ASSIGNMENT-1
LINEAR ALGEBRA

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QUESTION-1

Goods	Services	P.B
0.2	0.7	Goods
0.8	0.3	Services

P_G and P_S are total annual amounts.

$$P_G = 0.2p_G + 0.7p_S$$
$$P_S = 0.8p_G + 0.3p_S$$

Arranging the equations

$$.8p_G - .7p_S = 0 \text{ —— } 1)$$
$$-.8p_G + .7p_S = 0 \text{ —— } 2)$$

Augmented matrix form

$$\begin{bmatrix} .8 & -.7 & 0 \\ -.8 & .7 & 0 \end{bmatrix}$$

Through row reduction, solve the matrix

$$\cancel{.8} \quad \cancel{-.7} = \cancel{0}$$
$$\cancel{-.8} + \cancel{.7} = \cancel{0}$$
$$0 \quad 0 \quad 0 \text{ —— } 3)$$

Replace eq ② by eq ③ in augmented matrix

$$\begin{bmatrix} .8 & -.7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Divide eq ① by ⑧ and replace it by new eq in above matrix

$$\begin{bmatrix} 1 & -.875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, the general solution is $\boxed{P_G = -.875 p_s}$ and P_S is free.

QUESTION 2

No, the ratio of prices will remain same for whatever currency. Equilibrium price would change only as it is being multiplied with a constant value.

QUESTION·3

(a)

Colums depict where the output goes and rows depict from where the input come from

Chemicals	Fuels	Machinery	P.B
·2	·8	·4	Chemicals
·3	·1	·4	Fuels
·5	·1	·2	Machinery

(b)

P_c, P_F and P_M show total annual output. System of equations for them is as:

$$P_c = .2p_c + .8p_F + .4p_M$$
$$P_F = .3p_c + .1p_F + .4p_M$$
$$P_M = .5p_c + .1p_F + .2p_M$$

Shifting right side to left.

$$.8p_c - .8p_F - .4p_M = 0$$
$$-.3p_c + .9p_F - .4p_M = 0$$
$$-.5p_c - .1p_F + .8p_M = 0$$

Augmented matrix

$$\begin{bmatrix} .8 & -.8 & -.4 & 0 \\ -.3 & .9 & -.4 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix}$$

Scanned with CamScanner

(c)

Reduced echelon form will be;

$$\begin{bmatrix} .8 & -.8 & -.4 & 0 \\ -.3 & .9 & -.4 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix}$$

Dividing R_1 by .8 and replacing R_1 by new R.

$$\begin{bmatrix} 1 & -1 & -.5 & 0 \\ -.3 & .9 & -.4 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix}$$

Multiply R_1 by .3 and add it to R_2.

$$\begin{bmatrix} 1 & -1 & -.5 & 0 \\ 0 & .6 & -5.5 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix}$$

Multiply R_1 by .5 and add it to R_3

$$\begin{bmatrix} 1 & -1 & -.5 & 0 \\ 0 & .6 & -5.5 & 0 \\ 0 & -.6 & 5.5 & 0 \end{bmatrix}$$

$R_1 + R_2$, R_2 is divided by .6 and $R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & -1.417 & 0 \\ 0 & 1 & -.917 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So, general solution is $p_c = -1.417p_m$, $p_F = -.917p_m$ while p_m is free.

QUESTION·4

(a)

A	E	M	T	P.B
·65	·30	·30	·20	A
·10	·10	·15	·10	E
·25	·35	·15	·30	M
0	·25	·40	·40	T

(b)

P_A, P_E, P_M and P_T are total annual amounts.
System of equations for them is as:

$$P_A = .65p_A + .30p_E + .30p_M + .20p_T$$
$$P_E = .10p_A + .10p_E + .15p_M + .10p_T$$
$$P_M = .25p_A + .35p_E + .15p_M + .30p_T$$
$$P_T = .25p_E + .40p_M + .40p_T$$

Shifting them to left side

$$.35p_A - .30p_E - .30p_M - .20p_T = 0$$
$$-.10p_A + .90p_E - .15p_M - .10p_T = 0$$
$$-.25p_A - .35p_E + .85p_M - .30p_T = 0$$
$$-.25p_E - .40p_M + .60p_T = 0$$

Augmented matrix is:

$$\begin{bmatrix} .35 & -.3 & -.3 & -.2 & 0 \\ -.10 & .9 & -.15 & -.1 & 0 \\ -.25 & -.35 & .85 & -.3 & 0 \\ 0 & -.25 & -.4 & +.6 & 0 \end{bmatrix}$$

(c)

$R_1 \times \frac{20}{7}$

$$\begin{bmatrix} 1 & -6/7 & -6/7 & -4/7 & 0 \\ -1/10 & 9/10 & -3/20 & -1/10 & 0 \\ -1/4 & -7/20 & 17/20 & -3/10 & 0 \\ 0 & -1/4 & -2/5 & 3/5 & 0 \end{bmatrix}$$

$(R_1 \times \frac{1}{10}) + R_2$

$$\begin{bmatrix} 1 & -6/7 & -6/7 & -4/7 & 0 \\ 0 & 57/70 & -33/140 & -11/10 & 0 \\ -1/4 & -7/20 & 17/20 & -3/10 & 0 \\ 0 & -1/4 & -2/5 & 3/5 & 0 \end{bmatrix}$$

$(R_1 \times \frac{1}{4}) + R_3$

$$\begin{bmatrix} 1 & -6/7 & -6/7 & -4/7 & 0 \\ 0 & 57/70 & -33/140 & -11/70 & 0 \\ 0 & -79/140 & 89/140 & -31/70 & 0 \\ 0 & -1/4 & -2/5 & 3/5 & 0 \end{bmatrix}$$

$R_2 \times \frac{70}{57}$

$$\begin{bmatrix} 1 & -6/7 & -6/7 & -4/7 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & -79/140 & 89/140 & -31/70 & 0 \\ 0 & -1/4 & -2/5 & 3/5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -21/19 & -14/19 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & -79/140 & 89/140 & -31/70 & 0 \\ 0 & -1/4 & -2/5 & 3/5 & 0 \end{bmatrix} \left(R_2 \times \frac{6}{7}\right) + R_1$$

$$\begin{bmatrix} 1 & 0 & -21/19 & -14/19 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & 0 & 359/760 & -629/1140 & 0 \\ 0 & -1/4 & -2/5 & 3/5 & 0 \end{bmatrix} \left(R_2 \times \frac{79}{140}\right) + R_3$$

$$\begin{bmatrix} 1 & 0 & -21/19 & -14/19 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & 0 & 359/760 & -629/1140 & 0 \\ 0 & 0 & -249/760 & 629/1140 & 0 \end{bmatrix} R_2 + \left(\frac{1}{4} \times R_4\right)$$

$$\begin{bmatrix} 1 & 0 & -21/19 & -728/359 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & -249/760 & 629/1140 & 0 \end{bmatrix} \left(R_1 + \frac{21}{19} R_3\right)$$

$$\begin{bmatrix} 1 & 0 & 0 & -728/359 & 0 \\ 0 & 1 & 0 & -572/1077 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & -249/760 & 629/1140 & 0 \end{bmatrix} R_2 + \frac{11}{38} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -728/359 & 0 \\ 0 & 1 & 0 & -572/1077 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} R_4 - \frac{249}{760} R_3$$

So, the general solution is

$$P_A = \frac{728}{359} P_T = 2.02\, P_T$$

$$P_E = \frac{572}{359} P_T = 1.59\, P_T$$

$$P_M = \frac{1258}{1077} P_T = 1.16\, P_T$$

P_T is free

If $P_T = 100$, $P_A = 202.7$, $P_E = 53.11$, $P_M = 1.16$

QUESTION.5

Total compounds = 4

For each compound, vector is

B_2S_3 $\begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$ Boron, Sulphur, Hydrogen, Oxygen

H_2O $\begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$

H_2BO_3 $\begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}$

H_2S $\begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$

Augmented matrix becomes

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_1/2 \\ \frac{3R_1}{2} - R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \quad \text{Swap } R_2 \text{ \& } R_3$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 0 & -3 & 0 & 0 \end{bmatrix} \quad R_2/2$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 0 & -3/2 & 1 & 0 \end{bmatrix} \quad R_4 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & -3/2 & 1 & 0 \end{bmatrix} \quad R_1 + \frac{R_3}{2}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & -\frac{3}{2} & 1 & 0 \end{bmatrix} \quad \frac{3}{2}R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \frac{3}{2}R_3 + R_4$$

$x_1 = \frac{1}{3}x_4$, $x_2 = 2x_4$, $x_3 = \frac{2}{3}x_4$, x_4 is free.

When $x_4 = 3$, $x_1 = 1$, $x_2 = 6$ and $x_3 = 2$

So balanced equation will be

$$\boxed{B_2S_3 + 6H_2O \longrightarrow 2H_3BO_3 + 3H_2S}$$