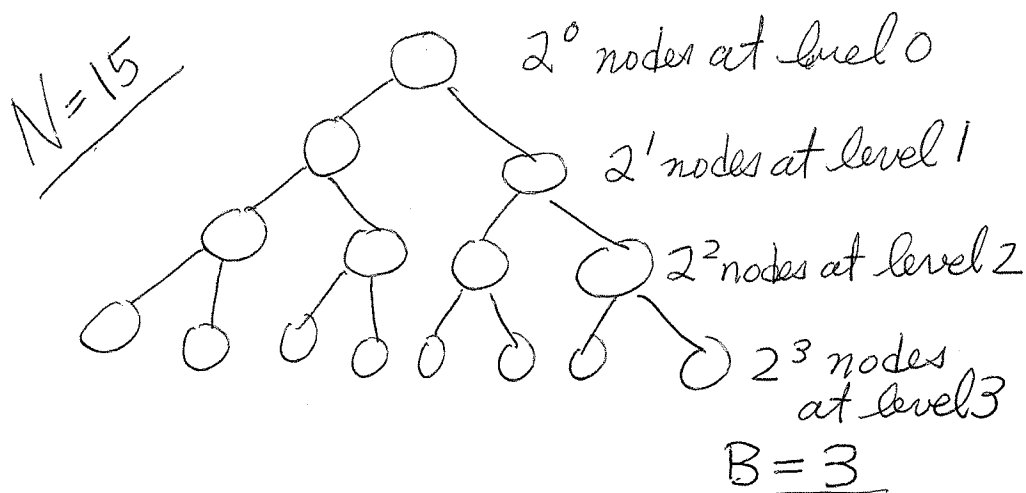


①

Binary Tree (Perfect Triangle)



2^k nodes at level k .

$$N = \text{total \# of nodes} = \sum_{k=0}^B 2^k = 2^{(B+1)} - 1$$

For the above example $2^{(3+1)} - 1 = 15$

How many levels? $B+1$

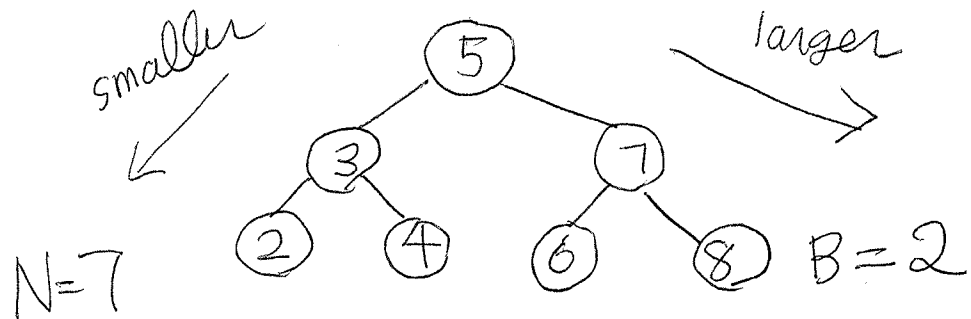
$$2^{(B+1)} - 1 = N$$

$$2^{(B+1)} = N+1$$

$B+1 = \log_2(N+1)$ levels
$= \# \text{ comparisons}$

②

Binary Decision Tree



To look for 2, 3 comparisons

To look for 6, 3 comparisons

To look for 5, 1 comparison

To look for 10, 3 comparisons & off the tree

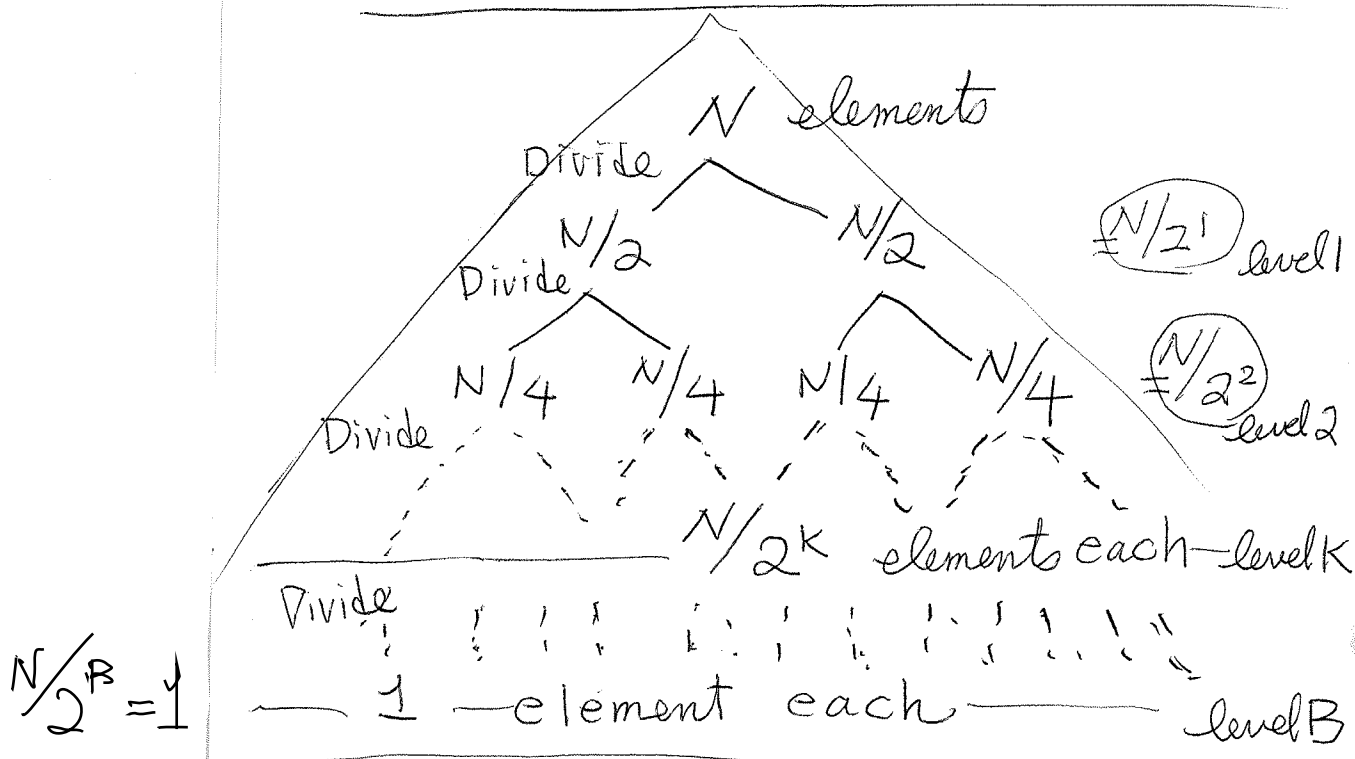
$$B+1 = \log_2(N+1)$$

$$= \log_2 8 = 3$$

= max number of comparisons

The best possible way to search through an ordered list.

③ Recursion Tree



At level B $N/2^B = 1$ element each

$$N = 2^B$$

$$B = \log_2 N$$

① If $N=8$, $B=3$.

\Rightarrow 4 levels in the tree ($B+1$)

\Rightarrow List was divided in halves via 3 levels (B) of recursion

② If $N=1024$, $B=10$. 11 levels in the tree

Draw a 3-ary tree & indicate # of nodes per level
How many paths? = #leaves