Statistical Science 2864A Project

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DESIGN AND ANALYSIS OF BEIJING PM2.5 DATA SET USING REGRESSION METHODS

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1. Introduction

In this project, we will be using R and multiple linear regression to model the relationship between two or more explanatory variables and a response variable by fitting a linear equation to observed data.

It contains following parts:

1. Description about the data set used this project
2. Calculate p-value for each predictor (Single parameter test)
3. Come out with some hypotheses that which predictors are not significant (Nested model + ANOVA)
4. Test for interactions
5. Variable selection (which variables to keep, based on previous results and AIC, BIC or PRESS test)
6. Model diagnostics on one well-fit model
    - 5.1 Linearity
    - 5.2 Normality
    - 5.3 Equal variance
    - 5.4 Independence using time series
7. Conclusion

2. Description of the Data Set

This hourly data set contains the PM2.5 data of US Embassy in Beijing. Meanwhile, meteorological data from Beijing Capital International Airport are also included. This data contains not available cells.

3. Design and Analysis

For this study, there are 43824 hourly data collected in Beijing including PM2.5 concentration(ug/m^3), dew point(â„ƒ), temperature(â„ƒ), pressure(hPa), combined wind direction, cumulated hours of snow, cumulated hours of rain from January 1st, 2010 to December 31st, 2014.

3.1 Calculate p-value for each predictor

Firstly, we put all these variables into the multiple linear regression model. The independent variables are dew point(â„ƒ), temperature(â„ƒ), pressure(hPa), combined wind direction, cumulated hours of snow, cumulated hours of rain and the dependent variable is PM2.5 concentration(ug/m^3). After fitting the data into the model by using R software tool can give us the p-value of each variable.

```r
df <- read.csv(file = 'PRSA_data_2010.1.1-2014.12.31.csv')
df=na.omit(df)
set.seed(2864)
index <- sample(1:nrow(df), 2000)
df=df[index, ]
nrow(df)
```

```
## [1] 2000
```

```r
head(df)
```

```
##          No year month day hour pm2.5 DEWP TEMP PRES cbwd    Iws Is Ir
## 12224 12224 2011     5  25    7    66   12   20 1014   SE 185.07  0  0
## 33448 33448 2013    10  25   15    25   -7   19 1025   NW   1.79  0  0
## 13028 13028 2011     6  27   19    53   15   28  999   SE  22.35  0  0
## 31240 31240 2013     7  25   15    30   15   36 1000   SE   7.15  0  0
## 14633 14633 2011     9   2   16    57   18   30 1008   SE   4.02  0  0
## 15579 15579 2011    10  12    2   221   13   14 1020   NE   1.78  0  0
```

```r
model1=lm(pm2.5 ~ DEWP+TEMP+PRES+cbwd+Iws+Is+Ir, data=df)
summary(model1)
```

```
##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Is + Ir,
##     data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -161.16  -52.60  -14.04   33.52  424.90
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1663.80214  332.84996   4.999 6.28e-07 ***
## DEWP           3.70727    0.24686  15.018  < 2e-16 ***
## TEMP          -6.22270    0.31006 -20.069  < 2e-16 ***
## PRES          -1.45221    0.32551  -4.461 8.60e-06 ***
## cbwdNE       -27.81937    6.52102  -4.266 2.08e-05 ***
## cbwdNW       -29.17477    5.36890  -5.434 6.19e-08 ***
## cbwdSE         6.02305    5.04614   1.194    0.233
## Iws           -0.29018    0.04062  -7.144 1.27e-12 ***
## Is            -2.00165    1.74745  -1.145    0.252
## Ir            -6.06433    1.25242  -4.842 1.38e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 79.75 on 1990 degrees of freedom
## Multiple R-squared:  0.2602, Adjusted R-squared:  0.2568
## F-statistic: 77.76 on 9 and 1990 DF,  p-value: < 2.2e-16
```

3.2 Some hypotheses that which predictors are not significant

By observing the summary table of the full model, we make the null hypothesis that the Is is not significantly important to explain this model.

```
#reduced model without Is
model2=lm(pm2.5 ~ DEWP+TEMP+PRES+Iws+cbwd+Ir, data=df)
summary(model2)
```

```
##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + Iws + cbwd + Ir, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -160.70  -52.56  -14.16   33.80  425.48
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1661.20329  332.86831   4.991 6.54e-07 ***
## DEWP           3.68358    0.24601  14.973  < 2e-16 ***
## TEMP          -6.17968    0.30780 -20.077  < 2e-16 ***
## PRES          -1.45020    0.32554  -4.455 8.86e-06 ***
## Iws           -0.29319    0.04054  -7.232 6.74e-13 ***
## cbwdNE       -27.69136    6.52057  -4.247 2.27e-05 ***
## cbwdNW       -29.05419    5.36828  -5.412 6.98e-08 ***
## cbwdSE         5.77988    5.04207   1.146    0.252
## Ir            -6.03967    1.25233  -4.823 1.52e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 79.76 on 1991 degrees of freedom
## Multiple R-squared:  0.2597, Adjusted R-squared:  0.2567
## F-statistic:  87.3 on 8 and 1991 DF,  p-value: < 2.2e-16
```

```
anova(model1,model2)
```

```
## Analysis of Variance Table
##
## Model 1: pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Is + Ir
## Model 2: pm2.5 ~ DEWP + TEMP + PRES + Iws + cbwd + Ir
##   Res.Df      RSS Df Sum of Sq      F Pr(>F)
## 1   1990 12656854
## 2   1991 12665200 -1   -8345.3 1.3121 0.2522
```

The large f statistics value in anova indicates there is no significant difference between the two models. Therefore, we fail to reject the null hypothesis that Is is significantly important to explain the model.

3.3 Test for interactions (2-way interaction)

```
model4=lm(pm2.5 ~
DEWP+TEMP+PRES+cbwd+Iws+Ir+Is+I(DEWP*TEMP)+I(DEWP*PRES)+I(DEWP*Iws)+I(DEWP*Ir
)+I(TEMP*PRES)+I(TEMP*Iws)+I(TEMP*Ir)+I(PRES*Iws)+I(PRES*Ir)+I(Iws*Ir)+I(Is*T
EMP)+I(Is*DEWP)+I(Is*PRES)+I(Is*Iws)+I(Is*Ir), data=df)
summary(model4)

##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is +
##     I(DEWP * TEMP) + I(DEWP * PRES) + I(DEWP * Iws) + I(DEWP *
##     Ir) + I(TEMP * PRES) + I(TEMP * Iws) + I(TEMP * Ir) + I(PRES *
##     Iws) + I(PRES * Ir) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
##     DEWP) + I(Is * PRES) + I(Is * Iws) + I(Is * Ir), data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -179.68  -49.14  -11.57   32.09  406.91
##
## Coefficients: (1 not defined because of singularities)
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     1.440e+03  5.057e+02   2.847 0.004460 **
## DEWP           -1.639e+02  2.893e+01  -5.664 1.70e-08 ***
## TEMP            6.369e+01  2.922e+01   2.179 0.029433 *
## PRES           -1.211e+00  4.960e-01  -2.441 0.014720 *
## cbwdNE         -2.457e+01  6.371e+00  -3.857 0.000118 ***
## cbwdNW         -2.643e+01  5.278e+00  -5.007 6.02e-07 ***
## cbwdSE          1.139e+01  5.069e+00   2.246 0.024789 *
## Iws            -2.183e+01  7.580e+00  -2.880 0.004020 **
## Ir              1.306e+02  2.807e+02   0.465 0.641719
## Is              9.093e+02  1.607e+03   0.566 0.571595
## I(DEWP * TEMP) -1.144e-02  2.414e-02  -0.474 0.635621
## I(DEWP * PRES)  1.656e-01  2.830e-02   5.852 5.67e-09 ***
## I(DEWP * Iws)  -2.519e-02  5.697e-03  -4.422 1.03e-05 ***
## I(DEWP * Ir)    3.998e-01  1.228e+00   0.325 0.744845
## I(TEMP * PRES) -6.968e-02  2.876e-02  -2.423 0.015493 *
## I(TEMP * Iws)   3.868e-02  7.010e-03   5.518 3.88e-08 ***
## I(TEMP * Ir)   -3.342e-01  1.208e+00  -0.277 0.782140
## I(PRES * Iws)   2.061e-02  7.412e-03   2.781 0.005467 **
## I(PRES * Ir)   -1.373e-01  2.737e-01  -0.502 0.616012
```

```
## I(Iws * Ir)     5.199e-02  3.980e-02   1.306 0.191637
## I(Is * TEMP)    2.407e+00  3.878e+00   0.621 0.534827
## I(Is * DEWP)   -1.439e+00  3.504e+00  -0.411 0.681402
## I(Is * PRES)   -8.956e-01  1.564e+00  -0.573 0.566965
## I(Is * Iws)     8.735e-02  9.275e-02   0.942 0.346407
## I(Is * Ir)             NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 77.48 on 1976 degrees of freedom
## Multiple R-squared:  0.3067, Adjusted R-squared:  0.2986
## F-statistic: 38.01 on 23 and 1976 DF,  p-value: < 2.2e-16
```

By observing the p-value of each predictior, we make the null hypothesis that DEWP * TEMP, DEWP * Ir, TEMP * Ir and PRES * Ir, Is * TEMP, Is * DEWP and Is * PRES are not significantly important to explain this model.

```
#reduced model without DEWP * TEMP, DEWP * Ir, TEMP * Ir and PRES * Ir, Is * TEMP, Is * DEWP and Is * PRES
model5=lm(pm2.5 ~ DEWP+TEMP+PRES+cbwd+Iws+Ir+Is+I(DEWP*PRES)+I(DEWP*Iws)+I(TEMP*PRES)+I(TEMP*Iws)+I(PRES*Iws)+I(Iws*Ir)+I(Is*Iws)+I(Is*Ir), data=df)
summary(model5)

##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is +
##     I(DEWP * PRES) + I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP *
##     Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * Iws) + I(Is *
##     Ir), data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -178.60  -49.25  -12.19   32.71  405.14
##
## Coefficients: (1 not defined because of singularities)
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)     1.588e+03  4.762e+02   3.335 0.000869 ***
## DEWP           -1.663e+02  2.445e+01  -6.804 1.35e-11 ***
## TEMP            5.604e+01  2.741e+01   2.044 0.041064 *
## PRES           -1.357e+00  4.667e-01  -2.909 0.003672 **
## cbwdNE         -2.502e+01  6.357e+00  -3.936 8.56e-05 ***
## cbwdNW         -2.639e+01  5.260e+00  -5.016 5.74e-07 ***
## cbwdSE          1.081e+01  5.043e+00   2.144 0.032180 *
## Iws            -2.304e+01  7.341e+00  -3.138 0.001724 **
## Ir             -7.215e+00  1.562e+00  -4.620 4.08e-06 ***
## Is             -1.221e+01  3.702e+00  -3.299 0.000989 ***
## I(DEWP * PRES)  1.678e-01  2.406e-02   6.977 4.10e-12 ***
```

```
## I(DEWP * Iws)  -2.480e-02  5.647e-03  -4.391 1.19e-05 ***
## I(TEMP * PRES) -6.216e-02  2.698e-02  -2.304 0.021320 *
## I(TEMP * Iws)   3.980e-02  6.868e-03   5.794 7.95e-09 ***
## I(PRES * Iws)   2.179e-02  7.178e-03   3.036 0.002430 **
## I(Iws * Ir)     4.407e-02  1.827e-02   2.412 0.015956 *
## I(Is * Iws)     1.431e-01  4.407e-02   3.247 0.001187 **
## I(Is * Ir)             NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 77.44 on 1983 degrees of freedom
## Multiple R-squared:  0.305,  Adjusted R-squared:  0.2994
## F-statistic: 54.38 on 16 and 1983 DF,  p-value: < 2.2e-16

anova(model4,model5)

## Analysis of Variance Table
##
## Model 1: pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is + I(DEWP *
##     TEMP) + I(DEWP * PRES) + I(DEWP * Iws) + I(DEWP * Ir) + I(TEMP *
##     PRES) + I(TEMP * Iws) + I(TEMP * Ir) + I(PRES * Iws) + I(PRES *
##     Ir) + I(Iws * Ir) + I(Is * TEMP) + I(Is * DEWP) + I(Is *
##     PRES) + I(Is * Iws) + I(Is * Ir)
## Model 2: pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is + I(DEWP *
##     PRES) + I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP * Iws) +
##     I(PRES * Iws) + I(Iws * Ir) + I(Is * Iws) + I(Is * Ir)
##   Res.Df      RSS Df Sum of Sq      F Pr(>F)
## 1   1976 11860707
## 2   1983 11890475 -7    -29768 0.7085 0.6649
```

- The large p-value in anova indicates there is no significant difference between the two models.

3.4 Variable selection (which variables to keep, based on previous results and AIC, BIC or PRESS test)

```
nullfit <- lm(pm2.5~1,data=df)
stepAppro_aic = step(nullfit,
                     scope = pm2.5 ~
DEWP+TEMP+PRES+cbwd+Iws+Ir+Is+I(DEWP*TEMP)+I(DEWP*PRES)+I(DEWP*Iws)+I(DEWP*Ir
)+I(TEMP*PRES)+I(TEMP*Iws)+I(TEMP*Ir)+I(PRES*Iws)+I(PRES*Ir)+I(Iws*Ir)+I(Is*T
EMP)+I(Is*DEWP)+I(Is*PRES)+I(Is*Iws)+I(Is*Ir),
                     direction = "forward",
                     trace = 0)

stepAppro_bic <- step(model4,
                      direction = "backward",
```

```
                      k=log(nrow(df)),
                      trace=FALSE)
stepAppro_aic

##
## Call:
## lm(formula = pm2.5 ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) +
##     cbwd + DEWP + I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP *
##     Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
##     Iws) + I(Is * PRES) + TEMP, data = df)
##
## Coefficients:
##    (Intercept)             Iws  I(TEMP * PRES)  I(DEWP * PRES)
cbwdNE
##      1.592e+03      -2.241e+01      -6.215e-02       1.676e-01      -
2.504e+01
##         cbwdNW          cbwdSE            DEWP    I(PRES * Ir)
PRES
##     -2.666e+01       1.101e+01      -1.661e+02      -7.146e-03      -
1.361e+00
##  I(DEWP * Iws)   I(TEMP * Iws)   I(PRES * Iws)     I(Iws * Ir)    I(Is *
TEMP)
##     -2.502e-02       3.932e-02       2.118e-02       4.398e-02
7.758e-01
##    I(Is * Iws)    I(Is * PRES)            TEMP
##      1.371e-01      -9.425e-03       5.602e+01

stepAppro_bic

##
## Call:
## lm(formula = pm2.5 ~ DEWP + PRES + cbwd + Iws + I(DEWP * PRES) +
##     I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP * Iws) + I(PRES *
##     Ir) + I(Is * PRES) + I(Is * Iws), data = df)
##
## Coefficients:
##    (Intercept)            DEWP            PRES          cbwdNE
cbwdNW
##     1765.35515      -117.36455        -1.53470       -25.06620       -
26.99562
##         cbwdSE             Iws  I(DEWP * PRES)   I(DEWP * Iws)  I(TEMP *
PRES)
##       11.35386        -0.69972         0.11974        -0.02789        -
0.00681
##  I(TEMP * Iws)    I(PRES * Ir)    I(Is * PRES)     I(Is * Iws)
##        0.02672        -0.00537        -0.01122         0.13292

anova(stepAppro_aic,stepAppro_bic)

## Analysis of Variance Table
##
```

```
## Model 1: pm2.5 ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) + cbwd + DEWP +
##     I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP * Iws) + I(PRES *
##     Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is * Iws) + I(Is *
##     PRES) + TEMP
## Model 2: pm2.5 ~ DEWP + PRES + cbwd + Iws + I(DEWP * PRES) + I(DEWP *
##     Iws) + I(TEMP * PRES) + I(TEMP * Iws) + I(PRES * Ir) + I(Is *
##     PRES) + I(Is * Iws)
##   Res.Df      RSS Df Sum of Sq      F    Pr(>F)
## 1   1982 11874882
## 2   1986 11994378 -4   -119497 4.9862 0.0005334 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(asbio)
```

```
## Loading required package: tcltk
```

```
#model selected by AIC
press(lm(formula = pm2.5 ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) +
    cbwd + DEWP + I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP *
    Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
    Iws) + I(Is * PRES) + TEMP, data = df)
)
```

```
## [1] 12018669
```

```
#model selected by BIC
press(lm(formula = pm2.5 ~ DEWP + PRES + cbwd + Iws + I(DEWP * PRES) +
    I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP * Iws) + I(PRES *
    Ir) + I(Is * PRES) + I(Is * Iws), data = df))
```

```
## [1] 12141580
```

- The PRESS statistic indicates that model selected by AIC is more preferred in this case. However, PRESS might not be approprite although the dataset is reduced already.

3.5 Model diagnostics on one well-fit model

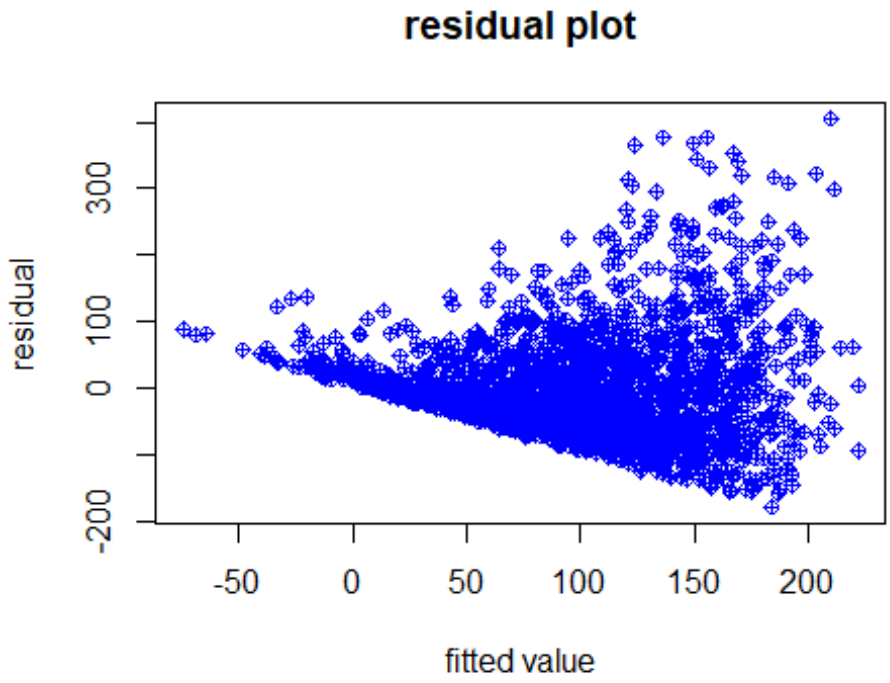
```
library(lmtest)
```

```
## Loading required package: zoo
```

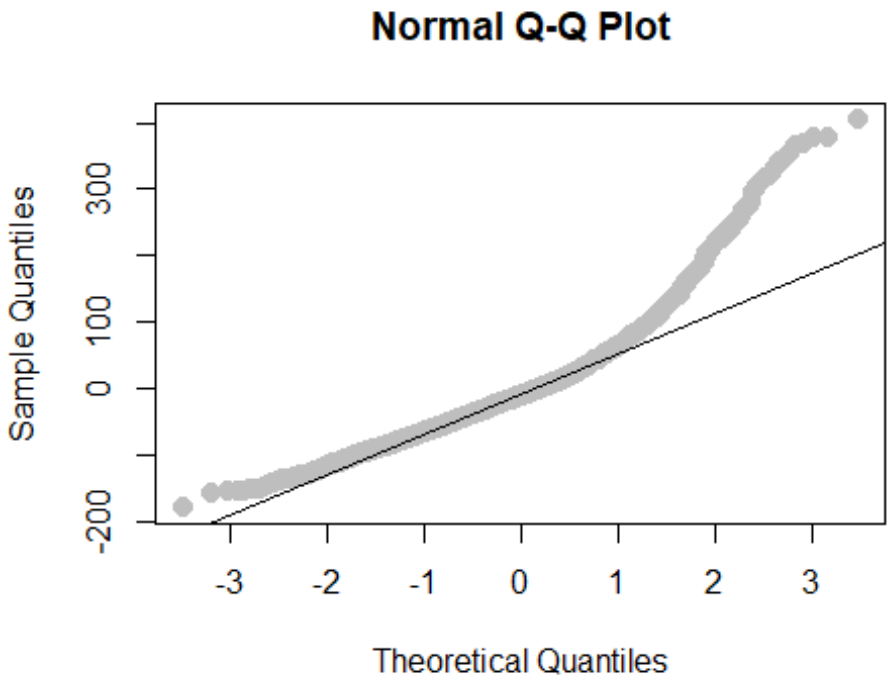
```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```r
#we will be using the model selected by AIC in later learning
model=lm(formula = pm2.5 ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) +
    cbwd + DEWP + I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP *
    Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
    Iws) + I(Is * PRES) + TEMP, data = df)
plot(fitted(model), resid(model),
     col = "blue", pch = 10,
     xlab = "fitted value",
     ylab = "residual",
     cex=1,
     main = "residual plot")
```

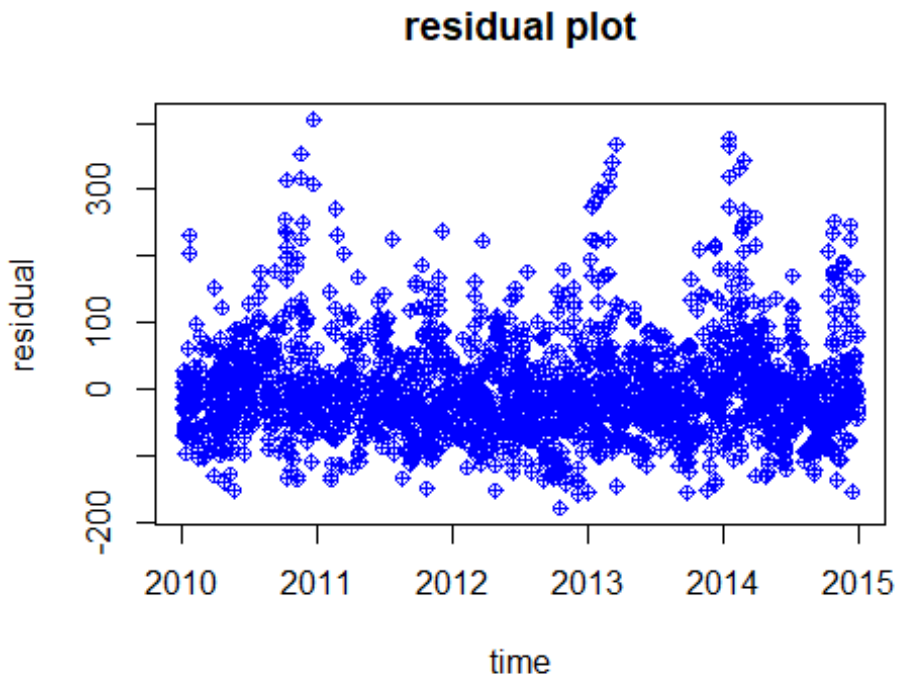



```r
qqnorm(resid(model), col = "grey",pch=20,cex=2)
qqline(resid(model))
```




```
bptest(model)

##
##  studentized Breusch-Pagan test
##
## data:  model
## BP = 210.02, df = 17, p-value < 2.2e-16

shapiro.test(resid(model))

##
##  Shapiro-Wilk normality test
##
## data:  resid(model)
## W = 0.91623, p-value < 2.2e-16

timeset=c()
residset=c()
i=1
while (i<nrow(df)){
  timeset=append(timeset,
as.Date(paste(df[i,"month"],df[i,"day"],df[i,"year"],sep="/"), "%m/%d/%Y"))
  residset=append(residset, df[i, "pm2.5"]-predict(model, df[i,]))
  i=i+1
}
plot(timeset, residset,
```

```
     col = "blue", pch = 10,
     xlab = "time",
     ylab = "residual",
     cex=1,
     main = "residual plot")
```




```
length(timeset)
```

```
## [1] 1999
```

Conclusion

- Linearity: The residuals distribute systematically and do not exhibit a mean of zero. The linearity assumption is violated.
- Equal Variance: The small p-value of the BP test indicates that the variance assumption is violated.
- Normality Assumption: The small p-value of SW test indicates that the normality assumption is violated. However, the logged model might hold the normality assumption.
- Independence Assumption: The residual plot against time, the value of random errors is independent. The normality assumption holds.

Reference List

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Yale University. 2012. "Multiple Linear Regression". (http://www.stat.yale.edu/Courses/1997-98/101/linmult.htm)

Guanghua School of Management, Peking University. 2017. "Beijing PM2.5 Data Set". (https://archive.ics.uci.edu/ml/datasets/Beijing+PM2.5+Data#)

Rui Zhu. 2020. "SS2864 Individual Project". (https://github.com/AimerAndern/SS2864IndividualProject)