounded and no optimal solution will be found.

When using `SDPB` to determine primal or dual feasibility, one can specify the options `--findPrimalFeasible` or `--findDualFeasible`. This will cause the solver to terminate immediately once the primal or dual errors are sufficiently small. This often occurs immediately after the primal or dual step lengths become equal to 1. A step length of 1 means that the solver has found a Newton step that exactly solves the primal or dual constraints, while preserving positive-semidefiniteness of X, Y. Sometimes a step length of 1 does not result

in sufficiently small primal/dual errors. This is indicative of numerical instabilities and usually means `precision` should be increased. The options `--detectPrimalFeasibleJump` and `--detectPrimalFeasibleJump` cause `SDPB` to terminate if a step length of 1 occurs without resulting in primal/dual feasibility. If desired, one can then restart the solver with a higher value of `precision`.

5.3 Output File

Listing 6: Contents of the output file `test_out/out.txt` corresponding to listing 4. Decimal expansions have been truncated for brevity. `Mathematica` uses `*^` instead of the character `e` for scientific notation. Thus, the output format is not quite suitable for import into `Mathematica` without modification. This could be changed in future versions.

```
terminateReason = "found primal-dual optimal solution";
primalObjective = 1.840265763132049246688040171730554200563585320302825564 6...;
dualObjective   = 1.8402657631320492466880401717292438808478490702030795792...;
dualityGap      = 3.560137187756362701499990596353350507234427431091688312 9...e-31;
primalError     = 5.809971353339813093497058173684136107954665719646557426 34...e-136;
dualError       = 1.468413781935381263316274702968131054671976778420256867 2...e-131;
Solver runtime  = 0;
```

The output file `test_out/out.txt` corresponding to listing 4 is shown in listing 6. It includes the reason for termination, the final primal/dual objective values, the final duality gap, the final primal/dual errors, and the total runtime. The vector y is saved in `test_out/y.txt`, and the two blocks of the x vector are saved in `test_out/x_0.txt` and `test_out/x_1.txt`. To write additional files for the matrices X and Y, add the option `--writeSolution="x,y,X,Y"`.

The value of y gives the solution to our optimization problem. The function

$$1 + x^4 + (-1.840265763084)\left(\frac{x^4}{12} + x^2\right) \tag{5.1}$$

is plotted in figure 1. The zero near $x = 1$ shows that y is optimal.

5.4 Checkpoints

Every `checkpointInterval`, `SDPB` saves a new checkpoint in a directory with the `.ck` extension. `SDPB` also saves a checkpoint after termination, provided the option `--noFinalCheckpoint` is not specified.

A checkpoint file encodes the values of x, X, y, Y. If `SDPB` detects an existing checkpoint file on startup, it will use those values of x, X, y, Y as initial conditions in the solver. Thus, `SDPB` can be stopped and started at will without losing progress.

A typical workflow for long-running computations on shared machines is to specify a moderate `checkpointInterval` (e.g. one hour) and a somewhat larger `maxRuntime` (e.g.
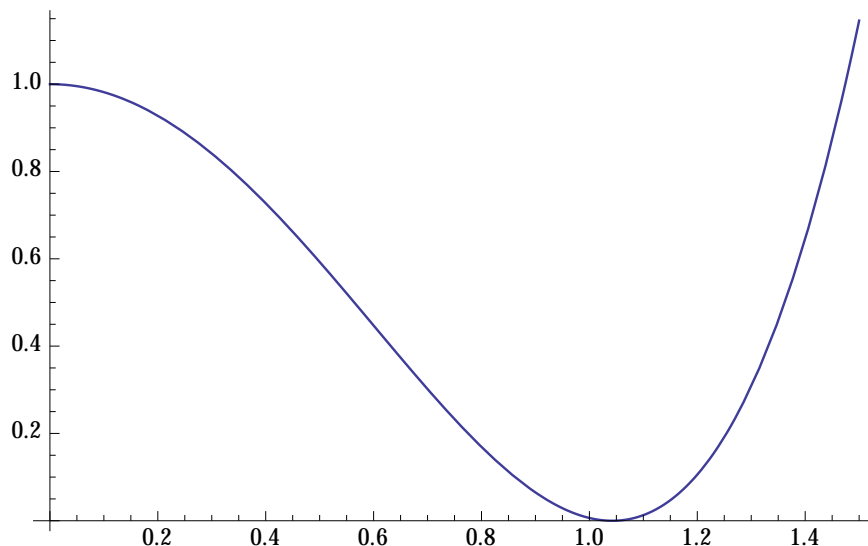
Figure 1: A plot of $1 + x^4 + y\left(\frac{x^4}{12} + x^2\right)$ with $y = -1.840265763084$ equal to its optimal value. The zero near $x = 1$ shows that $-y$ cannot be further increased without violating the positivity constraint.

12 hours). SDPB will terminate after 12 hours and can then be restarted without losing progress. If SDPB is killed prematurely, then at most 1 hour of progress will be lost. This pattern of restarting gives other users chances to run their processes. It can be sustained indefinitely, allowing extremely long computations.

Checkpoints are written in binary format to conserve space and speed up loading and unloading. If you specify the `--writeSolution="x,y,X,Y"` option, the output directory can also be used to restart a computation with the `-i` option. It will not be bitwise identical to restarting from a binary checkpoint, but it should be very, very, very close.

Text checkpoints can be useful if you want to solve a different system by starting closer to previously solved system. You can also use it to continue a calculation with a different number of cores, or even on a different machine. Using the previous input as an example,

```
$ ./build/sdpb -s test/test --noFinalCheckpoint --procsPerNode=1 --writeSolution="x,y,X,Y"
$ ./build/sdpb -s test/test --noFinalCheckpoint --procsPerNode=1 -i test/test\_out
```

the second calculation will start from the end of the first calculation.

6 Attribution

If you use SDPB in work that results in publication, please cite [1]. Depending on how SDPB is used, the following sources might also be relevant:

- The first use of semidefinite programming in the bootstrap [3].

- The generalization of semidefinite programming methods to arbitrary spacetime dimension [4].

- The generalization of semidefinite programming methods to arbitrary systems of correlation functions [5].

7   Acknowledgements

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