National University of Sciences & Technology
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Project Report

Time Value of Money

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1. Introduction

The Time Value of Money (TVM) is a fundamental concept in engineering economics that explains how the value of money changes over time. It helps engineers, managers, and decision-makers evaluate future cash flows, compare project alternatives, and determine whether long-term investments are financially sound.

1.1. Definition of TVM

The Time Value of Money (TVM) states that a specific amount of money today is worth more than the same amount in the future because today's money can be invested to earn returns. It supports calculations such as present value, future value, interest rates, and annuities.

1.2. Why Money Today ≠ Money Tomorrow

Money today is not equal to future money because of:

- Interest and investment opportunities
- Inflation reducing future purchasing power
- Risk and uncertainty in future payments
- Opportunity cost of not investing money today

1.3. Importance in Engineering Economics

TVM helps engineers:

- Compare project alternatives
- Evaluate long-term project feasibility
- Calculate present value and future value of cash flows
- Make cost-effective equipment and investment decisions
- Estimate financial outcomes over a project's lifetime

1.4. Applications in Real Life

The concept of TVM is used in:

- Savings and investment growth

- Loan and mortgage repayment planning
- Installment and credit card calculations
- Retirement planning
- Cost-benefit analysis of engineering projects

1.5. Example

If you invest Rs. 10,000 at 10% annual interest, after one year it becomes:
Future Value = 10,000 × (1 + 0.10) = Rs. 11,000.
This shows that money increases in value over time, demonstrating the Time Value of Money.

2. Interest

2.1. Simple Interest

2.1.1. Concept

Simple interest is the most basic form of interest used in financial calculations.
It is calculated only on the original principal amount (P) throughout the entire loan or investment period.
This means that interest does not compound i.e each year generates the same amount of interest.

Key characteristics:

- Interest remains constant every period.
- Total amount increases linearly with time.
- Easy to calculate and commonly used for short-term financial agreements.

Simple interest is typically used when the time period is short, or when contracts explicitly specify “non-compounding interest.”

2.1.2. Formula

The formula for simple interest is:

$$\mathbf{SI = P \times i \times n}$$

Where:

- SI = Simple Interest
- P = Principal (initial investment or loan)
- i = Interest rate per period
- n = Number of periods

The total amount to be paid or received at the end of the period:

$$A=P+SI$$

2.1.3. Example

Suppose you invest Rs. 10,000 at 5% annual interest for 3 years.

$$SI=10{,}000\times0.05\times3=1{,}500$$

Total amount after 3 years:

$$A=10{,}000+1{,}500=11{,}500$$

2.1.4. Use case

Simple interest is commonly used in:

- Short-term loans
- Car loans
- Personal loans
- Small-term deposits

It is suitable for financial calculations where interest does not compound.

2.2. Compound Interest

2.2.1. Concept

Compound interest is the interest calculated on the initial principal as well as the accumulated interest from previous periods. Unlike simple interest, where interest is earned only on the principal, compound interest allows money to grow exponentially because each period's interest adds to the principal for the next period.

This concept reflects the principle that "interest earns interest", making it fundamental in savings, investments, loans, and financial planning. The longer the time period and the more frequent the compounding, the greater the future value of the investment.

2.2.2. Formula

- Future Value (if principal is known):

  $$\mathbf{F = P\ (1 + i)^n}$$

- Principal (if future value is known):

  $$\mathbf{P} = \frac{F}{(1+i)^n}$$

- Compound Interest Earned:

  $$\mathbf{CI = F - P}$$

- Effective Annual Rate (EAR) if interest is compounded m times per year:

  $$\mathbf{EAR = (1 + \frac{\mathit{i}}{\mathit{m}})^m - 1}$$

Where:

P = Principal (initial investment)
i = Interest rate per period
n = Total number of periods
m = Number of compounding periods per year
F = Future Value

2.2.3. Example

Suppose you invest Rs. 20,000 at 10% annual interest for 4 years, compounded annually.

Step 1: Calculate Future Value

$F = 20{,}000 \times (1 + 0.10)^\wedge 4$

$F = 20{,}000 \times 1.4641 = 29{,}282$

Step 2: Calculate Compound Interest Earned

$CI = 29{,}282 - 20{,}000 = 9{,}282$

2.2.4. Use case

Compound interest is used widely in finance and investments:

- Savings accounts
- Fixed deposits
- Mutual funds and retirement plans
- Mortgages and home loans
- Credit card balances
- Bonds, securities, and business investments

Advantages:

- Higher returns over time compared to simple interest
- Encourages long-term investment
- Reflects real economic growth

2.3. Comparison Between Simple and Compound Interest

Simple interest and compound interest are two fundamental methods for calculating interest, but they differ significantly in how they operate over time. Simple interest is calculated only on the original principal amount throughout the duration of the investment or loan. The formula for simple interest is $SI = P \times i \times n$, where P is the principal, i is the interest rate per period, and n is the number of periods. The total amount at the end of the term is simply the sum of the principal and the interest ($A = P + SI$). Because simple interest does not account for interest earned on previously accumulated interest, it grows in a linear manner. This method is typically used for short-term loans or investments where a straightforward calculation is sufficient.

Compound interest, on the other hand, is calculated on the principal plus any interest that has been earned in previous periods, meaning that interest itself earns interest. The formula for compound interest is $CI = P \times (1 + i)^n - P$, and the total amount is $F = P \times (1 + i)^n$. This method leads to exponential growth of the investment or loan balance over time and is ideal for long-term investments, savings plans, and financial instruments where reinvesting interest maximizes returns. Compound interest is widely used in savings accounts, fixed deposits, retirement funds, mortgages, and bonds.

The key difference between simple and compound interest lies in their growth patterns and applications. Simple interest grows linearly and is easier to calculate, making it suitable for short-term financial decisions. Compound interest grows exponentially and provides higher returns over the same period, making it essential for long-term financial planning. Understanding both concepts is critical for making informed financial decisions, comparing investment alternatives, and evaluating loans and savings options.

Feature	**Simple Interest (SI)**	**Compound Interest (CI)**
Interest Calculated On	Principal only	Principal plus accumulated interest
Growth Pattern	Linear	Exponential

Formula (Interest Earned)	SI = P x i x n	CI = P x $(1 + i)^n$ - P
Total Future Amount	A = P + SI	F = P x $(1 + i)^n$
Suitable For	Short-term loans	Long-term investments
Advantage	Easy to calculate and understand	Provides higher returns over time

3. Interest Formulas

3.1. Present Value (PV) Formulas

3.1.1. Single payment present worth

The present value of a single future payment is the amount that must be invested today to achieve a specific future value.

Formula:

$$\mathbf{PV} = \frac{F}{(1+i)^n}$$

Where:

PV = Present Value

F = Future Value

i = Interest rate per period

n = Number of periods

3.1.2. PV of annuity

The present value of an annuity is the sum of present values of equal periodic payments.

Formula:

$$\mathbf{P= A} \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Where:

A = Periodic payment

i = Interest rate per period

n = Number of periods

3.1.3. PV of gradient

When payments increase by a fixed amount each period (arithmetic gradient), the present value is:

Formula:

$\mathbf{P = G[}\frac{1}{i} - \frac{n}{(1+i)^{n}-1}\mathbf{]}$ (for arithmetic gradient)

Where:

G = Gradient amount per period

i = Interest rate per period

n = Number of periods

3.1.4. Example

Suppose you are to receive Rs. 10,000 after 3 years and the interest rate is 10% per year.

$PV = 10{,}000 \div (1 + 0.10)^3$

$PV = 10{,}000 \div 1.331 = Rs.\ 7{,}514$

This shows that Rs. 7,514 invested today at 10% per year will grow to Rs. 10,000 in 3 years.

3.2. Future Value (FV) Formulas

3.2.1. Single payment future worth

The future value of a single present payment is the amount it will grow to after n periods at a given interest rate.

Formula:

$$\mathbf{FV = P\ (1 + i)^n}$$

Where:

FV = Future Value

P = Principal / Present Value

i = Interest rate per period

n = Number of periods

3.2.2. FV of annuity

The future value of an annuity is the sum of all payments compounded to the end of the period.

Formula:

$$\mathbf{F= A\ [}\frac{(1+i)^n - 1}{i}\mathbf{]}$$

Where:

A = Periodic payment

i = Interest rate per period

n = Number of periods

3.2.3. FV of gradient

For payments increasing by a fixed amount each period, the future value is:

Formula:

$$\mathbf{FV = G\ [\frac{(1+i)^{n} - i \times n - 1}{i^{2}}](1 + i)^n}$$

Where:

G = Gradient per period

i = Interest rate per period

n = Number of periods

3.2.4. Example

Suppose you invest Rs. 5,000 annually for 4 years at 10% interest.

$FV = 5{,}000 \times [((1 + 0.10)^4 - 1) \div 0.10]$

$FV = 5{,}000 \times (0.4641 \div 0.10)$

$FV = 5{,}000 \times 4.641 = \text{Rs. } 23{,}205$

This shows that Rs. 5,000 invested annually at 10% for 4 years will accumulate to Rs. 23,205.

3.3. Table Values / Interest Factor Table

n (Periods)	i=5%	i=10%	i=15%
1	0.952	0.909	0.870
2	0.907	0.826	0.756
3	0.864	0.751	0.658
4	0.823	0.683	0.572
5	0.784	0.621	0.497

4. Cash Flow

4.1. Concept

In engineering economics, cash flow refers to the movement of money into or out of a project, business, or investment over time. It represents actual monetary transactions that occur at different points during the life cycle of a project. Cash flows may be receipts (inflows) such as savings, revenues, or returns, or payments (outflows) such as operating costs, maintenance costs, capital investment, or taxes.

Because engineering decisions are spread over several years, cash flows do not occur all at once; instead, they occur periodically (monthly, quarterly, yearly). Therefore, cash flow analysis helps determine when money is received or paid and how much it is worth at each point in time. Since money has a time value, two amounts occurring at different times cannot be compared directly. Cash flow analysis, combined with TVM (present value/future value), provides a systematic way to convert and compare them.

Cash flow is crucial in evaluating the economic feasibility of engineering alternatives because it allows decision-makers to:

- Measure the economic performance of projects over time
- Assess profitability and payback capability
- Compare alternatives on a common financial basis
- Plan long-term financing, budgeting, and investment decisions

A structured cash flow model forms the foundation for techniques like Present Worth (PW), Future Worth (FW), Internal Rate of Return (IRR), and Payback Analysis. Hence, accurate identification and timing of cash flows is essential for reliable project evaluation in engineering economics.

4.2. Types of Cash Flows

4.2.1. Single Payment

4.2.1.1. Concept

A Single Payment Cash Flow refers to a situation where a lump sum amount is either received or paid once at a specific time in the future. There are no repeated cash flows - only one inflow or outflow.

Single payments commonly occur in engineering projects when:

- A machine is purchased at the start of the project (one-time outflow)
- A loan is repaid at a future date (one-time outflow)
- A salvage value is received at the end of asset life (one-time inflow)
- A bond matures and returns principal (one-time inflow)

To make meaningful economic decisions, this single future amount is converted to its equivalent present value (PV) or future value (FV) using the time value of money.

4.2.1.2. Formula

1. Future Value (FV) of a Present Payment

$$\mathbf{F = P\,(1 + i)^n}$$

Where:

- P = Present amount
- F = Future equivalent
- i = Interest/discount rate
- n = Number of periods

2. Present Value (PV) of a Future Payment

$$\mathbf{P} = \frac{F}{(1+i)^n}$$

4.2.1.3. Example

A construction firm will receive Rs. 200,000 after 3 years from a supplier contract. The discount rate is 10% per year. What is the present value of this future amount?

Given:

$F = 200,000, i = 0.10, n = 3$

Using:

$P = F / (1 + i)^n$

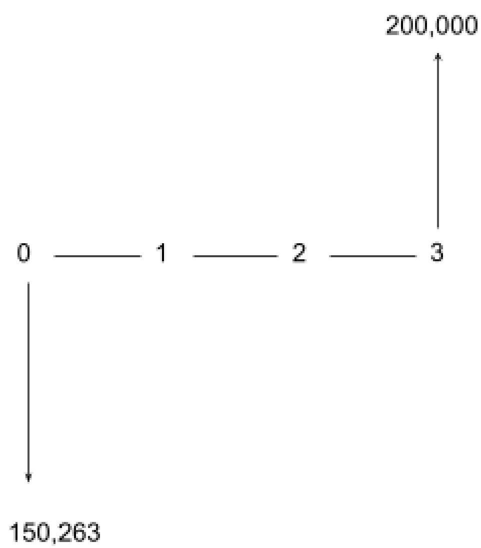
$P = 200,000 / (1.10)^3$

$P = 200,000 / 1.331 = 150,263$

Interpretation:

Receiving Rs. 200,000 three years later is economically equivalent to receiving Rs. 150,263 today.

4.2.1.4. Diagram (from supplier's point of view)



4.2.2. Uniform Payment

4.2.2.1. Concept

A Uniform Payment Cash Flow, also known as an Annuity, refers to a series of equal payments made or received at regular time intervals (e.g., yearly, quarterly, monthly).

Examples include:

- Annual maintenance cost for equipment
- Loan installments paid yearly
- Lease payments for machinery
- Equal returns from a contract

Since the payments are identical and periodic, the time value of money must be used to convert them into their present worth (PW) or future worth (FW).

Uniform payments occur commonly in engineering analysis when costs or revenues remain constant throughout the project's life.

4.2.2.2. Formula

1. Present Worth (PW) of Uniform Series

$$\mathbf{P= A \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$$

2. Future Worth (FW) of Uniform Series

$$\mathbf{F= A \left[\frac{(1+i)^n - 1}{i}\right]}$$

Where:

- A = Uniform annual payment
- i = Interest/discount rate
- n = Number of years
- P = Present worth
- F = Future worth

4.2.2.3. Working

To solve a uniform payment problem:

- Identify if the question requires present worth or future worth.
- Use the appropriate formula (PW or FW).
- Substitute the values of A, i, and n
- Compute the final answer and provide interpretation.

4.2.2.4. Examples

Example 1: Present Worth

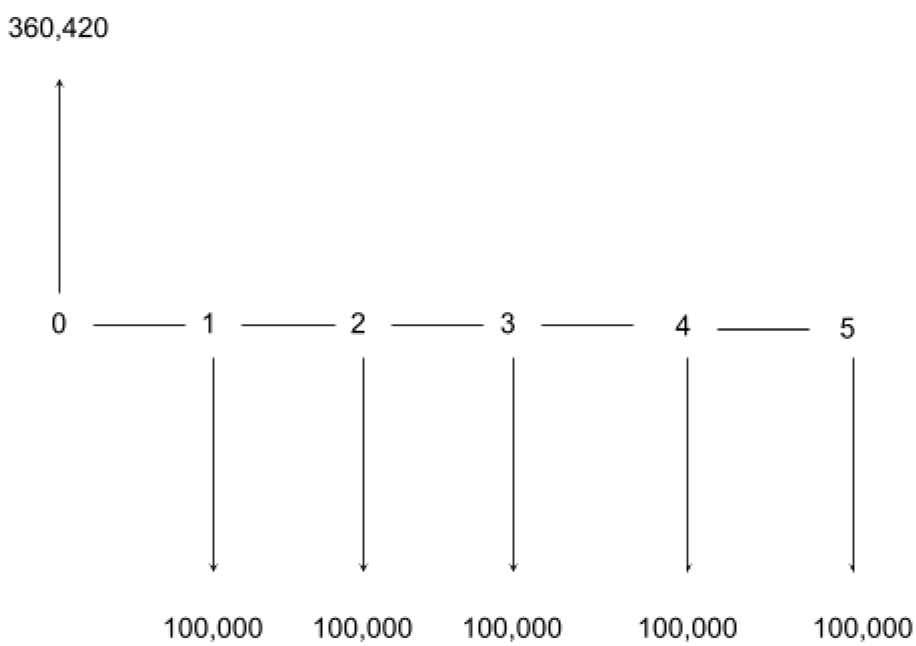
A factory pays Rs. 100,000 every year for 5 years as maintenance. The discount rate is 12%. Find the present worth.

Given:

$A = 100,000,\ i = 0.12,\ n = 5$

$P = 100,000(((1.12)^{n} - 1)/(0.12(1.12)^{n}))$
$= 100,000((1.763 - 1)/0.12(1.763))$
$= 100,000(0.7623/0.2115)$
$P = 360,420$
Interpretation:
The value of paying Rs. 100,000 every year for 5 years is equivalent to Rs. 360,420 today.
Example 2: Future Worth
A company invests Rs. 50,000 every year for 4 years in an energy system. Interest rate is 10%. Calculate the future worth at the end of Year 4.
Given:
$A = 50,000,\ i = 0.10,\ n = 4$
$F = 50,000(((1.10)^{4} - 1)/0.10)$
$= 50,000((1.4641 - 1)/0.10)$
$= 50,000(4.641)$
$F = 232,050$
Interpretation:
By the end of 4 years, the total future equivalent value becomes Rs. 232,050.

4.2.2.5. Diagram



4.2.3. Gradient Series

4.2.3.1. Concept

It is not common for cash flows in engineering economic analysis to stay the same throughout. To diminish the complexity of the analysis, the flow of costs or benefits increasing (or seldom decreasing) with a constant amount each year can be handled still through Gradient Series.

4.2.3.2. Arithmetic Gradient

Specifically, a sequence of cash flows in which the difference between each subsequent payment is constant is described as an Arithmetic Gradient. For instance, the annual maintenance costs rising by Rs. 15,000 each year, or the annual increment of Rs. 10,000 in the employee's salary, is a typical case of an arithmetic gradient.

With this model in place, engineers and managers can easily add "future increases" to their costs by simply transforming those increased future amounts into one present value. This makes it possible to compare with other options in a very meaningful way thus making investment choosing rational. On the other hand, if this transformation is not done, the future rising costs would lose their value and the resulting assessments would be wrong.

To illustrate how far the gradient analysis is particularly useful:

1. Maintenance budgeting (the costs go up because of wear and tear and usage)
2. Salary increments and promotions
3. Utility costs going up because of inflation or expansion
4. Operational costs in factories, plants, or software systems

The arithmetic gradient is observed from Year 2 because the cash flow for Year 1 does not belong to the gradient. The incremental increase starts thereafter.

4.2.3.3. Formula

The Present Worth (PW) of a gradient series is given as:

$$\mathbf{P = G[}\frac{1}{i} - \frac{n}{(1+i)^{n}-1}\mathbf{]}$$

Where:

G = Constant amount of gradient (annual increase)

i = Interest/discount rate

n = Number of years

P = Present worth equivalent of increasing gradient amounts

This equation translates all the future increases into one today's cash equivalent and can, therefore, be compared with other investment possibilities.

The significance of the equation is that it comprises both:

1. The time value of money, and
2. The impact of values that are constantly increasing.

4.2.3.4. Example

A software company expects annual maintenance expenses to start at Rs. 0 in Year 1, then increase by Rs. 12,000 every year for the next 5 years (Years 2-6). The discount rate is 9%. Find the present worth of this gradient series.

Given

G=12,000,i=0.09,n=6

Substituting in:

P=12,000$(1/0.09\ -\ 6/((1.09)^{6}$- 1)

$1.09^{6} = 1.677\ -1\ = 0.677$

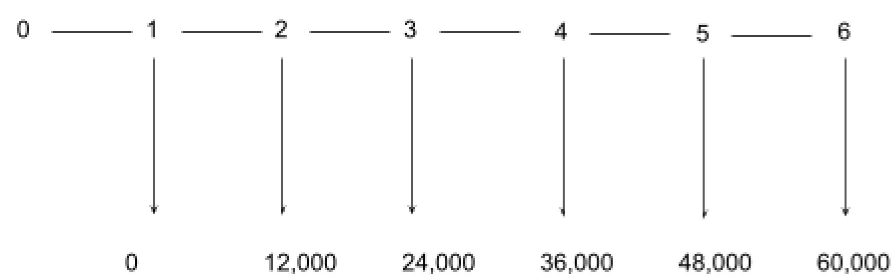
So:

$P\ = 12000(11.11 - 8.86)$

$P\ =\ 27,000$

Interpretation

The rising future expense stream is economically equivalent to paying **Rs. 27,000 today**.



4.2.3.5. Merits & demerits

Merits

1. Assists in assessing credible cost trends, where expenditures continuously increase over time.

2. Transforms increasing future cash flows into a single comparable amount (present value).
3. Extremely helpful in budget planning, salary predicting, and equipment maintenance scheduling.
4. Appropriate for long-term engineering asset life-cycle cost assessment.
5. Simplifies the process of making a choice when several options with rising costs are compared.

Demerits

1. Assumes a perfectly uniform increase, which might not be the case in case of fluctuating markets.
2. The outcome is super sensitive to the discount rate; even tiny mistakes in interest rate lead to big changes.
3. Demands a trustworthy forecasting of slope; uncertainty diminishes precision.
4. Not applicable when cash flows go up in irregular or nonlinear ways, for instance, through exponential growth.

4.2.3.6. Geometric Gradient Series

In numerous situations, the costs or revenues are not rising by a fixed monetary amount but rather by a fixed percentage every year. Such behavior is referred to as a Geometric Gradient Series.
A Geometric Gradient is portrayed as a cash flow series in which each payment is altered by an amount equal to a constant percentage rate (g) of the previous year's payment. An annual 8% increase in a company's revenue or a 5% rise in operating costs due to inflation would then classify these cash flows as subject to a geometric gradient phenomenon.
One of the main areas where geometrical gradients are of great value is:

1. Cost evaluation adjusted for inflation
2. Forecasting sales or revenue growth
3. Rate hikes of utility services determined by a percentage basis
4. Price uplift over a long period as a result of market phenomenon

Formula

For a geometric gradient series where the first payment is A1 at the end of year 1, and each payment grows by rate g per year, the **Present Worth (PW)** at interest rate i over n years is:

$$\mathbf{PW}=\frac{A_1\{1-(1+i)^{-n}(1+f)^{n}\}}{i+f}\ \mathbf{for\ i \neq f}$$

Where:
A_1 = cash flow in Year 1
i = interest/discount rate
f = geometric growth rate per year
n = number of years
PW = present worth of the geometric gradient series
This formula works when i≠g.

Examples

Revenue from a newly launched training program is expected to be **Rs. 220,000 in the first year**, and then grow by **6% each year** for **5 years**. The discount rate is **10%**.
Given: A1=220,000, g=0.06, i=0.10, n=5

PW=220000 $\times\ 1-(1.06/1.10)^{5}/(0.10-0.06)$

PW=220,000$\times\ 4.40=968,000$
Interpretation
The expected growth revenue over 5 years is equivalent to **Rs. 968,000 today**.

Merits

1. Theoretical models must be implemented to statistics percentages as growth or decline for instance in inflation and sales growth.
2. In case of values changing in proportions rather than by a fixed amount the arithmetic gradient can take as being less accurate than the other method.
3. This technique makes a significant contribution for long-term financial planning as well as when inflation or growth rates are already known or are simply assumed.

Demerits

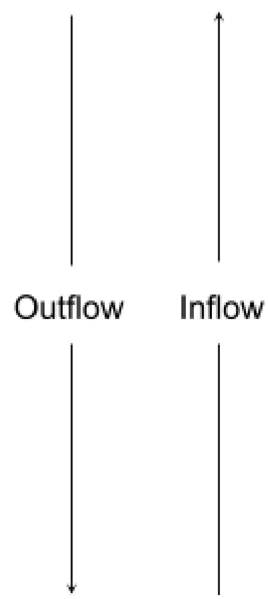
4. It is necessary to make an estimate of both the interest rate (i) and the growth rate (g), which might be uncertain.
5. If the g value is close to or even higher than the i value, the calculation of the present value becomes very delicate and may lead to unreasonable results.
6. Very small errors in the assumed growth rate can have a very large influence on the result.
7. The method is harder to calculate and understand than simple arithmetic gradients.

4.3. Cash Flow Diagrams

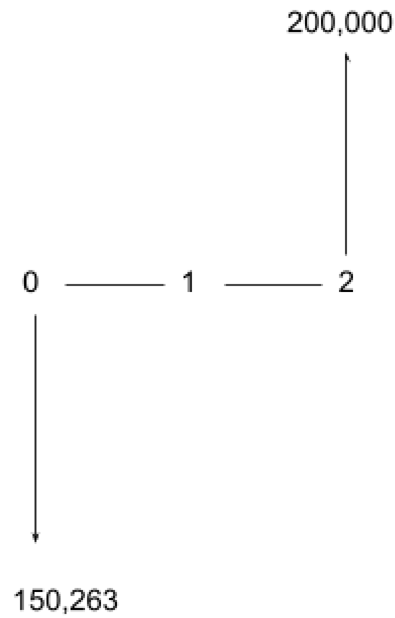
4.3.1. Timeline representation

0 ——— 1 ——— 2 ——— 3 ——— 4 ——— 5 ——— 6

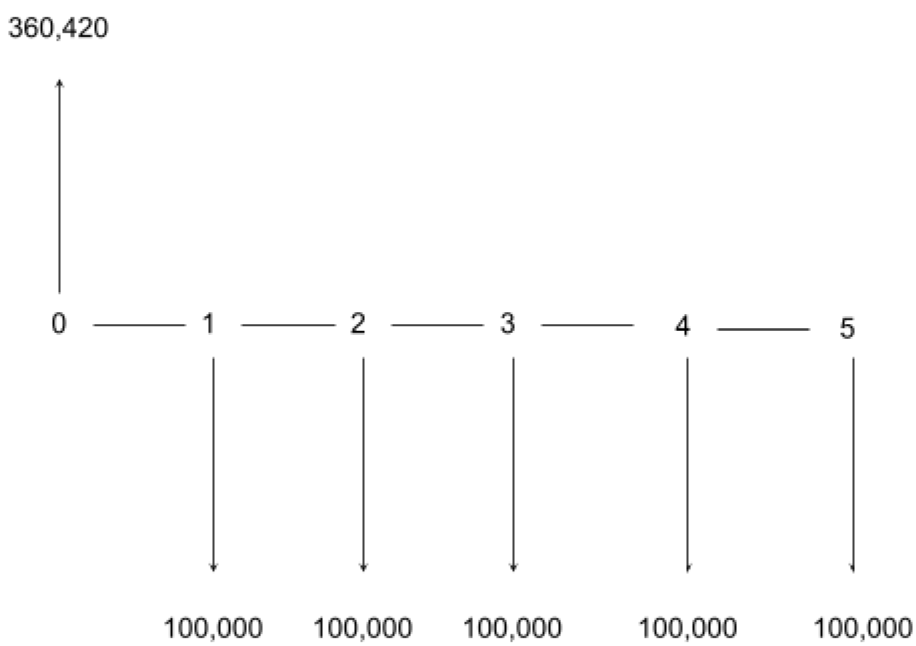
4.3.2. Arrows for inflow/outflow



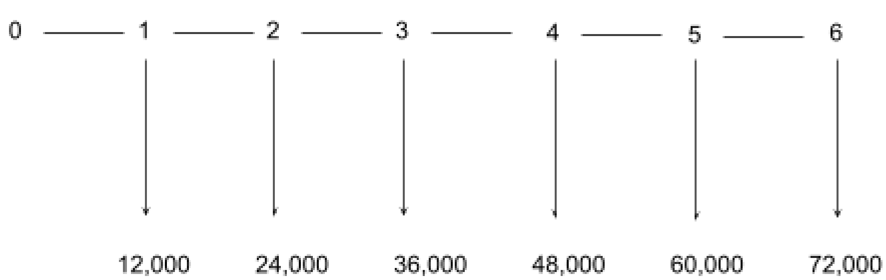
4.3.3. Single payment diagram



4.3.4. Uniform series diagram



4.3.5. Gradient series diagram



5. Payback Analysis

5.1. Concept

Payback Analysis is a method of assessing investments that computes the time needed to recover the initial cost of a project through the annual cash inflows. To put it another way, it tells the time to “break even.”
Industries have adopted it massively for prompt decisions, particularly in cases where cash is short or the projects have an uncertain or risky nature. A shorter payback period is regarded as a better investment opportunity.

5.2. Payback period formula

When annual inflows are equal:

$$Payback\ Period = Initial\ Investment / Annual\ Cash\ flow$$

When annual inflows are unequal, cumulative inflows are used to identify the exact recovery year.

5.3. Example

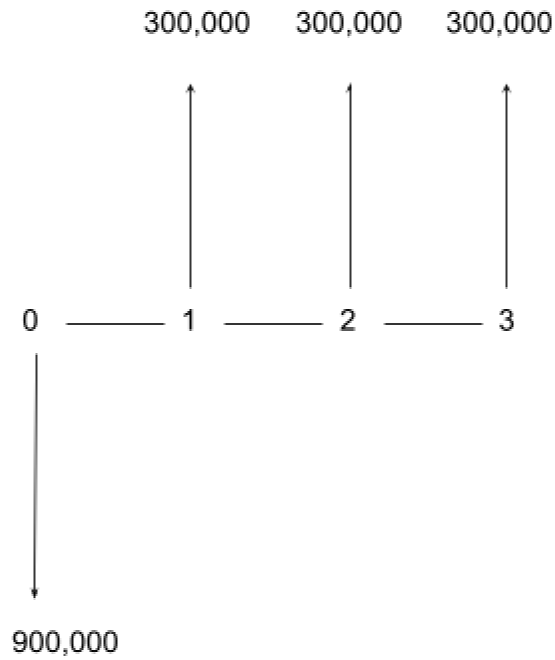
Example 1

A food processing unit installs an automatic packaging system costing **Rs. 900,000**. Annual savings are **Rs. 300,000**.

$$Payback\ Period = 900,000/300,000 = 3\ years$$

Interpretation

The investment breaks even in 3 years, and after that all annual savings are profit.



Example 2

A manufacturing firm installs solar panels costing **Rs. 700,000**. Annual reduction in electricity cost = **Rs. 175,000**.

$Payback\ Period\ =\ 700,000/175,000\ =\ 4\ years$

Interpretation:

The investment will recover its cost in 4 years, providing savings afterward.

5.4. Merits & Demerits

Merits

1. Simple to calculate and grasp.
2. Handy for choosing projects with rapid capital recovery.
3. Minimizes risk by not committing to uncertain investments for long periods.
4. Suitable for sectors with rapid technological obsolescence.

Demerits

1. Doesn't incorporate time value of money.
2. Disregards cash inflows occurring after the payback period.
3. Cannot assess the total profitability of the project.

4. May prevent acceptance of important long-term projects.

6. Present Worth Analysis (PW)

6.1. Concept

Present Worth (PW) analysis that is also referred to as Present Value (PV) analysis is a basic method in engineering economics applied to turn future cash flows into their present day equivalent value. Due to the fact that money has a time value, receiving Rs. 100 three years from now will be considered less valuable than receiving Rs. 100 today. PW analysis adopts this time value technique by discounting the future receipts and payments to the present moment.

PW analysis is primarily for:

1. To determine the economic feasibility of a project
2. To put several alternatives on the same monetary basis and comparison
3. To choose between investment options that mutually exclude each other
4. To enhance the financial accounting of costs and benefits throughout the life of the project

A positive PW tells that the project is economically acceptable, and on the other hand, a negative PW indicates the rejection of the project.

6.2. Formula

The general formula to convert a future cash flow F occurring after n years to its present worth is:

$$\mathbf{P} = \frac{F}{(1+i)^n}$$

Where:

P = Present worth

F = Future value

i = Interest/discount rate

n = Number of years

For multiple future cash flows:

$$P = \sum_{t=1}^{n} \frac{F_t}{(1+i)^t}$$

This formula ensures that all future amounts are expressed in today's value, making comparison meaningful.

Uniform Payment:

To find the present worth P of equal annual payments A:

$$\mathbf{P} = \mathbf{A}\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

6.3. Example

A machine requires Rs. 450,000 initially and generates Rs. 150,000 per year for 4 years. The discount rate is 12%.

Given
Initial Cost = Rs. 450,000
Annual Returns = Rs. 150,000
i=0.12,n=4

$$PW = -\ 450,000\ + \sum_{t=1}^{4} 150,000/(1.12)^{t}$$

$$PW = -\ 450,000 + (133,929 + 119,600 + 106,800 + 95,300)$$

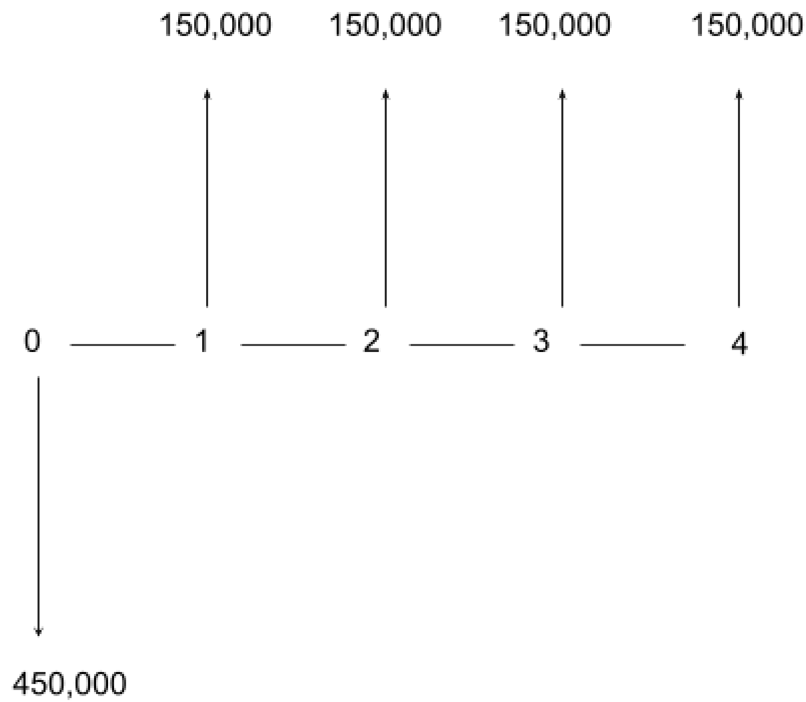
$$PW = -\ 450,000 + 455,629 = 5,629$$

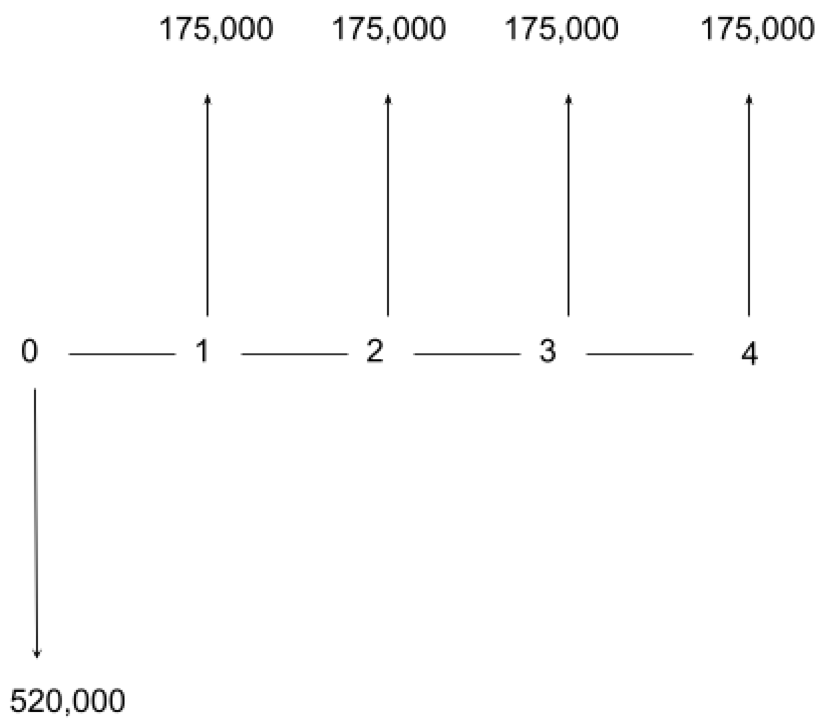
Interpretation
PW is **positive**, which means the project is economically acceptable.

6.4. PW comparison of alternatives

PW is often used to select between two or more alternatives. The project with the highest PW is preferred.
Example Comparison
Two machine options are available with interest rate of 12%:

Alternative	Initial Cost	Annual Return	Life (yrs)	PW Result
A	450,000	150,000	4	+5,629
B	520,000	175,000	4	+11,650





Decision:

- Alternative A has lesser **positive PW**, Alternative B has greater **positive PW**.
- Therefore, **Alternative B is the better choice.**

6.5. Merits & Demerits

Merits

1. Takes into account the time value of money, thus yielding results that are realistic and trustworthy.
2. Accounts for all money movements throughout the life of the project.
3. Creates a unified currency measurement, which is very simple to compare among different choices.
4. Perfect for long-term planning and big capital investment decision-making.
5. Functions especially good if project durations are not the same.

Demerits

1. Necessitates a proper discount rate, which could be tricky to forecast.
2. Very much influenced by the interest rate scenario (even minor fluctuations can lead to wrong results).
3. Needs estimation of future expenses and revenues which can be tricky due to uncertainty.
4. A bit more complicated than the simpler techniques such as payback period analysis.

7. Rate of Return (ROR) Analysis

7.1. Concept

The effective interest rate earned on an investment or paid on a loan over a specific time period is known as the rate of return, or ROR. It illustrates how quickly money increases in relation to the initial investment. By comparing the return to the minimum acceptable rate of return (MARR), ROR in engineering economics aids in determining if a project is appealing. The project is typically deemed acceptable if the computed rate of return is higher than or equal to MARR.

7.2. IRR definition

The interest rate that produces the following is known as the Internal Rate of Return (IRR):

$$PW = 0 \qquad \text{or} \qquad FW = 0$$

IRR is the rate at which the present value of cash inflow and outflow are equal.It is a project's break-even interest rate.

7.3. How to find ROR

The rate of return can be found in different types of situations:

7.3.1. Single payment case

If we are given present amount P,future amount F, and number of periods n, we use the following formula:

$$\mathbf{F=P\ (1+i)^{n} \quad or \quad i=(F/P)^{1/n}-1}$$

We can also use the interest table to find the factor F/P and read the corresponding i.

7.3.2. Uniform series (annuity) case

If we are given uniform annual payment A and either P or F and n, we use the following annuity relations:

$$\mathbf{P= A\ [\tfrac{(1+i)^{n}-1}{i(1+i)^{n}}] \quad and \quad F= A\ [\tfrac{(1+i)^{n}-1}{i}]}$$

We often solve by trial-and-error or interpolation between two tabulated values of the factor because " **i**"exists inside the formula.

7.3.3. Given principal, future value, and interest rate - find number of periods

In this case, we use the following formula:

$$\mathbf{F=P\ (1+i)^{n} \quad or \quad n=\ [\tfrac{log(F/P)}{log(1+i)}]}$$

This indicates how long it will take for money to increase at a specific rate.

7.3.4. Given principal (or future value), interest rate, and number of years - find annual payment A

For loans and sinking funds, we can use the following formulas:

$$\mathbf{A= P\ \ [\tfrac{i\,(1+i)^{n}}{(1+i)^{n}-1}] \quad and \quad A= F\ [\tfrac{i}{(1+i)^{n}-1}]}$$

7.4. Examples

7.4.1. Example 1 - Finding the Rate of Return for a Single Payment

Aimen invests Rs. 80,000 in a government bond. After 6 years, she receives Rs. 1,15,000. Find the annual rate of return on her investment.

Given:

P = 80,000

F = 1,15,000

n = 6 years

Solution:

We will use the formula: **F=P (1+i)^n**

So, $1 + i = (F / P)^{1/n}$

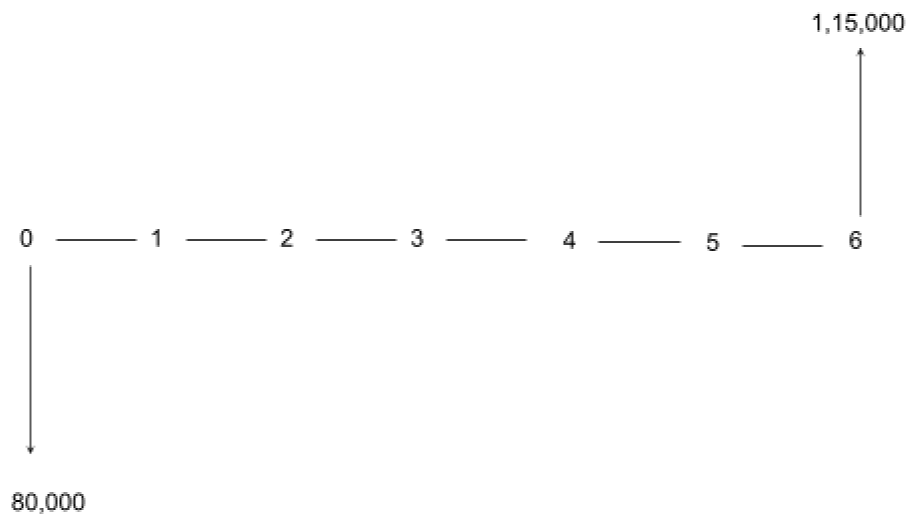
$1 + i = (115000 / 80000)^{1/6}$

$1 + i = (1.4375)^{1/6}$

Based on calculator: $(1.4375)^{1/6} \approx 1.0614$

So, i = 0.0614 or 6.14% per year

Aimen earns an effective annual rate of return of about **6.14%** on this bond.



7.4.2. Example 2 - Finding IRR for an Annuity (Using Interpolation)

Ayesha plans to buy a small apartment. She can invest Rs. 200,000 now and then deposit Rs. 40,000 at the end of each year for 5 years. At the end of 5 years she expects to sell the apartment for Rs. 500,000. Find the approximate rate of return on her investment.

Given:

Initial investment (purchase) at year 0: P = Rs. 200,000

Rental income at end of each year for 5 years: A = Rs. 40,000

Sale value of apartment at end of year 5: F = Rs. 500,000

Find the rate of return i such that Net Present Worth (NPW) A = Rs. 40,000

Solution:

Present worth of uniform series (annuity) = $\mathbf{A}\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

Present value of single future amount = $\frac{F}{(1+i)^n}$

Here, n=5

So, net present worth = NPW = -200,000 + 40,000 * $\frac{(1+i)^5-1}{i\,(1+i)^5} + \frac{500,000}{(1+i)^5}$

We want to find the interest rate for which NPW = 0.We can't solve this algebraically so we try different values of i and then interpolate.

Trying i= 30%:

First we will find $(1+i)^5$:

$(1+0.30)^5 = 1.30^5 \approx 3.71293$

Finding Present worth of annuity = 40,000 $[\frac{3.71293-1}{0.30 * 3.71293}]$

$= 40,000\ [\frac{2.71293}{1.11388}]$

$\approx 40,000 * 2.4356 \approx 97,424$

Finding Present worth of sale value = $\frac{500,000}{3.71293} \approx 134,650$

Finding NPW = −200,000+97,424+134,650 = +32,074

Trying i= 40%:

First we will find $(1+i)^5$:

$(1+0.40)^5 = 1.40^5 \approx 5.37824$

Finding Present worth of annuity = 40,000 $[\frac{5.37824-1}{0.40 * 5.37824}]$

$= 40,000\ [\frac{4.37824}{2.15130}]$

$\approx 40,000 * 2.035 \approx 81,400$

Finding Present worth of sale value = $\frac{500,000}{5.37824} \approx 93,000$

Finding NPW =−200,000+81,400+93,000=−25,600

So, IRR lies between 30% and 40% as for i=30% we got a positive NPW and the project was very attractive, but for i=40% we got a negative NPW and the project was not attractive.

We have, i_1=30%, NPW_1=+32,074

And i_2=40%, NPW_2= -25,600

If we use straight line interpolation:

$$i_{IRR} = i_1 + \frac{NPW_1}{NPW_1 - NPW_2}(i_2 - i_1)$$

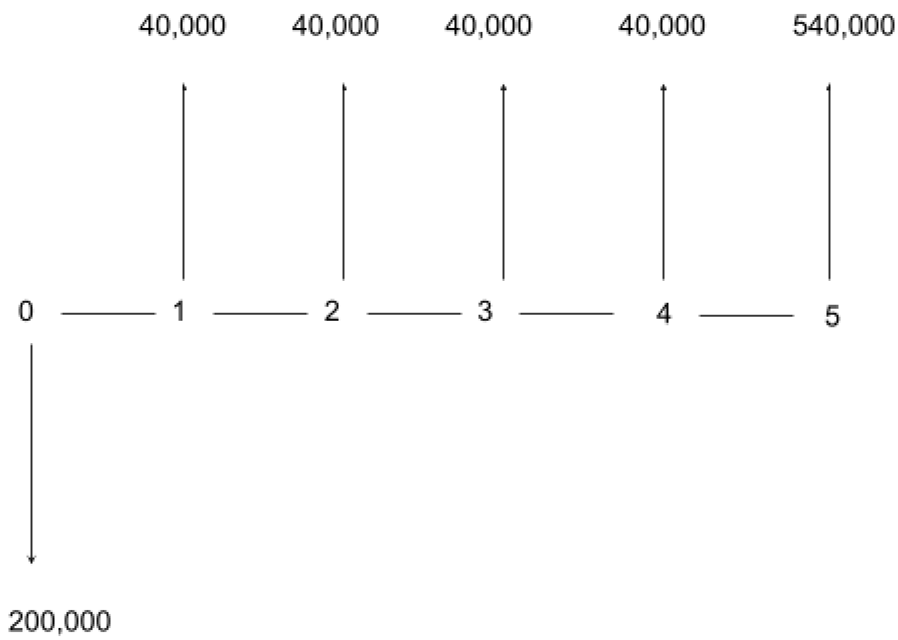
$$\approx 30\% + \frac{32,074}{32,074-(-25,600)}(40\% - 30\%)$$

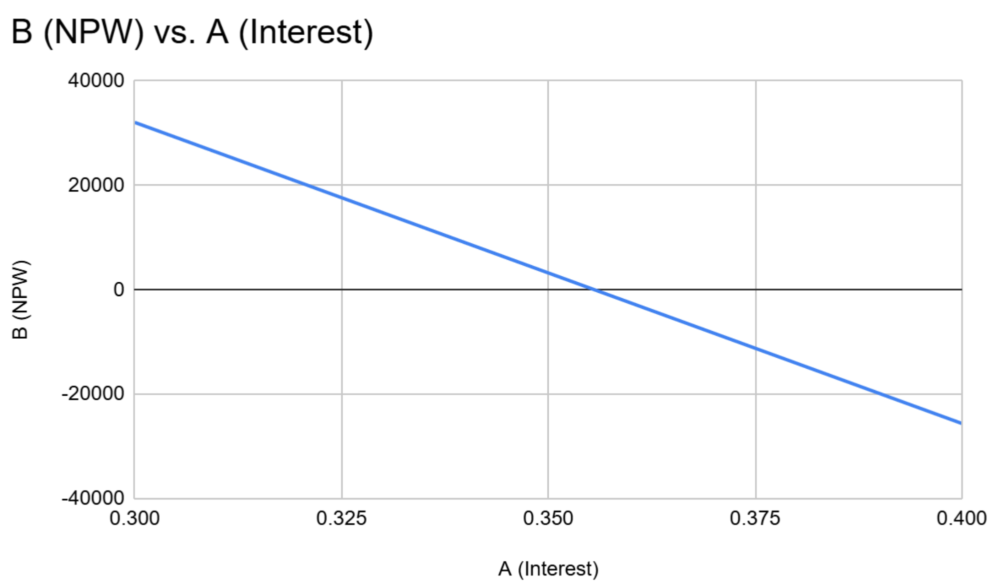
$$= 30\% + \frac{32,074}{57,674}(10\%)$$

$$\approx 30\% + 0.556 \times 10\%$$

$$\approx 30\% + 5.56\% \approx 35.6\%$$

So, the rate of return of Ayesha's investment is 35.6% per year.





7.4.3. Example 3 - Finding Number of Years from Given P, F and i

Hira deposits Rs. 25,000 into a savings account that pays 9% interest compounded annually. She wants the account to grow to Rs. 1,00,000. How many years will this take?

Given:

P = 25,000

F = 100,000
i = 0.09

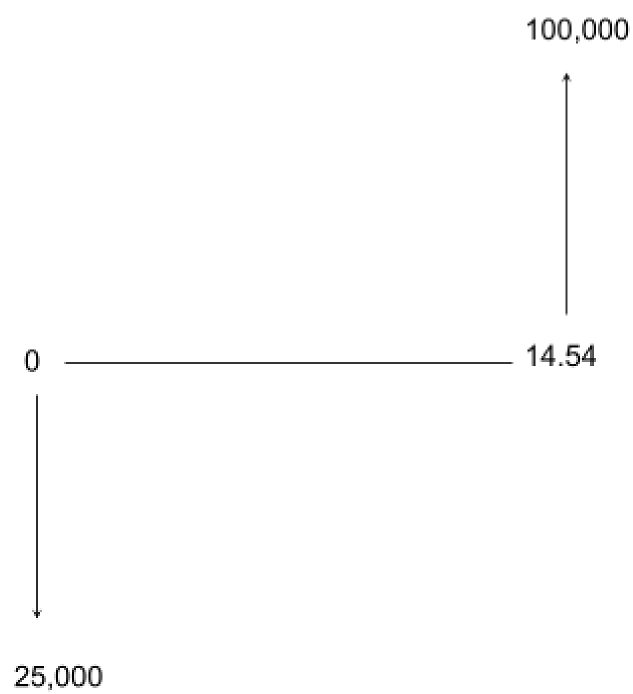
Solution:
Using $F=P(1+i)^n$:

$$100{,}000=25{,}000(1.09)^n$$
$$\frac{100{,}000}{25{,}000}=(1.09)^n$$
$$4=(1.09)^n$$

Taking logs:

$$n=\frac{\log 4}{\log 1.09}\approx\frac{0.6021}{0.0414}\approx 14.54 \text{ years}$$

So, Hira's money will take about 14.5 years to grow from Rs. 25,000 to Rs. 1,00,000 at 9% interest.



7.4.4. Example 4 - Finding Annual Payment A for a Required Future Amount

Maheen wants to create a fund for her higher studies. She needs Rs. 8,00,000 after 6 years. The bank offers 11% interest compounded annually. How much must she deposit every year (equal annual payments) to reach this amount?

Given:
F = 8,00,000
i = 0.11

n = 6
A = ?

Solution:

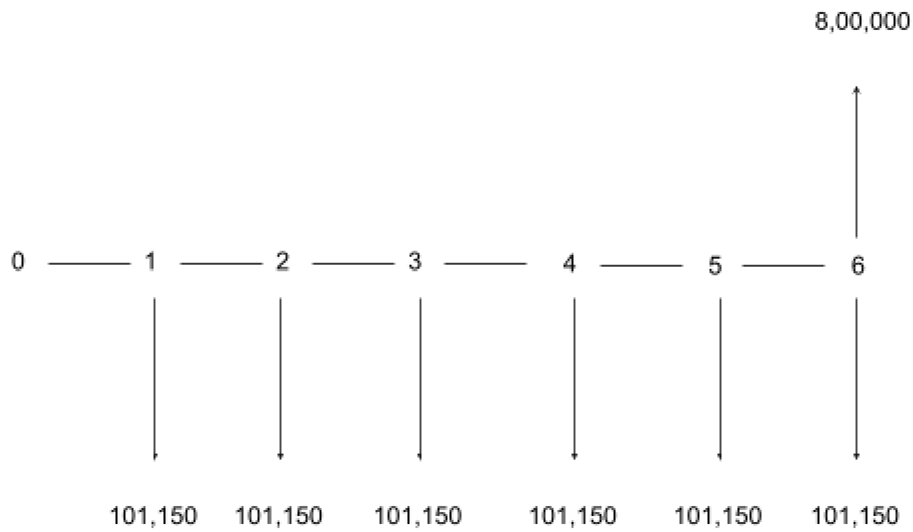
Using the uniform series future worth relation: F= A $\left[\frac{(1+i)^n - 1}{i}\right]$

So, A= F $\left[\frac{i}{(1+i)^n - 1}\right] = \frac{8,00,000 \times 0.11}{(1.11)^6 - 1}$

$(1.11)^6 \approx 1.870$

So, A= $\frac{88,000}{1.870-1} = \frac{88,000}{0.870} \approx 1,01,150$

So, Maheen must deposit about Rs. 1,01,150 every year for 6 years to have Rs. 8,00,000.



7.5. Merits & Demerits

7.5.1. Merits

- It is expressed as a percentage, which is simple to comprehend and compare with MARR or bank rates.
- It is useful for ranking different projects that have different initial investments.

7.5.2. Demerits

- For uneven cash flows, its calculation is complex and requires the usage of trial and error or any software.
- Making decisions might be complicated by some cash flow patterns that provide multiple IRRs or no real solution.
- IRR makes the somewhat unrealistic assumption that all intermediate cash flows are reinvested at the same rate.

8. Future Worth Analysis (FW)

8.1. Concept

Future Worth (FW) analysis transforms all project cash flows into an equivalent sum at a specific future date, typically the project's completion. Rather than asking, "What is everything worth today?" "What will everything be worth at the end of the study period?" is the question we pose (PW analysis). When calculated at the lowest acceptable rate of return, a project is deemed acceptable if its FW of benefits minus FW of expenses is positive.

8.2. FW Formulas

We utilize the following formulas for the future worth analysis:

8.2.1. Single Payment

To find the future value of a present amount we use the following formulas:

$$\mathbf{F = P\,(1+i)^n \quad or \quad F/P = (1+i)^n}$$

8.2.2. Uniform Series (Annuity)

To find the future value of equal annual payment A we use the following formulas:

$$\mathbf{F = A}\left[\frac{(1+i)^n - 1}{i}\right]$$

8.3. Example

8.3.1. Example 1 - FW of a Single Investment plus Annuity

Zara invests Rs. 1,50,000 today in a solar energy project and plans to invest an additional Rs. 40,000 at the end of each year for 4 years. The expected rate of return is 12% per year. Find the future worth of all her investments at the end of Year 4.

Given:
P = 1,50,000 at t = 0
A = 40,000 per year (Years 1–4)
i = 0.12
n = 4

Solution:
Finding future worth of single payment P = $P(1+i)^n$ = 1,50,000 * $(1.12)^4$
$(1.12)^4 \approx 1.5735$

$\approx 1,50,000 \times 1.5735 = 2,36,025$

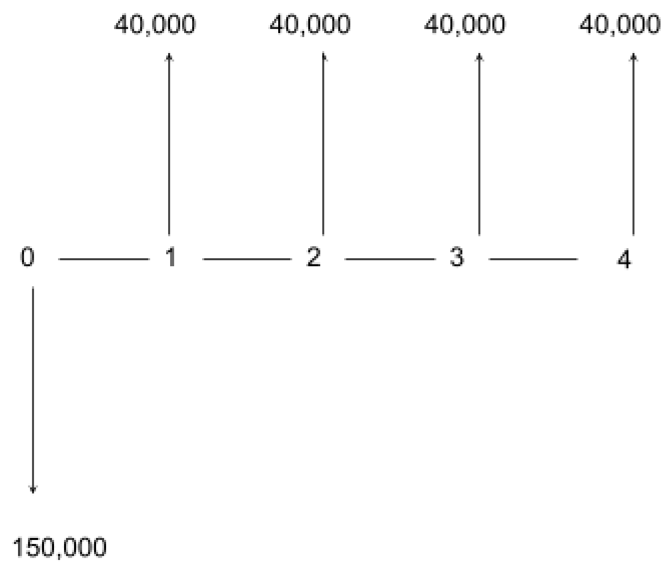
Finding future worth of annuity A = A $[\frac{(1+i)^n - 1}{i}]$ = 40,000* $[\frac{(1.12)^4 - 1}{0.12}]$

$(1.12)^4 - 1 = 0.5735$

= 40,000 * $[\frac{0.5735}{0.12}]$ $= 40,000 \times 4.779 \approx 1,91,160$

FW=FP+FA=2,36,025+1,91,160=4,27,185

So, at 12% interest, Zara's total investment will be worth about Rs. 4,27,185 at the end of 4 years.



8.3.2. Example 2 - FW Comparison of Two Alternatives

Fatima is choosing between two water-filter systems for a hostel. The study period is 5 years, and the interest rate is 10% per year.

Alternative A : Cheaper system

- Initial cost now: Rs. 2,00,000
- Annual operating cost: Rs. 40,000 per year (Years 1–5)
- No salvage value

Alternative B : Expensive system

- Initial cost now: Rs. 2,80,000
- Annual operating cost: Rs. 25,000 per year (Years 1–5)
- Expected salvage value at end of year 5: Rs. 30,000

Which alternative is better using Future Worth analysis?

Solution:

FW of Alternative A

Future worth of initial cost = $2,00,000(1.10)^5$

$(1.10)^5 = 1.61051$

$= 2,00,000 \times 1.61051 = 3,22,102$

Future worth of annual operating costs (outflows) = 40,000* $[\frac{(1.10)^5 - 1}{0.10}]$

$(1.10)^5 - 1 = 0.61051$

$= 40,000 \times 6.1051 = 2,44,204$

Total FW of costs for A (no salvage)= FWA= FPA+FAA= 3,22,102+ 2,44,204= 5,66,306

FW of Alternative B

Future worth of initial cost = $2,80,000(1.10)^5 = 2,80,000 \times 1.61051 =$ 4,50,942.8

Future worth of operating costs = 25,000* $[\frac{(1.10)^5 - 1}{0.10}]$ =25,000 $\times 6.1051 = 1,52,627.5$

Future worth of salvage value (inflow)

Salvage is already at year 5, so no conversion: FS=30,000F_S = 30,000FS=30,000

Total FW of costs for B (costs - salvage)= FWB=FPB+FAB−FS

FWB=4,50,942.8+1,52,627.5−30,000=5,73,570.3FW_B = 4,50,942.8 + 1,52,627.5 - 30,000 = 5,73,570.3FWB=4,50,942.8+1,52,627.5−30,000=5,73,570.3

Decision

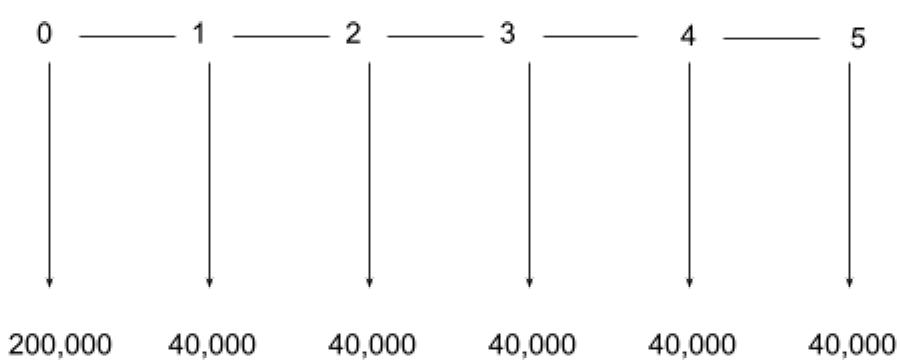
FWA ≈ 5,66,306

FWB ≈ 5,73,570

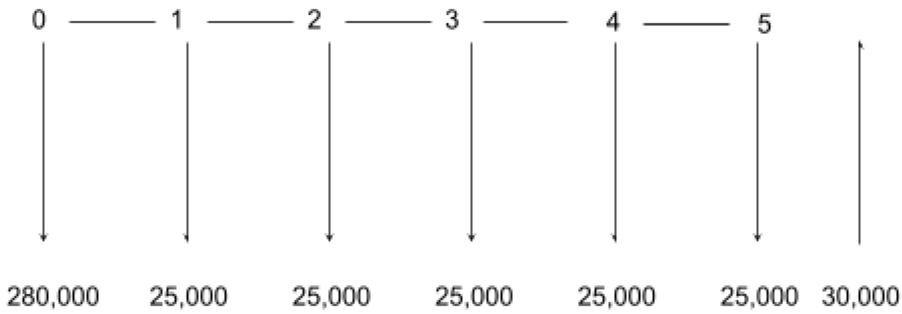
Since we are comparing **costs**, the **smaller future worth of cost** is preferred.

Therefore, **Alternative A** is slightly better than B at 10% interest.

Alternative A:



Alternative B:



8.4. FW comparison of alternatives

When utilizing FW to compare two or more engineering options:

- Select a typical study period, which is often the least frequent multiple of project lives.
- At the conclusion of the research period, use the relevant formulas to convert all of the cash inflows and outflows of each alternative to a single figure.
- Choose the project with the highest FW of net benefits for revenue-based projects (benefits > costs).
- Choose the option with the lowest FW of total cost for cost-only projects (e.g., choosing between designs that offer the same service).
- Since the FW and PW methods are merely different points on the same timeline, they always provide the same result.

8.5. Merits & Demerits

8.5.1. Merits

- It takes compound interest and the time value of money into account.
- It helps with long-term planning and goal-setting by giving a clear picture of the entire worth at a selected future date.
- It is particularly useful when a project inherently terminates at a later date (e.g., required fund target, replacement time).

8.5.2. Demerits

- It requires precise projections of future expenses, revenues, and interest rates - all of which may be uncertain
- Because decisions are frequently made in terms of current money, some people believe that future worth is less intuitive than present worth.
- Results can vary greatly if the interest rate assumption is altered, just like with any discounted cash-flow method.

9. Conclusion

In engineering economics, the Time Value of Money (TVM) is crucial because it acknowledges that the value of money varies over time. Engineers can assess and analyze options logically and consistently by utilizing notions like present worth, future worth, rate of return, and payback period. TVM assists in determining which alternative yields the best returns, how long it takes for investments to recoup, and whether projects are economically viable. These same ideas are used in real life for budgeting, retirement planning, investments, loans, and savings. In general, an understanding of TVM facilitates more effective long-term planning, efficient resource allocation, and better financial decision-making in both engineering and ordinary financial scenarios.

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