Introduction

A Bermudean option is a type of exotic options contract that can only be exercised on predetermined dates. In other words, it gives the holder of the option the right to exercise on a specific set of dates during the life of the option. The name is quite telling because it is a blend of the two other popular vanilla options, namely the American option and European option, and it got its name from the place Bermuda, which lies between these two places.
In a frictionless complete market, the price of a Bermudean option is given by :

$$V_0(s) = \sup_{t \in \mathbb{T}} \mathbb{E}\left[e^{-rt} g(S_t) \mid S_0 = s\right]$$

where $\mathbb{T} = \{0 < t_1 < ... < t_N = T\}$ is the set of the predetermined dates at which we can exercise the option, r the fixed interest rate, g the payoff function and S the underlying asset.
Despite its widespread significance, the valuation of early-exercise features remains a difficult problem in many important settings where there is no analytical solution, even for relatively simple models.
However, binomial, lattice and finite-difference methods can be used to generate numerical solutions to pricing problems with one or two sources of uncertainty. For this project, we will study :

- The Nested Monte Carlo approach using **Random Tree Method** developed by Broadie and Glasserman in [2]
- The **Finite Difference method** presented by Guyon and Henry-Labordère in [5]

We will develop the mathematical foundations of each solution along with the detail of their numerical implementation in the case of a Bermudean put option.