TUTORIAL 2 on Z-transforms

1. Determine the z-transforms of the following sequences and their associated ROCs,

Determine the z-transforms of the following *finite-duration* signals.

(a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\}$

(b) $x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

(c) $x_3(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 5, 7, 0, 1\}$

(d) $x_4(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

(e) $x_5(n) = \delta(n)$

(f) $x_6(n) = \delta(n - k), k > 0$

(g) $x_7(n) = \delta(n + k), k > 0$

(i) $x(n) = \{3, 0, 0, 0, 0, \underset{\uparrow}{6}, 1, -4\}$

1. Using the property of z-transform, determine the z-transforms of the following signal

Find the z-transform and the associated ROC for each of the following sequences:

(a) $x[n] = \delta[n - n_0]$

(b) $x[n] = u[n - n_0]$

(c) $x[n] = a^{n+1}u[n + 1]$

(d) $x[n] = u[-n]$

(e) $x[n] = a^{-n}u[-n]$

2. Using the property of z-transform, determine the z-transforms of the following signal.

$x[n] = n\,\{ u[n] - u[n-4]\}$

3. Determine the z-transforms of the following signals and the corresponding pole-zero patterns.

   (a) $x(n) = (1 + n)\, u(n)$

   (b) $x(n) = (a^n + a^{-n})\, u(n)$, a real

   (c) $x(n) = [3(2^n) - 4(3^n)]u(n)$

4. Find the Z-transform of the following signal and its ROC.

$$x(n)=\begin{cases}\cos(n\theta_0) & n\geq 0\\ 0 & n<0\end{cases}$$

5. Find the impulse response of the system described by the difference equation,
   y [n] = x [n] + 2x [n -1] – 4x [n-2] + x [n -3]

6. By using the partial-fraction expansion, determine the inverse z-transform :
   a)
   $$X(z)=\frac{3-\frac{5}{6}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)},\quad \frac{1}{4}<|z|<\frac{1}{3}.$$
   b)
   $$X(z)=\frac{3}{z-2}\qquad |z|>2$$

7. Determine the causal signal *x (n)* if its z-transform *X(z)* is given by:
   $$(a)\ \ X(z)=4+3(z^2+z^{-2})\qquad 0<|z|<\infty$$
   $$(b)\ \ X(z)=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{3}{1-\frac{1}{3}z^{-1}}\qquad |z|>\frac{1}{2}$$
   $$(c)\ \ X(z)=\frac{1}{1+3z^{-1}+2z^{-2}}\qquad |z|>2$$
   d)
   $$X(z)\ =\ \frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{2}z^{-1}}$$

8. By using the partial fraction expansion, determine the causal signal *x (n)*and its ROC if its z-transform *X(z)* is given by:
   $$(a)\quad \frac{4z^2+8z}{4z^2-5z+1}$$
   b)
   $$X(z)=\frac{1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}\qquad \text{ROC}:|z|>\frac{1}{2}$$

9. Compute the convolution x[n] of the following signals by using Z-transforms:

1. Compute the z-transforms of the signals to be convolved.

$$X_1(z) = Z\{x_1(n)\}$$

(time domain $\longrightarrow$ z-domain)

$$X_2(z) = Z\{x_2(n)\}$$

2. Multiply the two z-transforms.

$$X(z) = X_1(z)X_2(z), \qquad (z\text{-domain})$$

3. Find the inverse z-transform of $X(z)$.

$$x(n) = Z^{-1}\{X(z)\}, \qquad (z\text{-domain} \longrightarrow \text{time domain})$$

(a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}, \qquad x_2(n) = \{\underset{\uparrow}{1}, 1, 1\}$

(b) $x_1(n) = (\frac{1}{2})^n u(n), \qquad x_2(n) = (\frac{1}{3})^n u(n)$

c) In addition, use also graphical method for determining the convolution of $x_1[n]$ and $x_2[n]$

$$x_1(n) = \{1, -2, 1\}$$

$$x_2(n) = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

11.

A causal discrete-time LTI system is described by:

$$y[n] - \tfrac{3}{4}y[n-1] + \tfrac{1}{8}y[n-2] = x[n]$$

where *x[n]* and *y[n]* are the input and output of the system, respectively.
(a) Determine the system function H (z).
(b) Find the impulse response *h[n]* of the system.

12. We want to design a causal discrete-time LTI system with the property that if the input is

$$x(n) = (\frac{1}{2})^n u(n) - \frac{1}{4}(\frac{1}{2})^{n-1} u(n-1)$$

Then the output is

$$y(n) = (\frac{1}{3})^n u(n)$$

(a) Determine the impulse response h (n) and the system function *H(z)* of a system that satisfies the foregoing conditions.
(b) Find the difference equation that characterizes this system.
(c) Determine if the system is stable.