Question 1: How do climate manipulations affect soil moisture and temperature?

For this we need two equations where we evaluate the effects of experimenal temperature (eT) and experimental preciptation (eP) treatments on soil moisture and temperature. We're hoping to nest year within site on the intercept and slopes:

$$y_i = \alpha_{site[year[i]]} + \alpha_{site[i]} + \beta_{1site[i]} eT_i + \beta_{2site[i]} eP_i + \beta_{3site[i]} eT_i eP_i + \epsilon_i \tag{1}$$

$$\alpha_{site[year]} \sim N(\mu_{sy}, \sigma_{sy}) \tag{2}$$

$$\mu_{sy} \sim N(\mu_y, \sigma_y) \tag{3}$$

$$\alpha_{site} \sim N(\mu_{site}, \sigma_{site}) \tag{4}$$

$$\beta_{1site} \sim N(\mu_{\beta 1}, \sigma_{\beta 1}) \tag{5}$$

$$\beta_{2site} \sim N(\mu_{\beta 2}, \sigma_{\beta 2}) \tag{6}$$

$$\beta_{3site} \sim N(\mu_{\beta 3}, \sigma_{\beta 3}) \tag{7}$$

Alternatively we could consider using 'site-years' where we combine the site and year coding into one variable:

$$y_i = \alpha_{siteyear[i]} + \alpha_{sp[i]} + \beta_{1sp[i]} T_i + \beta_{2sp[i]} P_i + \beta_{3sp[i]} T_i P_i + \epsilon_i \tag{8}$$

$$\alpha_{siteyear} \sim N(\mu_{sy}, \sigma_{sy}) \tag{9}$$

Question 2: Does these effects differ from non-experimental data?

So right now we have data from Duke Forest (but probably just soil moisture) and from Harvard Forest (soil moisture and O'Keefe data).

A few things to do here ...

1. Post on ECOLOG to ask for more long-term soil moisture data, ideally with phenology but we'll take what we can get
2. Think on best model and how to model temperature as y variable ... here's one idea where y could be daily moisture data across multiple years and T would be MAT and P would be % different than mean for that year:

$$y_i = \alpha_{doy[i]} + \beta_{1sp[i]} T_i + \beta_{2sp[i]} P_i + \beta_{3sp[i]} T_i P_i + \epsilon_i \tag{10}$$

$$\alpha_{doy} \sim N(\mu_{doy}, \sigma_{doy}) \tag{11}$$

Question 3: How do these effects interact to affect plant phenology (budburst, leafout, flowering)?

First we need to use seasonal or annual temperature (T) and soil moisture (S) data to predict phenology (so here y is DOY):

$$y_i = \alpha_{site[year[i]]} + \alpha_{sp[i]} + \beta_{1sp[i]} T_i + \beta_{2sp[i]} S_i + \beta_{3sp[i]} T_i S_i + \epsilon_i \tag{12}$$

$$\alpha_{sp} \sim N(\mu_{sp}, \sigma_{sp}) \tag{13}$$

$$\beta_{1sp} \sim N(\mu_{\beta 1}, \sigma_{\beta 1}) \tag{14}$$

$$\beta_{2sp} \sim N(\mu_{\beta 2}, \sigma_{\beta 2}) \tag{15}$$

$$\beta_{3sp} \sim N(\mu_{\beta 3}, \sigma_{\beta 3}) \tag{16}$$

We can then combine equations from Question 1 (which predict MAT and soil moisture on an annual scale, we hope) with equations from Questions 3 (at the annual scale) to answer: how much does 1 degree change in target temperature (eT) affect phenology, if this were the only effect of the experiment? Similarly, how much does 50% change in precipitation (eP) affect phenology, if this were the only effect of the experiment? And how big a change does each make acknowledging that they both change moisture and temperature together? We can do this by plugging in different values of eT (e.g., all 1 C, then try with all 2 C) and eP to caculate different outcomes of moisture and temperature which we can evaluate in the eqations in Question 3 to assess changes in phenology.
For notation questions (I may have our notation wrong), see also: 12.5 in Gelman & Hill, pages 262-265.