Neural Prosthesis - Homework 2

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1 Exercise 1

1.1 Question a.

We use an 2D Cartesian electrostatic analysis to solve this problem. This problem is split in two symmetrical problems. There is two different regions of interest : air and the shaped one. Permittivities are respectively $\epsilon_{air} = \epsilon_0$ and $\epsilon_{shaped} = 2.\epsilon_0$, where $\epsilon_0 = 8.85 \times 10^{-12} F/m$ is the vacuum permittivity.

In this problem, an element creating a positive flux is immersed at the bound between air and the shaped region. The positive flux, in the electrostatic problem, is a positive voltage. We have set this voltage to $1V$ to have a simple problem.

Thus, the boundary conditions used for this problem are exactly the same as in the subject. The boundary condition on the axis of symmetry does not need to be specified to the Finite Element Method solver, it will be implicit while solving the problem.

Arbitrary dimensions have been set for the geometry, to respect more or less the shape given in the subject. The spacing of the mesh has been tuned to have more accuracy around the immersed element. The resulting geometry, for both left and right parts, is the one shown in figure 1.

The mesh contains 230 nodes. We solve this problem and plot the equipotential lines, as shown in figure 2. We can see that solutions for left and right side are exactly the same. We also can observe that the voltage propagates more easily in the air than in the shaped region, due to the difference of permittivity. By increasing the permittivity of the shaped region, we make this phenomena more evident.

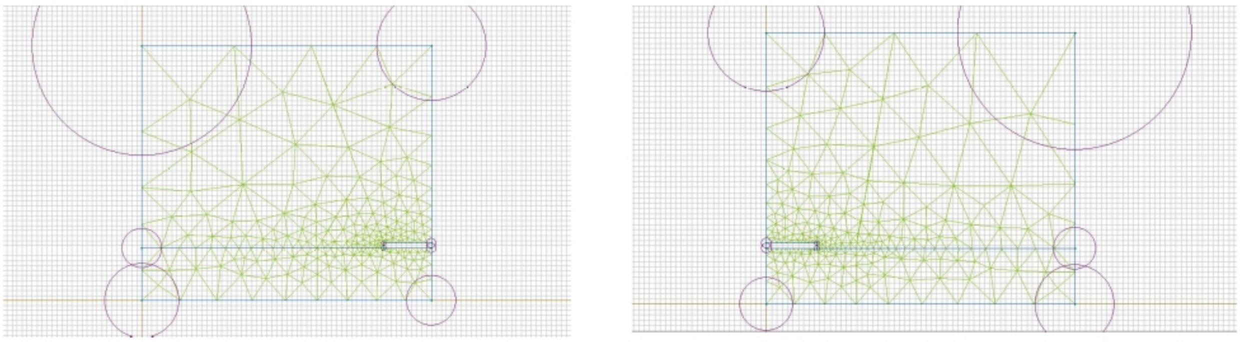

Figure 1: Left and right geometries

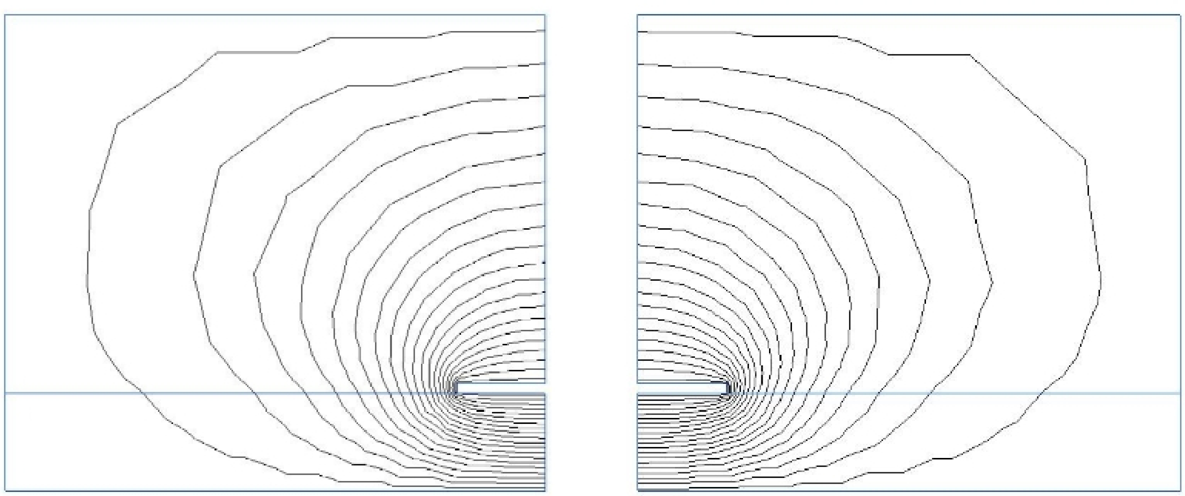

Figure 2: Left and right equipotential lines

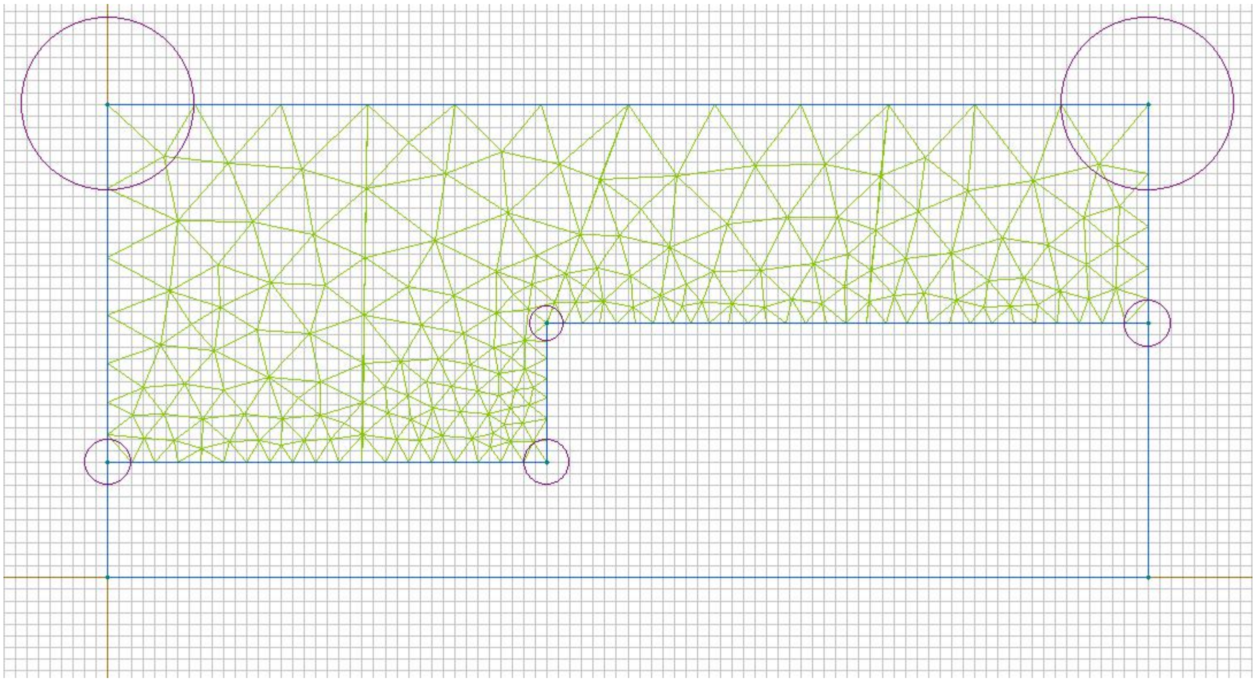

Figure 3: Geometry of the body of revolution

1.2 Question b.

For this question, we need to solve an axisymmetric electrostatic problem for the body of revolution. Boundary condition for the upper bound of the geometric model is a null flux. The surface of the body is a positive flux (set to $1V$). Both the right and left bounds have a null variation of flux, which does not need to be inputted in the problem.

The body is immersed in air, which is modeled as a region with the same permittivity as for the first problem. Once again, the spacing of the mesh has been tuned to be smaller next to the body, and wider in the external part of the air region, and to keep the number of nodes under 255.

The geometry used and the solution found by the FEM solver are presented respectively in figures 3 and 4. We can see that the equipotential lines are not perfectly fitting the shape of the body.

2 Exercise 2

2.1 Question a.

We are now interested in solving a DC conduction problem, in order to measure the variation of potential along an axon stimulated by a monopolar point source. The electrode is surrounded by muscle tissue modelled as a region with a mean resistivity $\rho_{muscle\ tissue} = 643\ \Omega.cm^{-1}$ (results of [1], found in table 1 of [2]). We work with an unmyelinated axon with an axial resistivity $\rho_{axon} = 100\ \Omega.cm^{-1}$. The axon diameter as been set to 10 μm (which is in the range of group A axon diameter for mammalian [3]).

Please note an inconsistency in the simulation : the myelin surrounding the axon is not considered in this DC conduction problem, but the axon in the motor nerve fiber are myelinated.

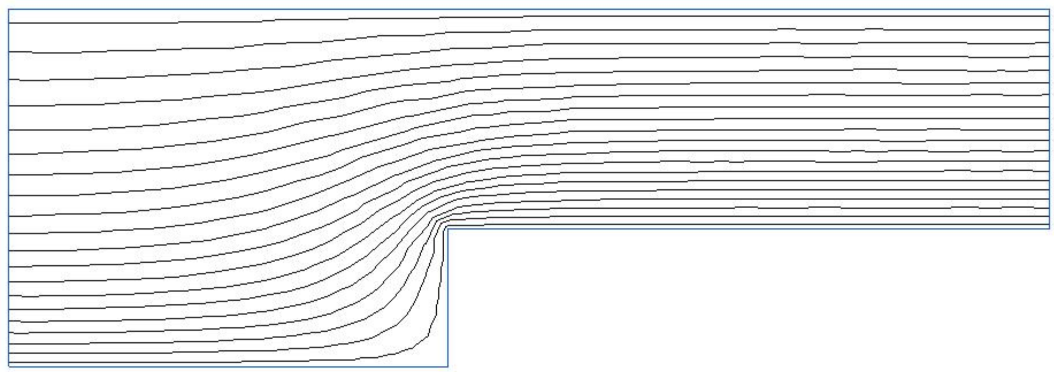

Figure 4: Equipotential lines for the body of revolution

The boundary conditions are set to have a null potential at the bound of the box, and to $V_{electrode} = 0.25V$ at the surface of a $1\mu m$-diameter point source centered at $1cm$ of the axon. This voltage value has been selected using binary search to obtain a peak voltage of $80mV$ at the center of the axon.

The problem bound box dimensions are $32cm \times 7cm$. The electrode is centered in $(16cm, 4cm)$ and the $32cm$-long axon is centered in $(16cm, 3cm)$. The mesh has then been tuned to have a symmetric diffusion of the current with a better accuracy between the axon and the electrode. The resulting mesh is presented in figure 5. Please note that some of the blue lines are only used to symmetrically balance the generated mesh.

We solve this problem with the FEM solver and plot the potential color graph shown in figure 6. The color is changed by step of potential value, with red being less of equal than $0.25V$ and blue being more than $0V$. We can see that the potential spreads more easily horizontally : this is due to the definition of the problem and to the dimension of the bounding box, which influence the voltage decrease. However, the dimensions are large enough to have quite good results in the center of the box.

We record potential values along the open circuit shown by green arrows on figure 6, and pass it to a Python script to be able to compare it with the following potential spread formula :

$$V_{axon}(x) = \frac{I_{stim}}{4\pi\sigma_{muscle\ tissue}\sqrt{h^2 + (x - x_{ps})^2}}$$

where :

- x is the position along the axis,
- I_{stim} is the stimulation current,
- $\sigma_{muscle\ tissue} = 0.15552 S.m^{-1}$ is the conductivity of the external medium,

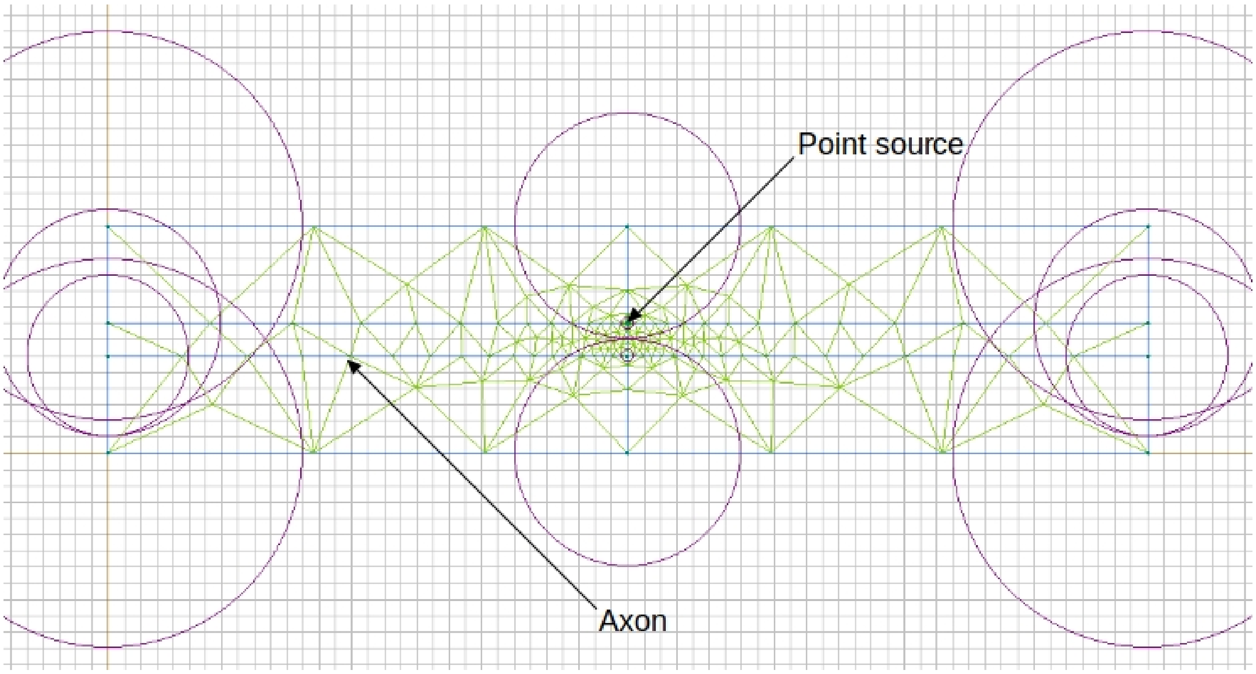

Figure 5: Mesh for the monopolar stimulation of the axon

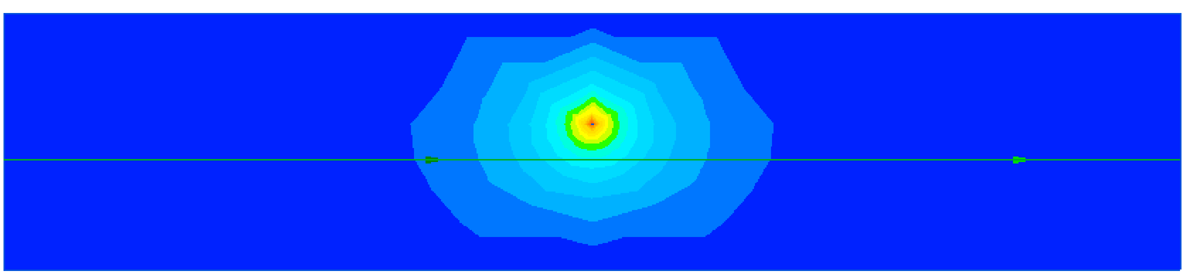

Figure 6: Potential color map for the monopolar stimulation of the axon

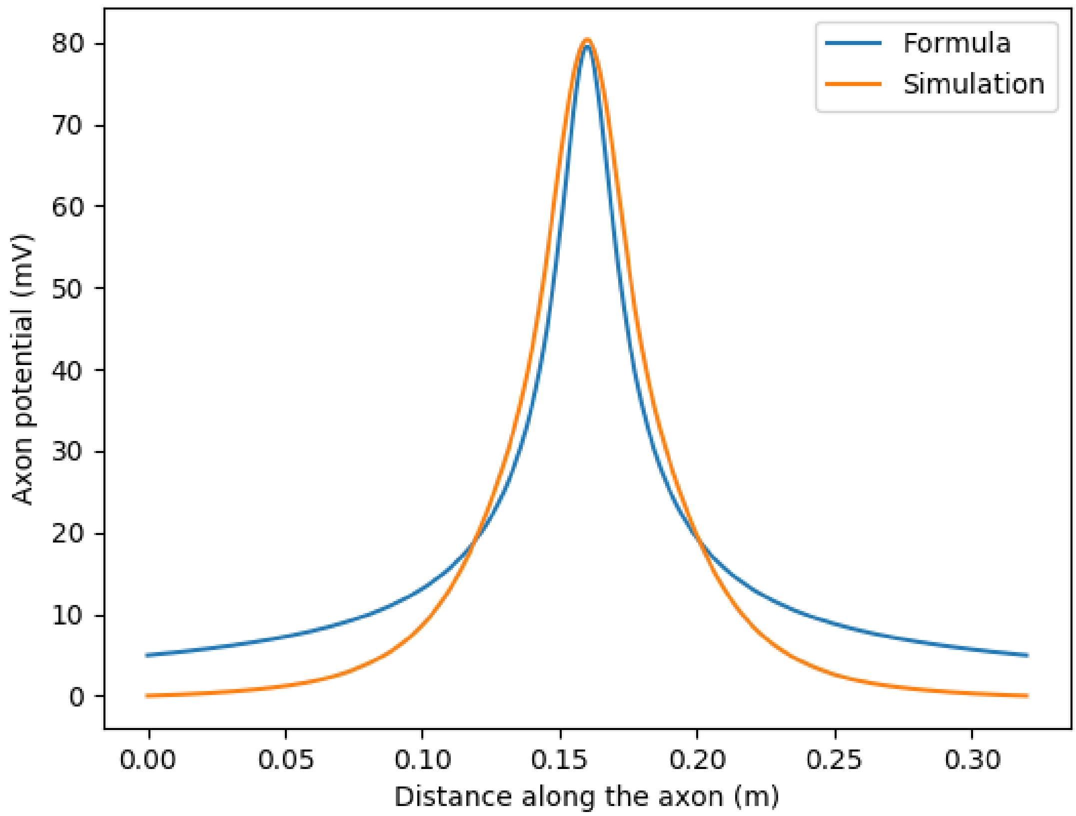

Figure 7: Potential depending of the position on the axon, comparison between formula and simulation results

- $h = 1cm$ is the distance between the point source and the axon,
- $x_{ps} = 16cm$ is the x-axis position of the point source

We use it to get the potential-position graph presented in figure 7. We use cubic spline as interpolation between data points.

Although the strong divergence between the two curves at the bounds of the axon ($0cm$ and $32cm$), due to the finite dimension of the problem and to the boundary conditions in the simulation, the voltage peak is quite well fitting with the formula.

2.2 Question b.

For the second question, we replace the monopolar stimulation by a bipolar stimulation. The single point source is replaced by to point sources spaced by $1mm$ and having opposite voltages, respectively $+U$ and $-U$. The boundary conditions at the surface of the point sources are the only conditions differing from the problem of question a.

Body and axon regions are set as in the previous problem, and the dimensions of the bounding box remains unchanged. The two figures 8 and 9 show the global mesh, and a zoom on the two electrodes.

We then use binary search to find the voltage U such as there is a positive and negative peak of amplitude $25mV$. We obtained this result with $U = 0.33V$, which means that the total tension between the two electrodes sum up to $0.66V$.

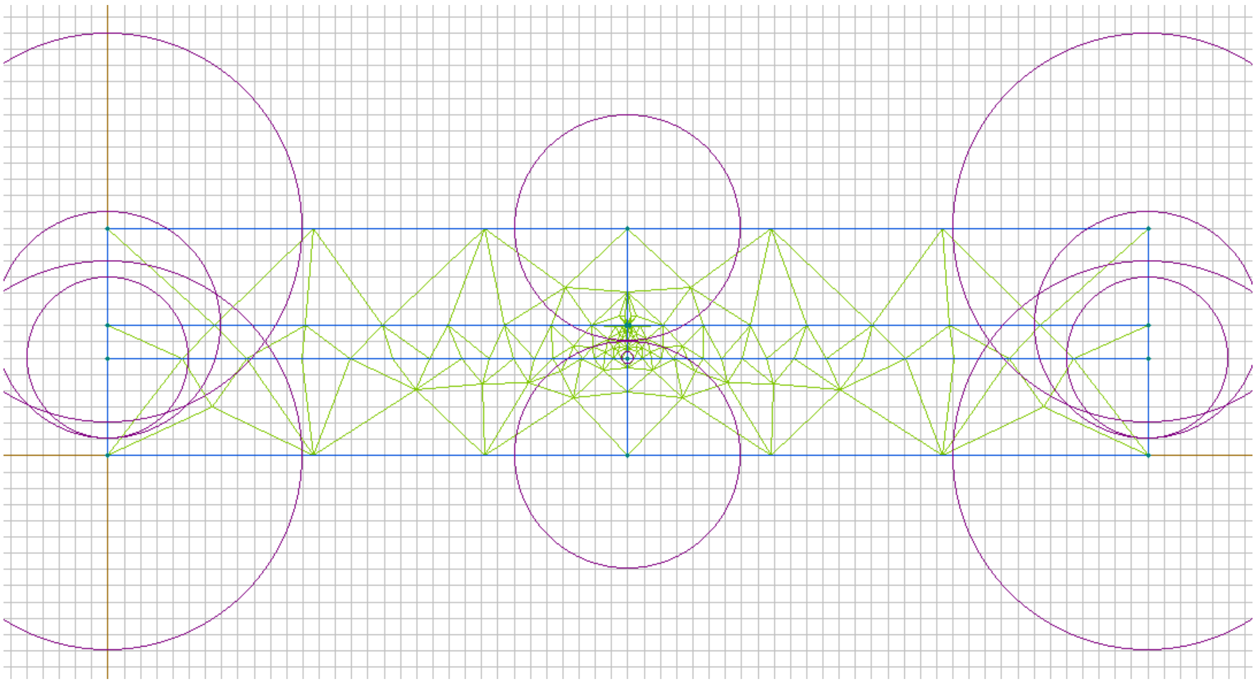

Figure 8: Mesh for the bipolar stimulation of the axon

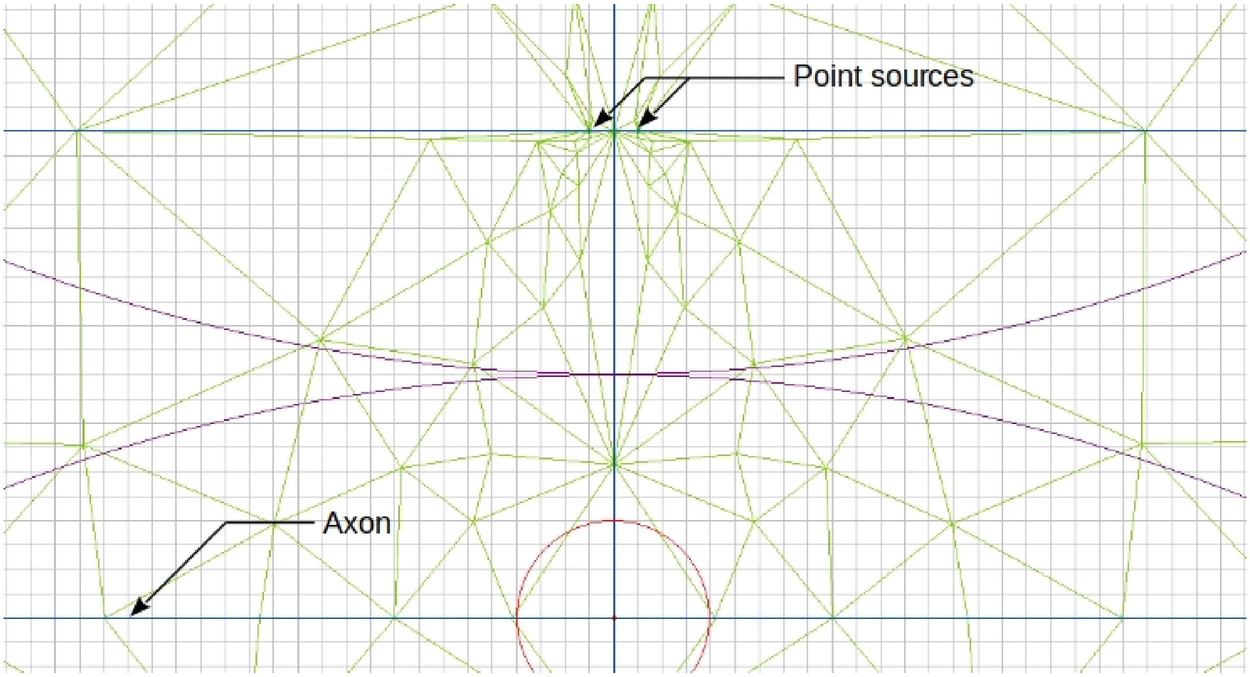

Figure 9: Mesh for the bipolar stimulation of the axon, zoom on the stimulation zone

After solving this problem, we obtain the potential color map presented in figure 10. We present a zoom on the stimulation region in figure 11. In comparison to the monopolar stimulation, we can see that the voltage is less spread : the stimulation is highly concentrated next to the electrodes.

This difference can be explained by comparing the current density between monopolar stimulation (figure 12) and bipolar stimulation (figure 13). In the monopolar stimulation, the current is spread all around the electrode, with no preference for a specific region, whereas in bipolar stimulation the current is focused between the two electrodes. This result in a sharp current peak between the electrodes, which makes possible a more selective stimulation.

Finally, we also compare the simulation results along the axon with a formula of the potential along the axon. This formula is derived from the one used in the previous problem, adapted to have two point source with opposite voltages :

$$V_{axon}(x) = \frac{I_{stim}}{4\pi\sigma_{muscle\ tissue}\sqrt{h^2 + (x - x_{ps1})^2}} - \frac{I_{stim}}{4\pi\sigma_{muscle\ tissue}\sqrt{h^2 + (x - x_{ps2})^2}}$$

where we introduce x_{ps1} and x_{ps2} the position of the two point sources.

The resulting potential-distance plot is presented in figure 14. Please note that cubic spline has been used as interpolation method. We can see a divergence between the simulation and the formula : the simulation peaks are more distant from the electrode position. This is probably due to the presence of boundary conditions in the simulation : the current delivered by the electrode is spread, which is not handled by the formula. However, this is resulting in relatively small errors, and does not change the qualitative results.

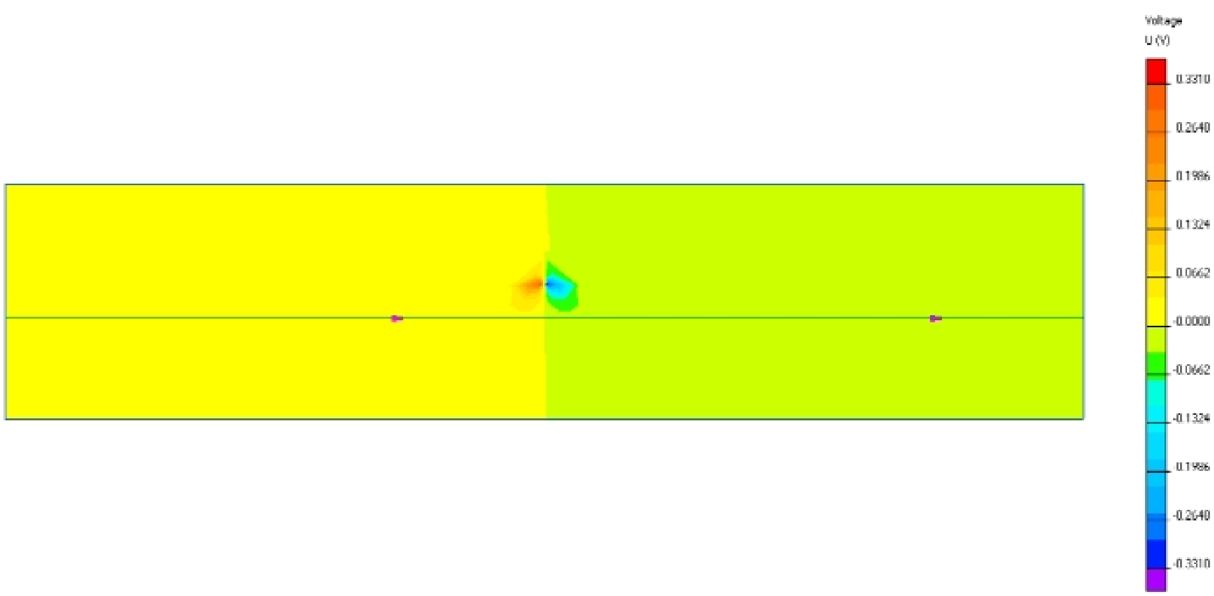

Figure 10: Potential color map for the bipolar stimulation of the axon

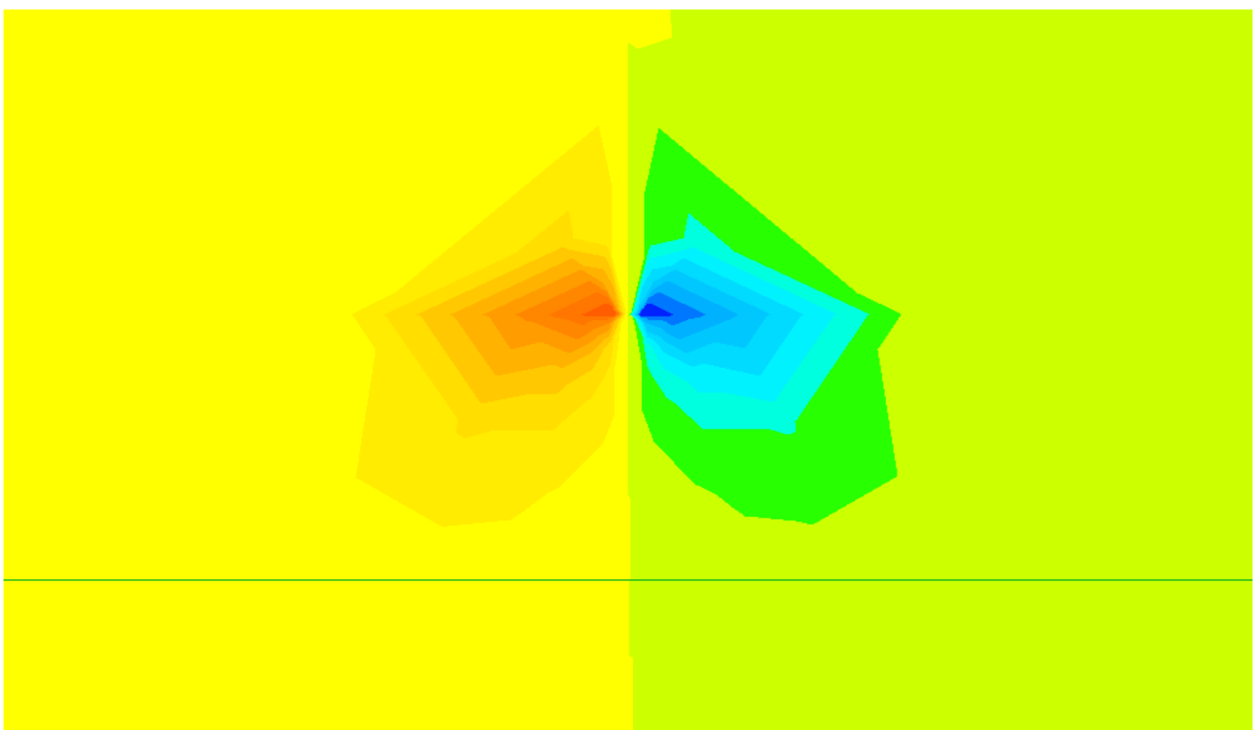

Figure 11: Potential color map for the bipolar stimulation of the axon, zoom on the stimulation zone

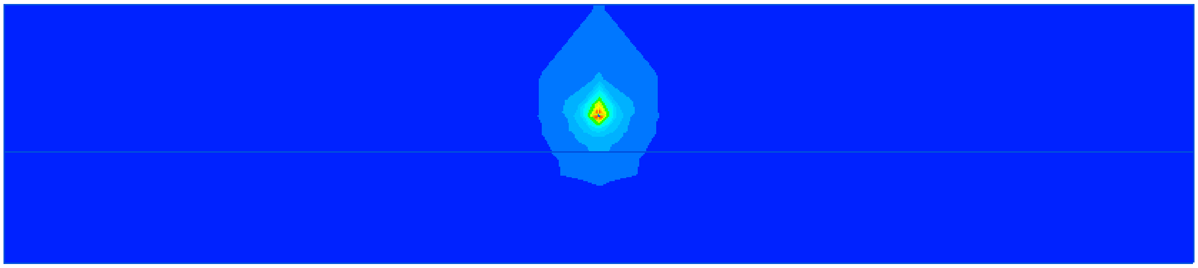

Figure 12: Current density color map for the monopolar stimulation of the axon

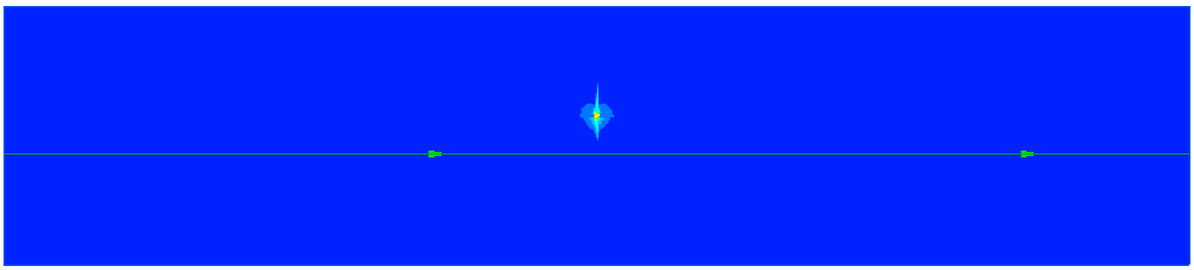

Figure 13: Current density color map for the bipolar stimulation of the axon

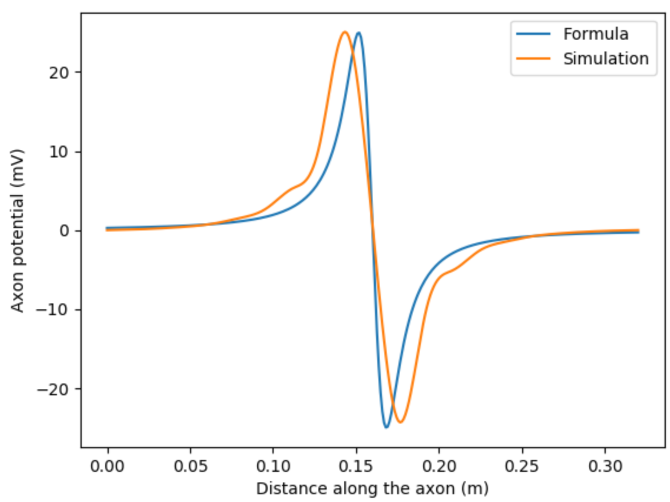

Figure 14: Potential depending of the position on the axon, comparison between formula and simulation results

References

[1] William Kaufman and Franklin D Johnston. "The electrical conductivity of the tissues near the heart and its bearing on the distribution of the cardiac action currents". In: *American Heart Journal* 26.1 (1943), pp. 42–54.

[2] Stanley Rush, JA Abildskov, and Richard McFee. "Resistivity of body tissues at low frequencies". In: *Circulation research* 12.1 (1963), pp. 40–50.

[3] JG Whitwam. "Classification of peripheral nerve fibres". In: *Anaesthesia* 31.4 (1976), pp. 494–503.