DIRACTION

Cracking the Coding Interviews

Assignment # 11

(Divide and Conquer + Prune and Search + Solving Recurrences)

Part I: (This should take 2.5 hours by one team) **37 points**

Solve all the following recurrences.

1. $T(n) = T(n-1) + n$

$= n + (n-1) + (n-2) + \cdots + 1$

$= \boxed{O(n^2)}$

2. $T(n) = T(n-1) + n^2$

$= n^2 + (n-1)^2 + (n-2)^2 + \cdots 3^2 + 2^2 + 1^2$

$= \boxed{O(n^3)}$

3. $T(n) = T(n/2) + n^2$

$= \cancel{2}n^2 + (n/2)^2 + (n/4)^2 + \cdots + 2^2 + 1^2$

$= O(2n^2) \Rightarrow \boxed{O(n^2)}$

4. $T(n) = T(n/2) + n^3$

$= n^3 + (n/2)^3 + (n/4)^3 + \cdots + 2^3 + 1^3$

$= O(2n^3) \Rightarrow \boxed{O(n^3)}$

5. $T(n) = 4T(n/2) + n^2$

$n^2, \frac{4n^2}{\cancel{4}}$ $\boxed{\text{same}}$

$n^2 \log_2 n \Rightarrow \boxed{O(n^2 \log_2 n)}$

6. $T(n) = 4T(n/2) + n^3$

$n^3, \frac{4n^3}{8} \Rightarrow n^3/2$ $\boxed{\text{dec}}$

$\boxed{O(n^3)}$

19. $T(n) = 3T(n/3) + n^{1/2}$ $\boxed{\text{inc}}$

$n^{1/2}, 3\sqrt{\frac{n}{3}} \Rightarrow \sqrt{3}\sqrt{n}$

$3^{\log_3 n} \Rightarrow n^{\log_3 3} \Rightarrow \boxed{O(n)}$

20. $T(n) = 2T(n/4) + n^{1/2}$

$n^{1/2}, 2\sqrt{\frac{n}{4}} \Rightarrow \frac{\cancel{2}\sqrt{n}}{\cancel{2}} = \sqrt{n}$ $\boxed{\text{same}}$

$\sqrt{n} \log_4 n \Rightarrow \boxed{O(\sqrt{n} \log_4 n)}$

21. $T(n) = 7T(n/3) + n^2$

$n^2, \frac{7n^2}{9}$ $\boxed{\text{dec}}$

$\boxed{O(n^2)}$

22. $T(n) = 7T(n/2) + n^2$

$n^2, \frac{7n^2}{4}$ $\boxed{\text{inc}}$

$7^{\log_2 n} \Rightarrow n^{\log_2 7} \Rightarrow \boxed{O(n^2)}$

23. $T(n) = 2T(n/4) + n^2$

$n^2, \frac{2n^2}{16} \Rightarrow \frac{n^2}{8}$ $\boxed{\text{dec}}$

$\boxed{O(n^2)}$

24. $T(n) = T(n-4) + n^2$

$n^2 + (n-4)^2 + (n-8)^2 + \cdots + 8 + 4 + 1$

$O\left(\frac{n^3}{4}\right) \Rightarrow \boxed{O(n^3)}$

7. T(n) = 27T(n/3) + n^3

$n^3, \frac{\cancel{27}n^3}{\cancel{27}} \Rightarrow n^3$ $\boxed{\text{same}}$

$n^3 \log_3 n \Rightarrow \boxed{O(n^3 \log_3 n)}$

8. T(n) = 27T(n/4) + n^3

$n^3, \frac{27n^3}{64}$ $\boxed{\text{dec}}$

$\boxed{O(n^3)}$

9. T(n) = 3T(n/3)+n

$n, \cancel{3}n/\cancel{3} = n$ $\boxed{\text{same}}$

$\boxed{O(n \log_3 n)}$

10. T(n) = 3T(n/3)+n^2

$n^2, \frac{3n^2}{27}$ $\boxed{\text{dec}}$

$\boxed{O(n^2)}$

11. T(n) = 2T(n/2) + 1

$1, \cancel{n/2}\ 2$ $\boxed{\text{inc}}$

$2^{\log_2 n} \Rightarrow n^{\log_2 2} \Rightarrow n$

$\boxed{O(n)}$

12. T(n) = 2T(n-1) + 1

$1, 2$ $\boxed{\text{inc}}$

$\boxed{O(2^n)}$

13. T(n) = 2T(n/4) + 1

$1, 2$ $\boxed{\text{inc}}$

$2^{\log_4 n} \Rightarrow n^{\log_4 2} \Rightarrow \boxed{O(n)}$

25. T(n) = 2T(n-4) + n^2

$n^2, 2(n-4)^2$ $\boxed{\text{inc}}$

$2^{n/4} \Rightarrow O(2^n)$

26. T(n) = T(n-1) + 1/n

// Read what is Harmonic Series

$\frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{3} + \frac{1}{2} + 1$

$\Rightarrow \log n$

$\Rightarrow \boxed{O(\log n)}$

27. T(n) = T(n-1) + lg n.

$\lg n, \lg(n-1)$ $\boxed{\text{same}}$

$n \lg n \Rightarrow \boxed{O(n \lg n)}$

28. T(n) = T(n/2) + T(n/4) + T(n/8) + 1

$1, 3$ $\boxed{\text{inc}}$

$\Omega(3^{\log_8 n}) \Rightarrow n^{\log_8 3} \Rightarrow \boxed{n^2}$

$O(3^{\log_2 n}) \Rightarrow n^{\log_2 3} \Rightarrow \boxed{O(n)}$

29. T(n) = T(n/2) + T(n/4) + T(n/8) + n

$n, \left(\frac{n}{2} + \frac{n}{4} + \frac{n}{8}\right) \Rightarrow \frac{7n}{8}$

$\boxed{O(n)}$ $\boxed{\text{dec}}$

30. T(n) = T(n/3)+T(2n/3) + n

$n, \left(\frac{n}{3} + \frac{2n}{3}\right) \Rightarrow \frac{\cancel{3}n}{\cancel{3}} = n$ $\boxed{\text{same}}$

$n \log_{2/3} n$

$\Rightarrow O(n \log_{2/3} n)$

31. T(n) = log n x T(n/log n) + 2n

Does not apply

logn is not constant

14. T(n) = 4T(n/2)+ n^2 lg n

$n^2 \lg n$, $\frac{4n^2}{4} \log \frac{n}{2}$ $\boxed{\text{Same}}$

$n^2 \lg n (\lg n)$ ⇒ $\boxed{O(n^2 \lg^2 n)}$

15. T(n) = 3T (n/2)+ n

n , $3n/2$ $\boxed{\text{inc}}$

$3^{\log_2 n}$ ⇒ $n^{\log_2 3}$ ⇒ n ⇒ $\boxed{O(n)}$

16. T (n) = 3T (n/3)+ n/2

$\frac{n}{2}$, $3 \frac{\frac{n}{2}}{3} = n/2$ $\boxed{\text{same}}$

$n/2 \log_3 n$ ⇒ $\boxed{O(n \log_3 n)}$

17. T(n) = $n^{1/2}$ T($n^{1/2}$) + n

Does not apply

$\sqrt{n}$ is not constant

18. T(n) = 3T(n-1) + 1

1 , 3 $\boxed{\text{inc}}$

3^n ⇒ $\boxed{O(3^n)}$

32. T (n) = 6T (n/3)+ n^2 log n

$\boxed{\text{dec}}$

$n^2 \log n$, $\frac{6n^2}{9} \log n/3$

$\boxed{O(n^2 \log n)}$

33. T (n) = T(n/4) + 5T(n/5) + T(n/3) + n^3

n^3, $\left(\frac{n^3}{64} + \frac{5n^3}{125} + \frac{n^3}{27}\right)$ $\boxed{\text{dec}}$

$\boxed{O(n^3)}$

34. T (n) = T(n/4) + 5T(n/5) + T(n/3) + n^2

n^2, $\left(\frac{n^2}{16} + \frac{5n^2}{25} + \frac{n^2}{9}\right)$ $\boxed{\text{dec}}$

$\boxed{O(n^2)}$

35. T (n) = T(n/4) + 5T(n/5) + T(n/3) + n

n, $\left(\frac{n}{4} + \frac{n}{5} + \frac{n}{3}\right)$ $\boxed{\text{dec}}$

$\boxed{O(n)}$

36. T (n) = T(n/4) + 5T(n/5) + T(n/3) + 1

1 , 3 $\boxed{\text{inc}}$

$3^{\log_3 n}$ ⇒ $n^{\log_3 3}$ ⇒ $\boxed{O(n)}$

37. T(N) = T($n^{1/2}$)+ N

n, $\sqrt{n}$ $\boxed{\text{dec}}$

$\boxed{O(n)}$

Some Hints

N, N/2 , N/4, ..., 1 ===> log N Step

$N^{1/2^0}$, $N^{1/2^1}$, $N^{1/2^2}$, $N^{1/2^3}$..., c

$N^{1/2^k} = c$

$\log N^{1/2^k} = \log c$

$\frac{1}{2^k} \log N = c$ cross multiply

$\log N = c2^k$

$\log \log N = \log c \sim O(1) + \log 2^k$

$\log \log N = k \log 2$

$\log \log N = k$