Cálculo II (Tristemonte)

(0) Calcular las derivadas de las siguientes funciones:

a) $f'(x) = (33-2x)^{\frac{4}{3}}$
b) $f(x) = e^{2x}$
c) $f(x) = 2^x$
d) $f(x) = \ln(7-x)$
e) $f(x) = \ln(x^2+3x+4)$
f) $f(x) = \ln(e^x + e^{-x})$
g) $f(x) = \ln(\cos(x) + \operatorname{sen}(x))$
h) $f(x) = \dfrac{\cos(x)}{\operatorname{sen}(x)}$

0) a) $(33-2x)^{\frac{4}{3}} \rightarrow u^{\frac{4}{3}}$ $\quad u = 33-2x$

$h(g(x)) \rightarrow f'(x) = h'(g(x)) \cdot g'(x)$

$h = -2x+33 \qquad h' = -2$

$g = u^{\frac{4}{3}} \qquad g' = \frac{4}{3} u^{\frac{4}{3}-1} = \frac{4}{3} u^{\frac{1}{3}}$

$\rightarrow f'(x) = \frac{4}{3}(33-2x)^{\frac{1}{3}} \cdot (-2)$

b) $f(x) = e^{2x} \rightarrow h'(g(x)) \cdot g'(x)$

$h = e^x \qquad h' = e^x$

$g = 2x \qquad g' = 2$

$f'(x) = e^{2x} \cdot (2)$

c) $2^x = f'(x) = 2^x \cdot \ln(2)$

d) $\ln(7-x)$

$f' = h'(g(x)) \cdot g'(x)$

$\rightarrow h = \ln(x) \rightarrow h' = \frac{1}{x}$

$g = 7-x \rightarrow g' = -1$

$\rightarrow f'(x) = \frac{1}{7-x} \cdot (-1) = -\frac{1}{7-x}$

e) $f(x) = \ln(x^2 + 3x + 4)$

$f'(x) = h'(g(x)) \cdot g'(x)$

$h = \ln(x) \Rightarrow h' = \frac{1}{x}$

$g = x^2 + 3x + 4 \qquad g' = 2x + 3$

$f'(x) = \frac{1}{x^2 + 3x + 4} \cdot 2x + 3$

$\frac{d}{dx}\left(\ln\left(x^2 + 3x + 4\right)\right)$

Pasos Gráfica Relacionado Ejemplos

$\frac{d}{dx}\left(\ln\left(x^2 + 3x + 4\right)\right)$

Solución

$\frac{2x + 3}{x^2 + 3x + 4}$

h) $f(x) = \frac{\cos(x)}{\operatorname{sen}(x)}$

$\frac{f(x)}{g(x)} = \frac{h' \cdot g - g' \cdot h}{g^2}$

$h = \cos(x) \qquad h'(x) = -\operatorname{Sen}(x)$

$g = \operatorname{Sen}(x) \qquad g' = \cos$

$f'(x) = \frac{-\operatorname{Sen} \cdot \operatorname{Sen} - \cos \cdot \cos}{\cos^2}$

(1) Dar las primitivas de las siguientes funciones:

a) $g(x) = x^3 - 5x$ c) $g(x) = \operatorname{sen}(2x)$ e) $g(x) = x^{3/2}$
b) $g(x) = e^{0.3x}$ d) $g(x) = 2x\cos(x^2)$ f) $g(x) = \sqrt{x+2}$

a) $g(x) = \frac{x^4}{3} - \frac{5x^2}{2}$

b) $g(x) = e^{0,3x} = e^{\frac{3}{10}x} = \frac{e^x}{\frac{3}{10}} = e^x : \frac{3}{10} = \frac{10e^x}{3}$

c) $g(x) = \operatorname{Sen}(2x) = -\frac{1}{2}\cos(2x)$

3) a) $\int e^{2x}\,dx$ d) $\int \frac{dx}{7-x}$ g) $\int \frac{\cos(x)-\operatorname{sen}(x)}{\cos(x)+\operatorname{sen}(x)}\,dx$

b) $\int 2^x\,dx$ e) $\int \frac{2x+3}{x^2+3x+4}\,dx$ h) $\int \frac{1}{\operatorname{sen}^2(x)}\,dx$

c) $\int \sqrt[3]{33-2x}\,dx$ f) $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

a) $\int e^{2x}\,dx =$

$u = 2x$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\int e^u \frac{du}{2} = \int e^u \frac{1}{2} \cdot du = \frac{1}{2}\int e^u\,du$

$= \frac{1}{2} \cdot e^u + c = \frac{1}{2} \cdot e^{2x} + c = \frac{e^{2x}}{2} + c$

b) $\int 2^x\,dx = \frac{2^x}{\ln(2)} + c$

c) $\int \sqrt[3]{33-2x}\,dx =$

$u = -2x+33$

$\frac{du}{dx} = -2x+33$

$du = (-2x+33)dx$

$du = -2\,dx$

$\frac{du}{-2} = dx$

$\int \sqrt[3]{u}\,\frac{du}{2} = \int \sqrt[3]{u} \cdot \frac{du}{2} = \int \sqrt[3]{u} \cdot \frac{1}{2} \cdot du$

$\frac{1}{2}\int \sqrt[3]{u} \cdot du = \frac{1}{2} \cdot \frac{3u^{\frac{4}{3}}}{4} + c = \frac{3(-2x+33)^{\frac{4}{3}}}{8} + c$

d) $\int \frac{dx}{7-x}$

$u = 7-x \rightarrow \frac{du}{dx} = 7-x$

$du = 7-x\,dx$

$du = -1\,dx$

$-du =$

$\int -\frac{du}{u} = \int -\frac{1}{u} \cdot du$

$= -1\int \frac{1}{u}\,du = -1\ln(|u|) + c = -1\ln(|7-x|) + c$

e) $\int \frac{2x+3}{x^2+3x+4}\,dx$

$u = x^2+3x+4$

$\frac{du}{dx} = x^2+3x+4$

$du = (2x+3)\cancel{dx}$

$\frac{du}{2x+3} =$

$\int \frac{2x+3}{u}dx = \int \frac{\cancel{2x+3}}{u} \cdot \frac{du}{\cancel{2x+3}}$

$\int \frac{1}{u} \cdot du = \ln(|x^2+3x+4|) + c$

(4) Sin realizar el cálculo de la integral, justificar las siguientes igualdades y desi

a) $\int_{-\pi}^{\pi} \operatorname{sen}(2x)\,dx = 0$ b) $\pi/6 \le \int_{\pi/6}^{\pi/2} \operatorname{sen}(x)\,dx \le \pi/3$

c) $\int_1^2 \sqrt{5-x}\,dx \ge \int_1^2 \sqrt{x+1}\,dx$

a) $\operatorname{Sen}(\pi) = 0$, $\operatorname{Sen}(-\pi) = 0$, verdader

$30°$

b) $\frac{\pi}{6} \le 90 - 30 \le 60$ c) $\sqrt{4} - \sqrt{6} \ge \sqrt{3} - \sqrt{2}$

$30 \le 60 \le 60$ ✓

(5) Calcular la derivada de las siguientes funciones donde sea posible:

a) $f(x) = \int_0^x \frac{\operatorname{sen}(t^2)}{1+\cos^2 t}\,dt$ b) $f(x) = \int_0^{x^2} \frac{e^{t^2}+1}{\sqrt{1-t^2}}\,dt$ c) $f(x) = \int_{\sqrt{x}}^{x^3} \frac{t+1}{\sqrt{1+2^t}}\,dt$

a) $F(x) = \int_0^x \frac{\operatorname{Sen}(x^2)}{1+\cos^2(x)}\,dx$

TFC2
No entendí enunciado

(6) Calcular las siguientes integrales usando el Teorema Fundamental del Cálculo:

a) $\int_1^2 2^x\,dx$ c) $\int_1^5 \frac{dx}{7-x}$ e) $\int_{\ln 2}^{\ln 3} \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

b) $\int_3^5 \sqrt[3]{33-2x}\,dx$ d) $\int_0^1 \frac{2x+3}{x^2+3x+4}\,dx$ f) $\int_0^{\pi/2} \frac{\cos(x) - \operatorname{sen}(x)}{\cos(x) + \operatorname{sen}(x)}\,dx$

a) $\int_1^2 2^x\,dx = \left.\frac{2^x}{\ln(2)}\right|_1^2 = \frac{2^2}{\ln(2)} - \frac{2^1}{\ln(2)} = \frac{2}{\ln(2)}$

b) $\int_3^5 \sqrt[3]{33-2x}\,dx =$

$u = 33-2x$

$\frac{du}{dx} = 33-2x$

$du = 33-2x\,dx$

$\frac{du}{-2} = dx$

$\int_3^5 \sqrt[3]{u} \cdot \left(\frac{du}{-2}\right) = -\frac{1}{2} \int_3^5 \sqrt[3]{u} \cdot du$

$-\frac{1}{2} \cdot \left.\frac{3u^{\frac{4}{3}}}{4}\right|_3^5$

2. Cambio de límites:

Cuando $x = 3$:

$u = 33 - 2(3) = 33 - 6 = 27$

Cuando $x = 5$:

$u = 33 - 2(5) = 33 - 10 = 23$

Perdon colgué

5) $-\frac{1}{2} \cdot \frac{3(33-2(5))}{4} = -\frac{69}{8}$

3) $-\frac{1}{2} \cdot \frac{3(33-2(3))}{4} = -\frac{27}{8}$

$= -\frac{27}{8} + \frac{69}{8} = \frac{42}{8} = \sim 5,25$

c) $\int_1^5 \frac{dx}{7-x}$

$u = 7 - x \to 7-5=2$
$7-1=6$

$\frac{du}{dx} = 7-x$
$du = -dx$
$-du = dx$

c) $\int_1^5 \frac{dx}{u} = \int_2^6 \frac{1}{u}\cdot(-du)$

$-1\int_u^6 \frac{1}{u}\cdot du = -1\ln(|u|)\Big|_2^6$

6) $-1\cdot\ln(|7-6|) = -1\cdot\ln(|1|)$

2) $-1\cdot\ln(|7-2|) = -\ln(|5|)$

$\to \int_1^5 \frac{dx}{7-x} = -\ln(|1|) + \ln(|5|)$

d) $\int_0^1 \frac{2x+3}{x^2+3x+4}\,dx$

$u = x^2+3x+4 \to 1^2+3(1)+4=8$
$0^2+3(0)+4=4$

$\frac{du}{dx} = x^2+3x+4$
$du = 2x+3\,dx$
$\frac{du}{2x+3} = dx$

$\int_0^1 \frac{2x+3}{u}\,dx = \int_4^8 \frac{\cancel{2x+3}}{u}\cdot\frac{du}{\cancel{2x+3}}$

$\int_4^8 \frac{1}{u}\cdot du = \ln(|u|)\Big|_3^8$

8) $\ln(|x^2+3x+4|) = \ln(|64+24+4|) = \ln(|92|)$

4) $\ln(|x^2+3x+4|) = \ln(|16+12+4|) = \ln(|32|)$

$\int_4^8 \frac{2x+3}{x^2+3x+4} = \ln(|92|) - \ln(|32|)$

(7) Calcular las siguientes integrales:

a) $\int x\,e^x\,dx$
b) $\int_{-1}^{1} (1-2x)\,e^{-2x}\,dx$
c) $\int x^2\cos(x)\,dx$
d) $\int_{\pi/4}^{\pi/2} \frac{x\,dx}{\mathrm{sen}^2(x)}$
e) $\int_3^9 x\ln(x-1)\,dx$
f) $\int \ln(x^2+1)\,dx$
g) $\int_0^2 x\ln(x^2+4)\,dx$
h) $\int e^{-x}\,\mathrm{sen}(2x)\,dx$
i) $\int_0^{2\pi} \cos^4(x)\,dx$

a) $\int x\,e^x\,dx$ Integración Por Parte ♥

$F(x) = x$ $F' = 1$ $g(x) = e^x$ $g'(x) = e^x$

$\to x\cdot e^x - \int 1\cdot e^x\,dx = xe^x - e^x + C$

$$\int_{-1}^{1} (1-2x)\ e^{-2x}\ dx$$

$$\int (F(x) \mp g(x))\, h(x)\, dx = \int F(x) \cdot h(x) \mp \int g(x) \cdot h(x)\, dx$$

$$\Rightarrow \int_{-1}^{1} 1 \cdot e^{-2x}\, dx - \int_{-1}^{1} 2x \cdot e^{-2x}\, dx$$

$$\int_{-1}^{1} e^{-2x}\, dx \ {}^{*} - 2\int_{-1}^{1} x\, e^{-2x}\, dx \ {}^{*}$$

* Se tiene:

$$-2\int_{-1}^{1} x e^{-2x}\, dx = -2 + \frac{x^2}{2} \cdot \left(-\frac{1}{2} \cdot e^{-2x}\right) = -2 + \frac{x^2}{2} \cdot \left(-\frac{e^{-2x}}{2}\right) = -2 - \frac{e^{-2x}\, x^2}{4}$$

$$-2 - \frac{e^{-2x}\, x^2}{4} \Bigg|_{-1}^{1} = -2 - \frac{e^{-2(1)}\, 1^2}{4} = -\cancel{2} - \frac{e^{-2}}{\cancel{4}_{2}} = -1 - \frac{e^{-2}}{2}$$

$$-1)\ -2 - \frac{e^{-2(-1)}(-1)^2}{4} = -\cancel{2} - \frac{e^{2}}{\cancel{4}} = -1 - \frac{e^{2}}{2}$$

$$\int_{-1}^{1} x e^{-2x}\, dx = -\cancel{1} - \frac{e^{2}}{2} + \cancel{1} - \frac{e^{-2}}{2} = -\frac{e^{2}}{2} - \frac{e^{-2}}{2}$$

$$* \int_{-1}^{1} e^{-2x}\, dx = -\frac{e^{-2x}}{2} \Bigg|_{-1}^{1} =$$

$$1)\ -\frac{e^{-2(1)}}{2} = -\frac{e^{-2}}{2}$$

$$-1)\ -\frac{e^{-2(-1)}}{2} = \frac{-e^{2}}{2}$$

$$\Rightarrow \int_{-1}^{1} e^{-2x}\, dx = -\frac{e^{2}}{2} + \frac{e^{-2}}{2}$$

$$\Rightarrow \int_{-1}^{1} (1-2x)\, e^{-2x}\, dx = -\frac{e^{2}}{2} + \frac{e^{-2}}{2} + \frac{e^{2}}{2} - \frac{e^{-2}}{2}$$

Creo q ✓

c) $\int x^2 \cos(x)\ dx$

Integral. P.P

$$\int F(x)\cdot g'(x)\ dx = F(x)\cdot g(x) - \int F'(x)\cdot g(x)dx$$

$$F(x) = x^2 \to F'(x) = 2x,\quad g(x) = \cos(x)$$

$$\to x^2\cdot \cos(x) - \int 2x\cdot \cos(x) dx = x^2\cdot\cos(x) - 2\int x\cdot\cos(x)dx$$

$$\int x^2 \cos(x) dx = x^2\cdot\cos(x) - 2\cdot \frac{x^2}{2}\cdot \operatorname{Sen}(x)$$ ✓

$$\int_{\pi/4}^{\pi/2} \frac{x\ dx}{\operatorname{sen}^2(x)}$$

P.P

$$\frac{1}{\operatorname{Sen}^2(x)} = \csc^2(x)$$

$$\int_{\frac{\pi}{4}}^{\pi/2} \frac{x}{\operatorname{Sen}^2(x)}\cdot dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{u}{\operatorname{Sen}^2(x)}\cdot dx$$

$u = x$

$\frac{du}{dx} = x$

$du = 1dx$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x dx}{\operatorname{Sen}^2(x)} dx = -x\cot(x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot(x)dx$$

*

$$-x\cot(x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \ln(|\operatorname{sen}|)$$

$\frac{\pi}{2}$

$*\ -\frac{\pi}{2}\cot\left(\frac{\pi}{2}\right) =$

$\frac{\pi}{4}\Big)\ -\frac{\pi}{4}\cot\left(\frac{\pi}{4}\right) =$

$$\to -\frac{\pi}{4}\underbrace{\cot\left(\frac{\pi}{4}\right)}_{1} + \frac{\pi}{2}\underbrace{\cot\left(\frac{\pi}{2}\right)}_{0} + \ln(|\operatorname{sen}(x)|)$$

(8) Calcular las siguientes integrales:

a) $\int_0^1 e^{\sqrt{x}}\,dx$ d) $\int \frac{1}{x\ln(x)}\,dx$ g) $\int e^x(1-e^x)^{-1}\,dx$

b) $\int \operatorname{sen}(\sqrt{x})\,dx$ e) $\int_0^1 \arccos(x)\,dx$ h) $\int \frac{dx}{x\sqrt{x-1}}$

c) $\int_0^1 (2x+1)\ln(x+1)\,dx$ f) $\int_0^1 x^3 e^{x^2}\,dx$ i) $\int \operatorname{sen}^3(x)\,dx$

$u=\sqrt{x} \rightarrow u^2=x$

$\frac{du}{dx}=u^2$

$dx=2u\,du$

$x=0,\ u=\sqrt{0}$

$x=1,\ u=\sqrt{1}$

$$\int_0^1 e^{\sqrt{x}}\,dx = \int_0^1 e^{x^{\frac{1}{2}}}\,dx$$

$$=\int_0^1 e^u\cdot 2u\,du = 2\int_0^1 e^u\cdot u\,du$$

integracion Por Partes

$F(x)=u$, $F'(x)=1$

$g(x)=e^u$, $g'(x)=e^u$

$$\rightarrow 2\left[u\cdot e^u-\int_0^1 1\cdot e^u\,du\right]$$

$$2\left[u\cdot e^u-e^u\right]_0^1$$

$$2\left[e^u(u-1)\right]_0^1 \leftarrow \text{F.c } e^u$$

$$2\left[\underbrace{e^1(1-1)}_{0}-e^0(0-1)\right]$$

$$2[0+1]=2$$

b) $\int \operatorname{sen}(\sqrt{x})\,dx$

$u=\sqrt{x} \rightarrow u^2=x$

$\frac{dx}{du}=u^2$

$dx=2u\,du$

$$=\int \operatorname{sen}(u)\,2u\,du$$

$$=2\int \operatorname{sen}(u)\cdot u\,du$$

Aplico sustitución

$F(x)=u$ $F'(x)=1$

$g(x)=\operatorname{sen}(u)$ $g'(x)=\cos u$

$\rightarrow$

$$2\left[\operatorname{sen}(u)\cdot u-\int 1\,\operatorname{sen}(u)\,du\right]$$

$$2\left[\operatorname{sen}(u)\cdot u-\int \operatorname{sen}(u)\,du\right]$$

$$2\left[\operatorname{sen}(u)\cdot u-\cos(u)\,du\right] = 2\operatorname{sen}(\sqrt{x})\cdot\sqrt{x}-2\cos(\sqrt{x})+c$$

c) $\int_0^1 (2x+1)\ln(x+1)\,dx$

$u = x+1 \rightarrow \frac{du}{dx} = x+1$ $\quad x=1,\ 1+1=2$

$x = u-1$ $\quad \frac{du}{du} = 1\frac{dx}{dx} \rightarrow$ $\quad x=0,\ 1+0=1$

$du = dx$

$\int_1^2 (2x+1)\ln(u)\,du = \int_1^2 \ln(u)(2(u-1)+1)$
$(2u-1)$

Por sustitución

$f(x) = \ln(u)$ $\quad f'(x) = \frac{1}{u}$

$g'(x) = u^2+u$ $\quad g'(x) = 2u-1$

quedando:

$\ln(u)\cdot(u^2-u)\Big|_1^2 - \int_1^2 \frac{1}{u}\cdot(u^2-u)\,du$ $\quad \rightarrow u^2-1$

$\left(\left(\ln(2)\cdot(4-2)\right)-\left(\ln(1)\cdot(1+1)\right)\right) - \int_1^2 \frac{u^2-ux}{ux}\,du$

$(\ln(2)\cdot 2) - 0 - \int_1^2 u^2-u\,du$

$\ln(2)\cdot 2 - \frac{u^2}{2}-u\Big|_1^2 = \ln(2)\cdot 2 - \left(\frac{4}{2}-2\right) - \left(\frac{1}{2}-1\right)$

$2\ln(2) - \frac{1}{2}$

d) $\int \frac{1}{x\ln(x)}\,dx$

$u = x\ln(x)$

$\frac{du}{dx} = x\ln(x) \rightarrow du = x\ln(x)\,dx$

$du = 1.\frac{1}{x}\,dx$

$\int \frac{1}{u}\,du$

$\ln(|u|) = \ln(|x\ln(x)|) + c$

(9) Trazar la región limitada por las curvas dadas y calcular su área:

a) $y = 4x^2, \quad y = x^2+3$

b) $y = \cos(x), y = \operatorname{sen}(x),\ x = 0,\ x = \pi/2.$

c) $y = |x|, \quad y = (x+1)^2 - 7, \quad x = -4$

$f(x) = 4x^2$, $g(x) = x^2+3$ $\qquad \int_a^b (g(x)-f(x))\,dx$ y $g > f$. (debe dar 4)

① obtengo el a y b de la integral:

$4x^2 = x^2+3$

$x^2-4x^2 = 3$

$3x^2 = 3$

$x = \pm 1 \rightarrow [-1, 1]$

$\rightarrow \int_{-1}^1 ((x^2+3)-4x^2)\,dx$

$$\int_{-1}^{1} x^2+3\,dx - \int_{-1}^{1} 4x^2\,dx$$

② teorema Fundamental del cálculo. Integro con las derivadas Primitivas.

$$\frac{x^3}{3}+3x\Big|_{-1}^{1} - 4\cdot\frac{x^3}{3}\Big|_{-1}^{1}$$

$$\left(\left(\frac{1^3}{3}+3(1)\right)-\left(\frac{(-1)^3}{3}-3\right)\right)-4\left(\left(\frac{(1)^3}{3}\right)-\frac{(-1)^3}{3}\right)$$

$$\left(\left(\frac{1}{3}+\frac{9}{3}\right)-\left(\frac{-1}{3}-\frac{9}{3}\right)\right)-4\left(\left(\frac{1}{3}\right)+\frac{1}{3}\right)$$

$$\left(\left(\frac{1}{3}+\frac{9}{3}\right)-\left(\frac{-1}{3}-\frac{9}{3}\right)\right)-4\left(\left(\frac{1}{3}+\frac{1}{3}\right)\right)$$

$$\frac{2}{3}+6 \quad \frac{8}{3}=4$$

(10) Calcular las siguientes integrales

a) $\displaystyle\int_2^4 \frac{x^2+4x+24}{x^2-4x+8}\,dx$

1) Simplifico las cuadráticas completando cuadrados

1) $x^2-4x+8=(x-x_v)^2+y_v=(x-2)^2+4$

$x_v=\frac{+4}{2\cdot 1}=2$, $y_v=2^2-8+8=4$

a) $x^2+4x+24=(x^2-4x+8)+8x+16=((x-2)^2+4)+8x+16$

2) Reescribo

$$\int_2^4\left(\frac{((x-2)^2+4)+8x+16}{(x-2)^2+4}\right)=\int_2^4 1\,dx+\int_2^4\frac{8x+16}{(x-2)^2+4}dx$$

$$= \int_2^4 1dx + \int_2^4 \frac{8x+16}{(x-2)^2+4}dx$$

$x=4 \to 4-2=2,\quad x=2 \to 2-2=0$

$u = x-2 \to \frac{du}{dx} = x-2$

$\to u-2$

$du = 1dx$

$$x\Big|_2^4 = 4-2 = \boxed{2}$$

$$\int_0^2 \frac{8(u-2)+16}{u^2+4}du = \int_0^2 \frac{8u+32}{u^2+4}$$

3) Simplifico aún más

$$8\int_0^2 \frac{u}{u^2+4}du + 32\int_0^2 \frac{1}{u^2+4}du$$

$v = u^2+4$

$\frac{dv}{du} = u^2+4$

$dv = 2u\,du$

quedando:

$$4\ln|u^2+4|\Big|_0^2$$

$$32 \cdot \frac{1}{4}\text{Arctg}\left(\frac{u}{2}\right)\Big|_0^2$$

quedando (No lo calcule xd)

$$2 + 4\ln(2) + 2\pi$$

b) $\int_0^2 \frac{x-1}{x^2+4}\,dx$ $\to \frac{P(x)}{q(x)}$ Notemos q: $P<q \to$ integración de F racionales usando Funciones simples.

Esta integral, se puede escribir como:

$\int_0^2 \frac{x-1}{(x-2)(x+2)}$ Entonces, puedo: $\frac{A_1}{(x-2)} + \frac{A_2}{(x+2)} = \frac{A_1(x+2)+A_2(x-2)}{(x-2)(x+2)}$

$$\to \frac{x-1}{\cancel{(x-2)(x+2)}} = \frac{A_1(x+2)+A_2(x-2)}{\cancel{(x-2)(x+2)}} \to x-1 = A_1(x+2)+A_2(x-2)$$

$$x-1 = A_1x+2A_1+A_2x-2A_2$$

Sistema $\begin{cases} x-1 = (A_1+A_2)x+(-2A_2+2A_1) \end{cases}$

$$\begin{cases} A_1 + A_2 = 1 \\ +2A_1 - 2A_2 = -1 \end{cases} \rightarrow \quad A_1 = 1 - A_2 \quad \rightarrow A_1 = 1 - \frac{4}{3} = -\frac{1}{3}$$

$1 - \frac{3}{4} = \frac{1}{4}$

$$2(1-A_2) - 2A_2 = -1$$

$$2 - 2A_2 - 2A_2 = -1$$

$$-4A_2 = -3 \rightarrow A_2 = \frac{4}{3}$$

(ta mal queda $\frac{3}{4}$)

quedando de la forma:

$$\frac{-\frac{1}{3}}{(x-2)} + \frac{\frac{4}{3}}{(x+2)}$$

$$\int_0^2 \frac{-\frac{1}{3}}{(x-2)} + \int_0^2 \frac{\frac{4}{3}}{(x+2)} = -\frac{1}{3} \cdot \int_0^2 \frac{1}{(x-2)} dx + \frac{4}{3} \cdot \int_0^2 \frac{1}{(x+2)}$$

$u = x-2$, $x = 2$, $2-2=0$; $x = 0$, $0-2=-2$

$\frac{du}{dx} = x-2$.

$du = dx$

$u = x+2$ $x=2$, $2+2=4$; $x=0$, $0+2=2$

$\frac{du}{dx} = x+2$, $du = dx$

$$-\frac{1}{3} \cdot \int_{-2}^{0} \frac{1}{u} du + \frac{4}{3} \cdot \int_2^4 \frac{1}{u} du$$

$$-\frac{1}{3} \ln(|u|)\Big|_{-2}^{0} + \frac{4}{3} \cdot \ln(|u|) du \Big|_2^4 -$$

¿?

$$-\frac{1}{3} \ln(|0-2|) - \frac{1}{3} \cdot \ln(|-4|) + \frac{4}{3} \ln(|4+2|) - \frac{4}{3} \ln(|2+2|)$$

$$-\frac{\ln(|-2|)}{3} - \frac{\ln(|-4|)}{3} + \frac{4\ln(|6|)}{3} - \frac{4\ln(|4|)}{3}$$

11) La sustitución $t = \tan\left(\frac{x}{2}\right)$, o equivalentemente, $x = 2\arctan(t)$, transforma cualquier integral que involucre sólo senos y cosenos vinculados por suma, producto o cociente, en la integral de una función racional. Verificar que con esta sustitución resulta

$$\cos(x) = \frac{1-t^2}{1+t^2}, \qquad \operatorname{sen}(x) = \frac{2t}{1+t^2} \quad \text{y} \quad dx = \frac{2}{1+t^2}dt.$$

Utilizar esta sustitución en los siguientes casos:

$$\text{a)} \int_0^{\pi/2} \frac{2}{1+\cos(x)}\,dx \qquad \text{b)} \int_{\pi/3}^{\pi/2} \frac{1}{\operatorname{sen}(x)}\,dx$$

$$\cos(x) = \frac{1 - \tan^2(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}, \quad \operatorname{Sen}(x) = \frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}, \quad dx = \frac{2}{1+\tan^2(\frac{x}{2})} dt$$

Se verifica poniendo $x = 2\arctan(t)$

quedando: (no pide resolver bien!)

$$\text{a)} \int_0^{\frac{\pi}{2}} \frac{2}{1+\frac{1-\tan^2(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}} + \frac{2}{1+\tan^2(\frac{x}{2})} dt \quad \text{y} \quad \text{b)} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}} + \frac{2}{1+\tan^2(\frac{x}{2})} dt$$

(13) Determinar si las siguientes integrales impropias convergen y en tal caso calcularlas.

a) $\int_0^{+\infty} \frac{1}{\sqrt{s+1}}\, ds$ b) $\int_0^{2} \frac{1}{(1-y)^{2/3}}\, dy$ c) $\int_{-\infty}^{0} x\, e^{-x^2}\, dx$

$$\int_0^{\infty} \frac{1}{\sqrt{x+1}}\, dx \rightarrow \lim_{t\to\infty} \int_0^{t} \frac{1}{\sqrt{u}}\, du$$

$u = x+1$ $x=t \Rightarrow t+1$

$\frac{du}{dx} = x+1$ $x=0$ $0+1$

$du = dx$

$$\lim_{t\to\infty} \int_1^{t+1} \frac{1}{\sqrt{u}}\, du$$

$$\lim_{t\to\infty} 2\sqrt{u}\Big|_1^{t+1} = \lim_{t\to\infty}\left(2\sqrt{t+1} - 2\sqrt{1}\right)$$

$$\lim_{t\to\infty}\left(2\sqrt{t+1}-2\right)$$

$2\sqrt{\infty} - 2 = +\infty$, diverge.

c) $\int_{-\infty}^{0} x\, e^{-x^2}\, dx$

$$\lim_{t\to-\infty} \int_t^{0} x e^{-x^2}\, dx$$