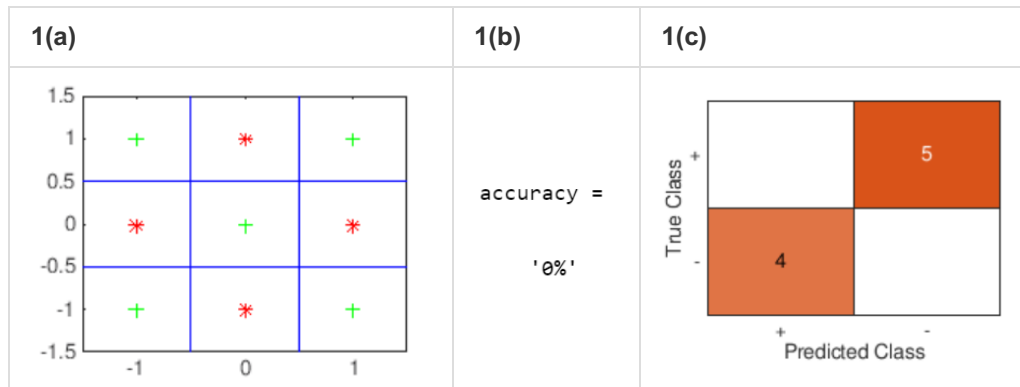


# ML hw1 0416235 劉昱劭

測試環境 MATLAB R2019a，可由 MATLAB 網頁版使用 R2019a 測試：<https://matlab.mathworks.com/>

作業連結縮網址：[http://bit.ly/MLhw1\\_0416235](http://bit.ly/MLhw1_0416235)

## Q1



- code

```
1 % Q1
2 %% 1(a)
3 A_plus = [0,0; 1,1; -1,1; 1,-1; -1,-1]
4 A_minus = [1,0; -1,0; 0,1; 0,-1]
5 A = [A_plus; A_minus];
6 x1 = A(:,1);
7 x2 = A(:,2);
8
9 [vx,vy] = voronoi(x1, x2);
10 fa = figure
11 plot(A_plus(:,1),A_plus(:,2),'g+',A_minus(:,1),A_minus(:,2),'r*',vx,vy,'b-')
12 xlim([min(x1)*1.5 max(x1)*1.5])
13 ylim([min(x2)*1.5 max(x2)*1.5])
14
15 %% 1(b)
16 Y = ['+' '+' '+' '+' '+' '-' '-' '-' '-'];
17 mdl = fitcknn(A,Y,'NumNeighbors',3)
18 [Y_pred,score,cost] = predict(mdl,A);
19 [C,order] = confusionmat(Y, Y_pred);
20
21 num_of_train = size(Y,1);
22 accuracy = trace(C)/num_of_train;
23 accuracy = strcat(num2str(accuracy*100),'%')
24
25 %% 1(c)
26 fc = figure
27 confusionchart(C, order);
28 C
```

## Q2

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<sol>

A symmetric matrix  $A_{n \times n}$  is positive definite

$$\rightarrow x^T A x > 0, \forall \text{ nonzero vector } x_{n \times 1} \dots \textcircled{1}$$

Let  $\lambda$  be an eigenvalue of  $A$ ,  
 $x$  be the corresponding vector.

$$\Rightarrow Ax = \lambda x$$

(multiply  $x^T$  on LHS)

$$x^T A x = \lambda x^T x = \lambda \|x\|^2$$

We know left-hand-side is positive by  $\textcircled{1}$   
then any eigenvalue  $\lambda$  of  $A$  must be positive.  
\*

### Q3

- 以  $\Sigma$  表示  $\text{var}(w)$

$$\text{Var} = \sigma^2 = \frac{1}{n} \sum_i (x_i \cdot a)^2 \quad 0416235$$

$$= \frac{1}{n} (\sum a)^T (\sum a)$$

$$= \frac{1}{n} a^T \sum \sum a$$

$$= a^T \left( \frac{\sum \sum}{n} \right) a$$

$$= a^T \Sigma a$$

\*

- 求  $\text{var}(w)$  的值

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3.  $W = a^T Z = \|a\| \|Z\| \cos \theta$ ,  $\theta$  是  $a$  和  $Z$  的夹角

$$0 \leq \theta \leq \pi \Rightarrow W \in [-\|a\| \|Z\| \cos \theta, \|a\| \|Z\| \cos \theta]$$

$$E(W) = \int_{\text{Domain}} w dw = \int_0^\pi \|a\| \|Z\| \cos \theta d\theta$$

$$= \|a\| \|Z\| \sin \theta \Big|_0^\pi$$

$$= 0$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\text{Var}(W) = \int_{\text{Domain}} [W - E(W)]^2 dW = \int_{\text{Domain}} W^2 dW = \int_0^\pi \|a\|^2 \|Z\|^2 \cos^2 \theta d\theta$$

$$= \|a\| \|Z\| \int_0^\pi \frac{1}{2} (1 + \cos 2\theta) d\theta$$

$$= \frac{1}{2} \|a\| \|Z\| \cdot \left[ \theta + \frac{1}{2} \sin 2\theta \right] \Big|_{\theta=0}^{\theta=\pi}$$

$$= \frac{1}{2} \|a\| \|Z\| \cdot (\pi - 0)$$

$$\Rightarrow \text{Var}(W) = \frac{\pi}{2} \|a\| \|Z\|$$

$$= \frac{\pi}{2} \sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{x_1^2 + x_2^2 + x_3^2}$$

## Q4

4(a)	4(b)
sample_mean =	sample_mean =
0.4927	0.5016
sample_std =	sample_std =
0.0926	0.0101

```

1 %% 4(a)
2 num_of_samples = 10
3 num_of_experiment = 20
4 S = rand(10000, 1);
5 experiment = zeros(num_of_experiment, 1);
6
7 for iteration = 1:num_of_experiment
8     samples = datasample(S, num_of_samples);
9     experiment(iteration) = experiment(iteration) + mean(samples);
10 end
11
12 sample_mean = mean(experiment)
13 sample_std = std(experiment)

```

```

1 %% 4(b)
2 num_of_samples = 1000
3 num_of_experiment = 50
4 S = rand(10000, 1);
5 experiment = zeros(num_of_experiment, 1);
6
7 for iteration = 1:num_of_experiment
8     samples = datasample(S, num_of_samples);
9     experiment(iteration) = experiment(iteration) + mean(samples);
10 end
11
12 sample_mean = mean(experiment)
13 sample_std = std(experiment)

```

## Q5

- 結果：

```

xhat =

    2.0000
    0.5000
    1.0000

```

```

1 A = [2 2 0; 1 0 1; 0 2 2; 1 0 1]
2 b = [5; 2; 3; 4]
3
4 xhat = A\b

```

## Q6

- 6(a)

- matrix 維度太大，請助教直接跑前 7 行就會產生  $S = [X, y]$ 。

```

1 % Q6
2 X = -1 + (1-(-1))*rand(1000, 2);
3 x1 = X(:, 1);
4 x2 = X(:, 2);
5 epsilon = normrnd(0,1, [1000,1]);
6 y= 2*x1.*x1 - 0.6*x2.*x2 + 1.5*x1.*x2 + x1 + 2*x2 + epsilon;
7 S = [X, y];
8 [f,gof2] = fit([x1, x2],y,'poly22');
9
10 %% 6(b) (c)
11 f
12 MeanAbsoluteError = mae(f(x1,x2)-y)
13
14 % plot
15 fc = figure
16 plot( f )
17 fc2 = figure
18 plot( f, [x1, x2], y )

```

- 6(b)

Linear model Poly22:

$$f(x,y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2$$

Coefficients (with 95% confidence bounds):

$p_{00} =$	-0.0188	(-0.131, 0.09346)
$p_{10} =$	1.003	(0.8989, 1.108)
$p_{01} =$	1.993	(1.883, 2.102)
$p_{20} =$	2.041	(1.837, 2.246)
$p_{11} =$	1.597	(1.411, 1.784)
$p_{02} =$	-0.7599	(-0.9675, -0.5523)

- 6(c)

MeanAbsoluteError =

0.7749

- 畫上  $f$  和  $S$  的 1000 個資料點

