SS3859 Group Project

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Calculate p-value for each predictor

```
df <- read.csv(file = 'PRSA data 2010.1.1-2014.12.31.csv')</pre>
df=na.omit(df)
set.seed(3859)
index <- sample(1:nrow(df), 2000)</pre>
df=df[index, ]
nrow(df)
## [1] 2000
head(df)
##
            No year month day hour pm2.5 DEWP TEMP PRES cbwd
                                                                 Iws Is Ir
                        5
                           25
                                                 20 1014
## 12224 12224 2011
                                 7
                                       66
                                            12
                                                           SE 185.07
                                                                       0
                                                                          0
## 33448 33448 2013
                       10
                           25
                                 15
                                       25
                                            -7
                                                 19 1025
                                                           NW
                                                                 1.79
                                                                       0
                                                                          0
## 13028 13028 2011
                        6
                           27
                                 19
                                       53
                                            15
                                                 28 999
                                                           SE
                                                               22.35
                                                                       0
                                                                          0
## 31240 31240 2013
                        7
                           25
                                 15
                                       30
                                            15
                                                 36 1000
                                                           SE
                                                                7.15
                                                                          0
                                                                      0
## 14633 14633 2011
                        9
                           2
                                 16
                                       57
                                            18
                                                 30 1008
                                                           SE
                                                                4.02
                                                                      0
                                                                          0
                          12
                                 2
                                      221
                                            13
                                                 14 1020
                                                                1.78 0
## 15579 15579 2011
                       10
                                                           NE
                                                                          0
model1=lm(pm2.5 ~ DEWP+TEMP+PRES+cbwd+Iws+Is+Ir, data=df)
summary(model1)
##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Is + Ir,
##
       data = df
##
## Residuals:
##
       Min
                10 Median
                                 3Q
                                        Max
## -161.16 -52.60 -14.04
                             33.52 424.90
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
                                       4.999 6.28e-07 ***
## (Intercept) 1663.80214 332.84996
                             0.24686 15.018
## DEWP
                  3.70727
                                              < 2e-16 ***
## TEMP
                 -6.22270
                             0.31006 -20.069 < 2e-16 ***
## PRES
                             0.32551 -4.461 8.60e-06 ***
                 -1.45221
## cbwdNE
                -27.81937
                             6.52102 -4.266 2.08e-05 ***
## cbwdNW
                -29.17477
                             5.36890 -5.434 6.19e-08 ***
## cbwdSE
                  6.02305
                             5.04614
                                       1.194
                                                 0.233
## Iws
                 -0.29018
                             0.04062 -7.144 1.27e-12 ***
## Is
                 -2.00165
                             1.74745 -1.145
                                                 0.252
                             1.25242 -4.842 1.38e-06 ***
## Ir
                 -6.06433
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 79.75 on 1990 degrees of freedom
## Multiple R-squared: 0.2602, Adjusted R-squared: 0.2568
## F-statistic: 77.76 on 9 and 1990 DF, p-value: < 2.2e-16
```

Some hypothese that which predictors are not significant

• By observing the summary table of the full model, we make the null hypothesis that the Is is not significantly important to explain this model.

```
#reduced model without Is
model2=lm(pm2.5 ~ DEWP+TEMP+PRES+Iws+cbwd+Ir, data=df)
summary(model2)
##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + Iws + cbwd + Ir, data = df)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -160.70 -52.56 -14.16
                             33.80 425.48
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 1661.20329 332.86831
                                      4.991 6.54e-07 ***
## DEWP
                 3.68358
                             0.24601 14.973 < 2e-16 ***
                             0.30780 -20.077 < 2e-16 ***
## TEMP
                 -6.17968
## PRES
                 -1.45020
                             0.32554 -4.455 8.86e-06 ***
                             0.04054 -7.232 6.74e-13 ***
## Iws
                 -0.29319
                             6.52057 -4.247 2.27e-05 ***
## cbwdNE
                -27.69136
                             5.36828 -5.412 6.98e-08 ***
## cbwdNW
                -29.05419
## cbwdSE
                 5.77988
                             5.04207 1.146
                                                0.252
## Ir
                             1.25233 -4.823 1.52e-06 ***
                 -6.03967
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 79.76 on 1991 degrees of freedom
## Multiple R-squared: 0.2597, Adjusted R-squared: 0.2567
## F-statistic: 87.3 on 8 and 1991 DF, p-value: < 2.2e-16
anova(model1, model2)
## Analysis of Variance Table
## Model 1: pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Is + Ir
## Model 2: pm2.5 ~ DEWP + TEMP + PRES + Iws + cbwd + Ir
    Res.Df
                 RSS Df Sum of Sq
                                       F Pr(>F)
##
## 1
      1990 12656854
      1991 12665200 -1 -8345.3 1.3121 0.2522
## 2
```

• The large f statistics value in anova indicates there is no significant difference between the two models. Therefore, we fail to reject the null hypothesis that Is is significantly important to explain the model.

Test for interactions(2 way interaction)

```
model4=1m(pm2.5 \sim
DEWP+TEMP+PRES+cbwd+Iws+Ir+Is+I(DEWP*TEMP)+I(DEWP*PRES)+I(DEWP*Iws)+I(DEWP*Ir
)+I(TEMP*PRES)+I(TEMP*Iws)+I(TEMP*Ir)+I(PRES*Iws)+I(PRES*Ir)+I(Iws*Ir)+I(Is*T
EMP)+I(Is*DEWP)+I(Is*PRES)+I(Is*Iws)+I(Is*Ir), data=df)
summary(model4)
##
## Call:
## lm(formula = pm2.5 \sim DEWP + TEMP + PRES + cbwd + Iws + Ir + Is +
##
       I(DEWP * TEMP) + I(DEWP * PRES) + I(DEWP * Iws) + I(DEWP *
       Ir) + I(TEMP * PRES) + I(TEMP * Iws) + I(TEMP * Ir) + I(PRES *
##
       Iws) + I(PRES * Ir) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
##
##
       DEWP) + I(Is * PRES) + I(Is * Iws) + I(Is * Ir), data = df)
##
## Residuals:
      Min
                10 Median
                                3Q
                                       Max
##
## -179.68 -49.14
                   -11.57
                             32.09
                                   406.91
##
## Coefficients: (1 not defined because of singularities)
                    Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                   1.440e+03 5.057e+02
                                         2.847 0.004460 **
                  -1.639e+02 2.893e+01 -5.664 1.70e-08 ***
## DEWP
## TEMP
                  6.369e+01 2.922e+01
                                        2.179 0.029433 *
## PRES
                  -1.211e+00 4.960e-01 -2.441 0.014720 *
## cbwdNE
                  -2.457e+01 6.371e+00 -3.857 0.000118 ***
                  -2.643e+01 5.278e+00 -5.007 6.02e-07 ***
## cbwdNW
                  1.139e+01 5.069e+00 2.246 0.024789 *
## cbwdSE
## Iws
                  -2.183e+01 7.580e+00 -2.880 0.004020 **
## Ir
                   1.306e+02 2.807e+02 0.465 0.641719
## Is
                  9.093e+02 1.607e+03
                                         0.566 0.571595
## I(DEWP * TEMP) -1.144e-02 2.414e-02 -0.474 0.635621
## I(DEWP * PRES)
                  1.656e-01 2.830e-02
                                          5.852 5.67e-09 ***
## I(DEWP * Iws)
                  -2.519e-02 5.697e-03 -4.422 1.03e-05 ***
## I(DEWP * Ir)
                   3.998e-01 1.228e+00
                                         0.325 0.744845
## I(TEMP * PRES) -6.968e-02 2.876e-02 -2.423 0.015493 *
## I(TEMP * Iws)
                   3.868e-02 7.010e-03
                                        5.518 3.88e-08 ***
## I(TEMP * Ir)
                  -3.342e-01 1.208e+00 -0.277 0.782140
## I(PRES * Iws)
                  2.061e-02 7.412e-03
                                         2.781 0.005467 **
## I(PRES * Ir)
                  -1.373e-01 2.737e-01 -0.502 0.616012
## I(Iws * Ir)
                  5.199e-02 3.980e-02 1.306 0.191637
## I(Is * TEMP)
                   2.407e+00 3.878e+00
                                         0.621 0.534827
## I(Is * DEWP)
                  -1.439e+00 3.504e+00
                                        -0.411 0.681402
## I(Is * PRES)
                  -8.956e-01 1.564e+00
                                        -0.573 0.566965
## I(Is * Iws)
                  8.735e-02 9.275e-02
                                          0.942 0.346407
## I(Is * Ir)
                         NA
                                    NA
                                             NA
                                                      NA
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 77.48 on 1976 degrees of freedom
## Multiple R-squared: 0.3067, Adjusted R-squared: 0.2986
## F-statistic: 38.01 on 23 and 1976 DF, p-value: < 2.2e-16
```

• By observing the p-value of each predictior, we make the null hypothesis that DEWP * TEMP, DEWP * Ir, TEMP * Ir and PRES * Ir, Is * TEMP, Is * DEWP and Is * PRES are not significantly important to explain this model.

```
#reduced model without DEWP * TEMP, DEWP * Ir, TEMP * Ir and PRES * Ir, Is *
TEMP, Is * DEWP and Is * PRES
model5=1m(pm2.5 \sim
DEWP+TEMP+PRES+cbwd+Iws+Ir+Is+I(DEWP*PRES)+I(DEWP*Iws)+I(TEMP*PRES)+I(TEMP*Iw
s)+I(PRES*Iws)+I(Iws*Ir)+I(Is*Iws)+I(Is*Ir), data=df)
summary(model5)
##
## Call:
## lm(formula = pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is +
##
               I(DEWP * PRES) + I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP *
               Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * Iws) + I(Is 
##
##
               Ir), data = df)
##
## Residuals:
               Min
##
                                   10 Median
                                                                      3Q
                                                                                     Max
## -178.60 -49.25 -12.19
                                                               32.71 405.14
## Coefficients: (1 not defined because of singularities)
##
                                           Estimate Std. Error t value Pr(>|t|)
                                         1.588e+03 4.762e+02 3.335 0.000869 ***
## (Intercept)
## DEWP
                                       -1.663e+02 2.445e+01 -6.804 1.35e-11 ***
## TEMP
                                        5.604e+01 2.741e+01 2.044 0.041064 *
                                       -1.357e+00 4.667e-01 -2.909 0.003672 **
## PRES
                                       -2.502e+01 6.357e+00 -3.936 8.56e-05 ***
## cbwdNE
                                       -2.639e+01 5.260e+00 -5.016 5.74e-07 ***
## cbwdNW
                                        1.081e+01 5.043e+00 2.144 0.032180 *
## cbwdSE
## Iws
                                       -2.304e+01 7.341e+00 -3.138 0.001724 **
## Ir
                                       -7.215e+00 1.562e+00 -4.620 4.08e-06 ***
                                       -1.221e+01 3.702e+00 -3.299 0.000989 ***
## Is
## I(DEWP * PRES) 1.678e-01 2.406e-02
                                                                                        6.977 4.10e-12 ***
## I(DEWP * Iws)
                                       -2.480e-02 5.647e-03 -4.391 1.19e-05 ***
## I(TEMP * PRES) -6.216e-02 2.698e-02 -2.304 0.021320 *
## I(TEMP * Iws)
                                                                                           5.794 7.95e-09 ***
                                         3.980e-02 6.868e-03
## I(PRES * Iws)
                                         2.179e-02 7.178e-03
                                                                                           3.036 0.002430 **
## I(Iws * Ir)
                                         4.407e-02 1.827e-02
                                                                                           2.412 0.015956 *
## I(Is * Iws)
                                         1.431e-01 4.407e-02
                                                                                           3.247 0.001187 **
## I(Is * Ir)
                                                        NA
                                                                                NA
                                                                                                  NA
                                                                                                                     NA
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 77.44 on 1983 degrees of freedom
## Multiple R-squared: 0.305, Adjusted R-squared: 0.2994
## F-statistic: 54.38 on 16 and 1983 DF, p-value: < 2.2e-16
anova(model4, model5)
## Analysis of Variance Table
##
## Model 1: pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is + I(DEWP *
                     TEMP) + I(DEWP * PRES) + I(DEWP * Iws) + I(DEWP * Ir) + I(TEMP *
##
                      PRES) + I(TEMP * Iws) + I(TEMP * Ir) + I(PRES * Iws) + I(PRES *
##
                      Ir) + I(Iws * Ir) + I(Is * TEMP) + I(Is * DEWP) + I(Is *
##
                     PRES) + I(Is * Iws) + I(Is * Ir)
##
## Model 2: pm2.5 ~ DEWP + TEMP + PRES + cbwd + Iws + Ir + Is + I(DEWP *
                     PRES) + I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP * Iws) +
##
                     I(PRES * Iws) + I(Iws * Ir) + I(Is * Iws) + I(Is * Ir)
##
##
               Res.Df
                                                     RSS Df Sum of Sq
                                                                                                                         F Pr(>F)
## 1
                     1976 11860707
## 2
                     1983 11890475 -7 -29768 0.7085 0.6649
```

 The large p-value in anova indicates there is no significant difference between the two models.

Variable selection (which variables to keep, based on previous results and AIC, BIC or PRESS test)

```
nullfit <- lm(pm2.5~1,data=df)</pre>
stepAppro aic = step(nullfit,
                      scope = pm2.5 \sim
DEWP+TEMP+PRES+cbwd+Iws+Ir+Is+I(DEWP*TEMP)+I(DEWP*PRES)+I(DEWP*Iws)+I(DEWP*Ir
)+I(TEMP*PRES)+I(TEMP*Iws)+I(TEMP*Ir)+I(PRES*Iws)+I(PRES*Ir)+I(Iws*Ir)+I(Is*T
EMP)+I(Is*DEWP)+I(Is*PRES)+I(Is*Iws)+I(Is*Ir),
                     direction = "forward",
                     trace = 0
stepAppro bic <- step(model4,</pre>
                      direction = "backward",
                      k=log(nrow(df)),
                      trace=FALSE)
stepAppro_aic
##
## Call:
## lm(formula = pm2.5 \sim Iws + I(TEMP * PRES) + I(DEWP * PRES) +
       cbwd + DEWP + I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP *
##
       Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
##
##
       Iws) + I(Is * PRES) + TEMP, data = df)
##
## Coefficients:
                              Iws I(TEMP * PRES) I(DEWP * PRES)
## (Intercept)
```

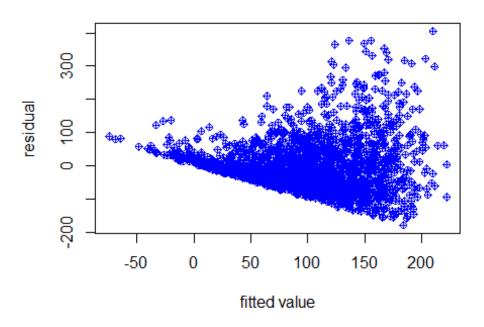
```
cbwdNE
##
        1.592e+03
                       -2.241e+01
                                       -6.215e-02
                                                        1.676e-01
2.504e+01
          cbwdNW
                           cbwdSE
                                             DEWP
                                                     I(PRES * Ir)
PRES
##
       -2.666e+01
                        1.101e+01
                                       -1.661e+02
                                                       -7.146e-03
1.361e+00
## I(DEWP * Iws)
                   I(TEMP * Iws) I(PRES * Iws)
                                                     I(Iws * Ir)
                                                                    I(Is *
TEMP)
##
      -2.502e-02
                        3.932e-02
                                        2.118e-02
                                                        4.398e-02
7.758e-01
     I(Is * Iws)
                    I(Is * PRES)
                                             TEMP
##
##
       1.371e-01
                      -9.425e-03
                                        5.602e+01
stepAppro_bic
##
## Call:
## lm(formula = pm2.5 ~ DEWP + PRES + cbwd + Iws + I(DEWP * PRES) +
       I(DEWP * Iws) + I(TEMP * PRES) + I(TEMP * Iws) + I(PRES *
##
       Ir) + I(Is * PRES) + I(Is * Iws), data = df)
##
## Coefficients:
                             DEWP
                                             PRES
##
      (Intercept)
                                                           cbwdNE
cbwdNW
##
      1765.35515
                                         -1.53470
                      -117.36455
                                                        -25.06620
26.99562
                                                   I(DEWP * Iws) I(TEMP *
##
          cbwdSE
                              Iws I(DEWP * PRES)
PRES)
##
        11.35386
                         -0.69972
                                          0.11974
                                                         -0.02789
0.00681
## I(TEMP * Iws)
                    I(PRES * Ir)
                                    I(Is * PRES)
                                                     I(Is * Iws)
##
         0.02672
                         -0.00537
                                         -0.01122
                                                          0.13292
anova(stepAppro_aic,stepAppro_bic)
## Analysis of Variance Table
##
## Model 1: pm2.5 ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) + cbwd + DEWP +
       I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP * Iws) + I(PRES *
##
##
       Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is * Iws) + I(Is *
       PRES) + TEMP
##
## Model 2: pm2.5 ~ DEWP + PRES + cbwd + Iws + I(DEWP * PRES) + I(DEWP *
       Iws) + I(TEMP * PRES) + I(TEMP * Iws) + I(PRES * Ir) + I(Is *
##
      PRES) + I(Is * Iws)
                RSS Df Sum of Sq
##
    Res.Df
                                            Pr(>F)
      1982 11874882
## 1
## 2
      1986 11994378 -4 -119497 4.9862 0.0005334 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

• The PRESS statistic indicates that model selected by AIC is more preferred in this case. However, PRESS might not be approprite although the dataset is reduced already.

Model diagnostics on one well-fit model

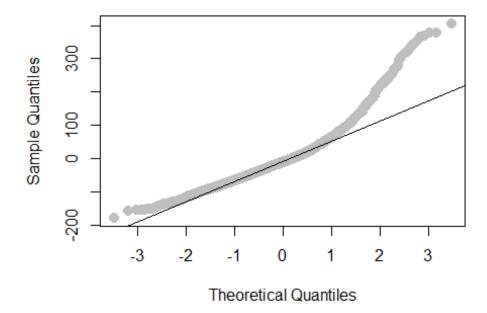
```
library(lmtest)
## Loading required package: zoo
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
#we will be using the model selected by AIC in later learning
model=lm(formula = pm2.5 ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) +
    cbwd + DEWP + I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP *
    Iws) + I(PRES * Iws) + I(Iws * Ir) + I(Is * TEMP) + I(Is *
    Iws) + I(Is * PRES) + TEMP, data = df)
plot(fitted(model), resid(model),
     col = "blue", pch = 10,
     xlab = "fitted value",
     ylab = "residual",
     cex=1,
    main = "residual plot")
```

residual plot



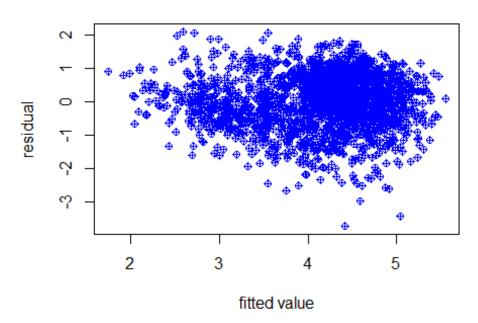
qqnorm(resid(model), col = "grey",pch=20,cex=2)
qqline(resid(model))

Normal Q-Q Plot



```
loggedModel=lm(formula = log(pm2.5) ~ Iws + I(TEMP * PRES) + I(DEWP * PRES) +
    cbwd + DEWP + I(PRES * Ir) + PRES + I(DEWP * Iws) + I(TEMP *
    Iws) + I(PRES * Iws) + I(Iws * Ir) + TEMP, data = df)
plot(fitted(loggedModel), resid(loggedModel),
    col = "blue", pch = 10,
    xlab = "fitted value",
    ylab = "residual",
    cex=1,
    main = "residual plot")
```

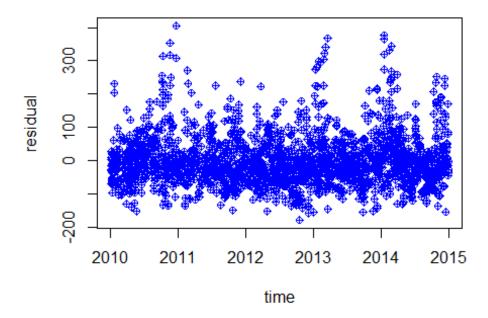
residual plot



```
bptest(model)
##
##
   studentized Breusch-Pagan test
##
## data: model
## BP = 210.02, df = 17, p-value < 2.2e-16
shapiro.test(resid(model))
##
##
    Shapiro-Wilk normality test
##
## data: resid(model)
## W = 0.91623, p-value < 2.2e-16
timeset=c()
residset=c()
```

```
i=1
while (i<nrow(df)){
   timeset=append(timeset,
as.Date(paste(df[i,"month"],df[i,"day"],df[i,"year"],sep="/"), "%m/%d/%Y"))
   residset=append(residset, df[i, "pm2.5"]-predict(model, df[i,]))
   i=i+1
}
plot(timeset, residset,
   col = "blue", pch = 10,
   xlab = "time",
   ylab = "residual",
   cex=1,
   main = "residual plot")</pre>
```

residual plot



```
length(timeset)
## [1] 1999
```

- Assumptions:
 - Linearity: The residuals distribute systematically and do not exhibit a mean of zero. The linearity assumption is violated.
 - Equal Variance: The small p-value of the BP test indicates that the variance assumption is violated.

- Normality Assumption: The small p-value of SW test indicates that the normality assumption is violated. However, the logged model might hold the normality assumption.
- Independence Assumption: The residual plot against time, the value of random errors are independent. The normality assumption holds.