Q1 Curious as to the relation of holes to negation

Holes are not a fundemental part of type theory, so there isn't a deep answer to this as far as I am aware

Ah I was thinking in terms of paths. It seems that paths cannot be equated if there is a hole int the way.

holes like those in the doughnut.

I still don't see a potential relation to negation. Do you have something in mind?

Well the relation seems to be at least to inequality. If there is a hole between two paths a to b then the paths can't be identified.

anyway, I don't know much about topology, but I think that is what's going on.

Oh, I see (I was thinking about holes as in ? in Agda). In this case, you can think of the shape of a space as somehow arising from all of the (possibly higher dimensional) holes that it has, approximately

And you can study that shape by analysing which (higher) paths can be identified

For simplicity, consider the circle. Here not all points are equal to the basepoint. So this is one way of looking at it.

Q3 Would that interpretation hold for a context/type like $\mathrm{base} : \mathbb{S}^1 \vdash \mathrm{loop} : \mathrm{base} = \mathrm{base}$?

e.g. when we fix both endpoints and can't apply path induction

Yes, in the sense that the Martin-Löf identity type can be used to work with HITs. You can't apply path induction without free endpoints though!

You can still talk about paths with free endpoints on the circle, for example

Q4 When we say "equality for a particular type" (as in the notes) is it similar to how we saw identifications for Π and Σ types in the previous lectures, where we actually manually consider the structure of the type?

Exactly!

Q5 Is observational equality related to decidable equality? Defining an instance of them seems somewhat similar.

They're related; if you want the types to be propositions, then you are confining yousself to equality on a set. That doesn't necessarily mean that it's discrete (decidable equality) because propositions are not always decidable, but discrete types are sets

But in the case that we're currently doing with coproducts, the types might not be sets

Because $\mathrm{Id}(a, b)$ won't in general be a proposition

Q6 Why is this considered observational equality if it ultimately reduces to the identity type? Shouldn't we invoke $\mathrm{Eq}\, A$ and $\mathrm{Eq}\, B$ as well?

We don't have useful definitions of $\mathrm{Eq}\, A$ and $\mathrm{Eq}\, B$, because they are arbitrary types.

We don't really have any good candidates for $\mathrm{Eq}\, A$ and $\mathrm{Eq}\, B$ because we have no idea what types they are. What $\mathrm{Eq}(A + B)$ is doing is basically reducing identifications on $A + B$ to identifications on either A or B.

Isn't the point of observational equality to be defined for all types?

I don't think so. It's usually defined for a specific type or class of types.

Could we take definitions of equality for those terms as arguments to our definition of the observational equality for the coproduct?

These arguments live in a universe, so you would need to define a function that can take any (codes of) types for these arguments.

Well, I guess they are binary type families, but the idea remains.

Q7 What does $P(x, p)$ mean?

P is a type family over points $x : A$ and paths $p : a = x$

You should think of it as being defined in the line that has the premises of the rule

Thank you. Is P general or particular?

P is an arbitrary such family

Q9 Is it fair to say E is sort of a candidate identity type family? e.g. $E(x)$ is 'acting like' $x = x$?

It's acting like $a = x$.

So a is fixed within E even though it is not explicitly an argument to the type?

a is a given term of type A, the type that E runs over. It's just like the intro rule for the based/one-sided id types.

Q10 Does the univalence axiom essentially assert that $\mathrm{Eq}\,\mathcal{U}$ (the observational equality on types) is equivalence? Following the pattern that we see here

If by 'observational equality on types' you mean any of the equivalent definitions of equivalence that we have considered, then that's exactly right, except that univalence also says that a specific map gives an equivalence between observational equality and equality on types

What do you mean by the specific map?

The map from equality to observational equality defined by path induction

ahh because it has to send refl to the identity equivalence?

Yes

Q11 Would injections between sets $A \to B$ be embeddings?

Yes

Q13 Is it fair to say that the fundamental theorem of identity types says how to show that a type is equivalent to the identity type?

Exactly!