计算方法Lab3

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实验结果：

$$A1 = \begin{pmatrix} \frac{1}{9} & \frac{1}{8} & \frac{1}{7} & \frac{1}{6} & \frac{1}{5} \\ \frac{1}{8} & \frac{1}{7} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} \\ \frac{1}{7} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{1}{1} \end{pmatrix}$$

k	$Y^{(k)}$	$X^{(k+1)}$	λ
0	(1，1，1，1，1)	(1，1，1，1，1)	1
1	(0.5625，-1，0.5625，-0.10714286，0.00446429)	(630，-1120，630，-120，5)	8.9286e-04
2	(0.49103047，-1，0.65863861，-0.1514674，0.00798141)	(146252.81249998，-297848.74999995，196174.68749995，-45114.37499998 2377.25446428)	3.357408751925828e-06
3	(0.49042606，-1，0.65975，-0.15209698，0.00804664)	(149112.77068582，-304047.40617087，200595.27517632，-46244.69166846 2446.55949575)	3.288960799218314e-06
4	(0.4904196，-1，0.65976195，-0.15210379，0.00804735)	(149157.10524439，-304141.80992685，200661.19404019，-46261.1227451 2447.53617196)	3.287939925919796e-06
5	(0.49041953，-1，0.65976208，-0.15210387，0.00804736)	(149157.58469441，-304142.83058563，200661.90651391，-46261.300272 2447.54671957)	3.287928892075115e-06
6	(0.49041953，-1，0.65976208，-0.15210387，0.00804736)	(149157.58984856，-304142.84155776，200661.91417298，-46261.30218039 2447.54683296)	3.287928773461167e-06
7	(0.49041953，-1，0.65976208，-0.15210387，0.00804736)	(149157.58990396，-304142.84167569，200661.91425531，-46261.3022009 2447.54683417)	3.287928772186288e-06

X(k+1)如下：
[0. 0. 0. 0. 0.]
[1. 1. 1. 1. 1.]
[630. -1120.   630. -120.   5.]
[146252.81249998 -297848.74999995 196174.68749995 -45114.37499998
    2377.25446428]
[149112.77068582 -304047.40617087 200595.27517632 -46244.69166846
    2446.55949575]
[149157.10524439 -304141.80992685 200661.19404019 -46261.1227451
    2447.53617196]
[149157.58469441 -304142.83058563 200661.90651391 -46261.300272
    2447.54671957]
[149157.58984856 -304142.84155776 200661.91417298 -46261.30218039
    2447.54683296]
[149157.58990396 -304142.84167569 200661.91425531 -46261.3022009
    2447.54683417]
矩阵最小特征值为：3.287928772186253e-06
对应的特征向量为：[0.49041953 -1.   0.65976208 -0.15210387 0.00804736]

共迭代8次完成该精度

所求矩阵1按模最小特征值为3.287928772186253e-06

特征向量为v=(0.49041953 , -1, 0.65976208, -0.15210387, 0.00804736)^T

$$A2 = \begin{pmatrix} 4 & -1 & 1 & 3 \\ 16 & -2 & -2 & 5 \\ 16 & -3 & -1 & 7 \\ 6 & -4 & 2 & 9 \end{pmatrix}$$

X(k+1)如下：
[0. 0. 0. 0.]
[1. 1. 1. 1.]
[0. 2. -0. 1.]
[-0.625 5.625 -2.375 3.5]
[-0.93333333 8.07777778 -3.43333333 5.04444444]
[-0.93621045 8.08992435 -3.44377579 5.05433287]
[-0.93671229 8.09381841 -3.44549   5.05681069]
[-0.93671884 8.09385612 -3.44551297 5.05683754]
矩阵最小特征值为：0.12355050358974673
对应的特征向量为：[-0.11573208 1.   -0.42569486 0.62477482]

k	Y$^{(k)}$	X$^{(k+1)}$	λ
0	(1, 1, 1, 1, 1)	(1, 1, 1, 1, 1)	1
1	(0, 1, -0, 0.5)	(0, 2, 0, 1)	0.5
2	(-0.11111111, 1, -0.42222222, 0.62222222)	(-0.625, 5.625, -2.375, 3.5)	0.177777777777778
3	(-0.11554333, 1, -0.42503439, 0.62448418)	(-0.93333333, 8.07777778, -3.43333333, 5.04444444)	0.123796423624815

k	Y^(k)	X^(k+1)	λ
4	(-0.11572549, 1, -0.42568702, 0.62476887)	(-0.93621045, 8.08992435, -3.44377579, 5.05433287)	0.123610550966466
5	(-0.11573181, 1, -0.42569401, 0.62477442)	(-0.93671229, 8.09381841, -3.44549, 5.05681069)	0.123551079273596
6	(-0.11573208, 1, -0.42569486, 0.62477482)	(-0.93671884, 8.09385612, -3.44551297, 5.05683754)	0.123550503638061

共迭代7次完成该精度

所求矩阵2按模最小特征值为0.12355050358974673

特征向量为v=(-0.11573208，1，-0.42569486，0.62477482)^T

结果分析

(a)

矩阵1按模最小特征值小于矩阵2，但是迭代次数大于矩阵2(8>7)，说明"A的按模最小特征值越接近于0，收敛越快"不正确。

(b)

运用double型浮点数，因此在预估每次迭代特征值时，未遇到比值无穷大或者过小情况。