LA

ASSIGNMENT·2

CHAP#2

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QUESTION

What is a matrix determinent ? What are the properties of determinent? Explain each property by the help of an example.

ANSWER

Matrix determinent:

Matrix determinant is a special number that can be calculated in a square matrix by multiplying diagnol entries first and then subtracting them.

Formula:

Determinant of a 2x2 matrix can be calculated as.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Determinent = $|A|$ = $ad - cb$.

Properties of Determinant:

1. ALL zero property:

If all entries in a matrix are zero, then determinant is also zero.

Example:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad |A| = (0)(0) - (0)(0)$$
$$|A| = 0$$

2. Identity matrix property:

Determinant of an identity matrix is 1.

Example:

$$\text{Identity matrix} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|I| = (1)(1) - (0)(0)$$
$$|I| = 1$$

3. Reflection Property:

Value of determinant remains unchanged even if rows and columns are interchanged.

Example:

$$A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}, \quad |A| = (5)(8) - (7)(6)$$
$$= 40 - 42 = -2$$

Now interchanging rows & columns (transpose of A)

$$A^t = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} = (5)(8) - (6)(7)$$
$$= 40 - 42 = -2$$

4. Switching Property:

Value of determinant is multiplied by -1 if any two rows or two columns of matrix are interchanged.

Example:

$$A = \begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix}, \quad |A| = (5)(3) - (2)(6)$$
$$= 15 - 12 = 3$$

Interchanging rows

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}, \quad |A| = (2)(6) - (5)(3)$$
$$= 12 - 15 = -3 \text{ (i.e } 3 \times -1)$$

Interchanging columns

$A = \begin{bmatrix} 6 & 5 \\ 3 & 2 \end{bmatrix}$, $|A| = (6)(2) - (3)(5)$
$= 12 - 15 \quad = -3 \quad$ (i.e 3×-1)

5. Scalar Multiple Property:

If entries of row or column of matrix are multiplied with some scalar say 'n', value of new determinant is 'n' times previous value of determinant.

Example:

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $|A| = (1)(4) - (2)(3)$
$= 4 - 6 \quad = -2$

Multiplying R2 of A with '2'

$2 \times [\, 2 \quad 4 \,] \rightarrow A = \begin{bmatrix} 1 & 3 \\ 4 & 8 \end{bmatrix}$, $|A| = (1)(8) - (4)(3)$
$= 8 - 12$
$= -4$

We can see that when rows or columns of determinant are multiplied by 2, determinant also gets multiplied by 2.

6. Repetition Property:

If entries in a row or column are same as other entries of row or column, determinant is zero.

Example:

$A = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$, $|A| = (2)(3) - (2)(3)$
$= 6 - 6 \quad = 0$

7. Sum Property:

In a determinant, each element in any row (or column) consists of sum of two terms, then determinant can be expressed as sum of two determinants of same order.

$$\begin{vmatrix} a_1 + u_1 & b_1 & c_1 \\ a_2 + u_2 & b_2 & c_2 \\ a_3 + u_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} u_1 & b_1 & c_1 \\ u_2 & b_2 & c_2 \\ u_3 & b_3 & c_3 \end{vmatrix}$$

Example:

$$\begin{vmatrix} 2+2 & 5 \\ 3+1 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 5 \\ 4 & 6 \end{vmatrix} = [(2)(6)-(3)(5)] + [(2)(6)-(1)(5)]$$

$$(4)(6)-(4)(5) = (12-15)+(12-5)$$

$$24-20 = -3+7$$

$$4 = 4$$

8. Triangle Property:

If all elements of determinant above or below the main diagnol consist of zeros, determinant is equal to product of diagnol entries.

Example:

$$A = \begin{bmatrix} 1 & 4 \\ 0 & 3 \end{bmatrix}, \quad |A| = (1)(3)-(0)(4)$$
$$= 3$$

9. Factor Property:

If a determinant becomes 0 on putting $x = \alpha$, then $(x-\alpha)$ is factor of determinant.

Example:

$$A = \begin{bmatrix} x & 1 & 4 \\ x+1 & 2 & 5 \\ x+2 & 3 & 6 \end{bmatrix}, \text{ then by putting } x = 1$$

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 5 \\ 3 & 3 & 6 \end{bmatrix} \Rightarrow |A| = 1(12-15) - 1(12-15) + 4(6-6)$$

$$= 1(-3) - 1(-3) + 4(0)$$

$$= -3+3+0$$

$$= 0$$

Hence $(x-1)$ is a factor of determinant.

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10. **Determinant of co-factor matrix:**

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

where C_{ij} denotes the co-factors of elements a_{ij} in Δ.