# WORD ALIGNMENT MODELS

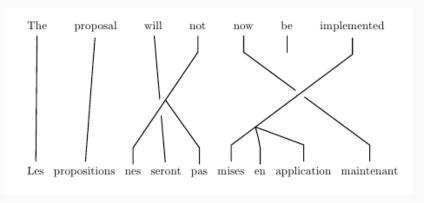
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### THE FIRST WORD ALIGNMENT



Brown et al. (1990).

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$$\Pr(f_1,...,f_J|e_1,...,e_I,\theta) = \sum_{a_1=1}^{I} ... \sum_{a_J=1}^{I} \Pr(f_1,...,f_J,a_1,...,a_J|e_1,...,e_I,\theta)$$

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Exact E-step is only tractable for a very limited set of models.

# IBM PAPERS (1990-1993)

Formulated a generative model of parallel sentence pairs

$$\begin{split} Pr(f|e) &= \sum_{a \in \mathcal{A}} Pr(a,f|e) \\ &= \sum_{a \in \mathcal{A}} \underbrace{Pr(a|e)}_{Prior} \underbrace{Pr(f|e,a)}_{Translation \ model} \end{split}$$

where f is a French sentence, e is an English sentence and  $\mathcal{A}$  is the set of all possible alignments for the sentence pair.

### ALIGNING WORDS IN A PARALLEL CORPUS

We're given corpus of translated sentence pairs  $D = \{(e_1, f_1), (e_2, f_2), (e_3, f_3), ...\}.$ 

We assume these sentence pairs are distributed *i.i.d.*.

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$$\begin{split} \Pr(D|\theta) &\approx \prod_{k \in D} \Pr(f_k|e_k, \theta) \\ &= \prod_{k \in D} \sum_{a_k \in \mathcal{A}} \Pr(a_k, f_k|e_k, \theta) \\ &= \prod_{k \in D} \sum_{a_k \in \mathcal{A}} \underbrace{\Pr(a_k|e_k, \theta)}_{\text{Prior}} \underbrace{\Pr(f_k|e_k, a_k, \theta)}_{\text{Translation model}} \end{split}$$

Бегемотика укусила собака

A dog bit the little hippopotamus

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### SIMPLIFYING ASSUMPTIONS

# Assumption 1

Each French word  $f_j$  is generated independently given the English word to which it is aligned  $e_{a_j}$ , i.e.

$$\Pr(\mathbf{f}, \mathbf{a} | \mathbf{e}) pprox \prod_{j=1}^{J} \Pr(\mathbf{a} | \mathbf{e}, \theta) \Pr(f_j | e_{a_j}, \theta).$$

### SIMPLIFYING ASSUMPTIONS

# Assumption 2

We'll parameterize the translation model  $Pr(f_j|e_{a_j}, \theta)$  with a table of conditional probabilities t(f|e).

E.g. for Russian to English translation the table t(f|dog) could be defined as

$$t(coбaкa|dog) = 0.5$$

$$t(co6aky|dog) = 0.3$$

$$t(кошка|dog) = 0.2.$$

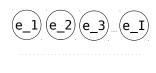
### SIMPLIFYING ASSUMPTIONS

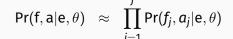
# Assumption 3

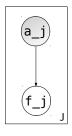
We'll simplify the 'prior'  $Pr(a|e, \theta)$  by assuming that  $a_j$  depends only on a subset of the other alignments, i.e.

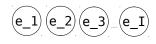
$$\Pr(\mathbf{f}, \mathbf{a}|\mathbf{e}) \approx \prod_{j=1}^{J} \Pr(a_j|\mathbf{a}_{\mathsf{subset}}, \mathbf{e}, \theta) \Pr(f_j|\mathbf{e}_{a_j}, \theta).$$

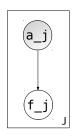
$$\begin{split} \Pr(\mathbf{f}, \mathbf{a} | \mathbf{e}, \theta) &= \Pr(\mathbf{a} | \mathbf{e}, \theta) \Pr(\mathbf{f} | \mathbf{e}, \mathbf{a}, \theta) \\ &= \prod_{j=1}^{J} \Pr(a_j | \mathbf{a}_1^{j-1}, \mathbf{f}_1^{j-1}, \mathbf{e}, \theta) \Pr(f_j | \mathbf{a}_1^{j}, \mathbf{f}_1^{j-1}, \mathbf{e}, \theta) \\ &\approx \prod_{j=1}^{J} \Pr(a_j | \mathbf{a}_1^{j-1}, \mathbf{f}_1^{j-1}, \mathbf{e}, \theta) \Pr(f_j | \mathbf{e}_{a_j}, \theta) \\ &\approx \prod_{j=1}^{J} \underbrace{\Pr(a_j | \mathbf{a}_{subset}, \mathbf{e}, \theta)}_{\text{prior model}} \underbrace{\Pr(f_j | \mathbf{e}_{a_j}, \theta)}_{\text{translation model}} \end{split}$$



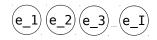


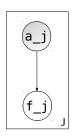






$$\Pr(f, a|e, \theta) \approx \prod_{j=1}^{J} \Pr(f_j, a_j|e, \theta)$$
$$= \prod_{j=1}^{J} \Pr(a_j|e) \Pr(f_j|e, a_j, \theta)$$

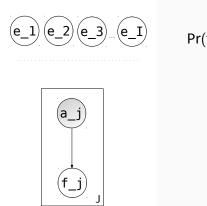




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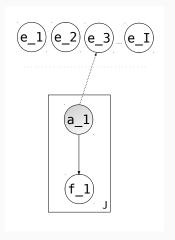


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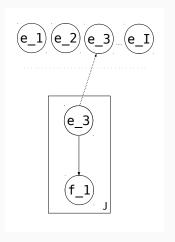
$$= \prod_{j=1}^{J} \Pr(a_j|e) \Pr(f_j|e, a_j, \theta)$$

$$\approx \prod_{j=1}^{J} \epsilon \Pr(f_j|e_{a_j}, \theta)$$

$$\propto \prod_{j=1}^{J} t(f_j|e_{a_j})$$



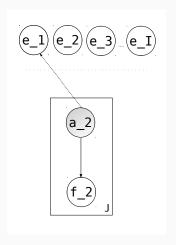
$$Pr(f, a|e, \theta) \approx \prod_{j=1}^{J} Pr(f_j, a_j|e_{a_j}, \theta)$$
$$= Pr(f_1, a_1 = 3|e_3, \theta) \dots$$



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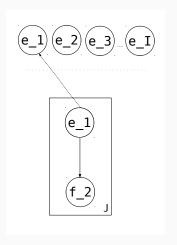
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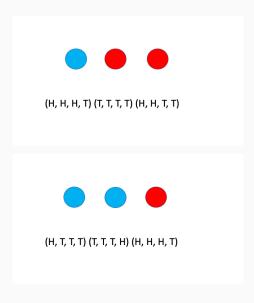
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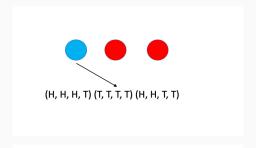
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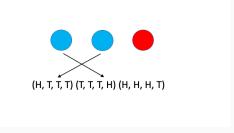
$$\approx t(f_1, |e_3) t(f_2|e_1) \dots$$

# COINS?

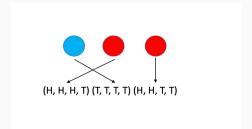


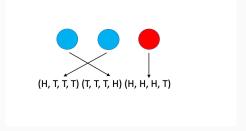
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- Explaining away: once an output word is explained by one input, then the other inputs become less important.
- (Assuming a uniform prior)

$$Pr(a_j = i | e, f) = \frac{Pr(f_j | e_i)}{\sum_k Pr(f_j | e_k)}$$

# LIMITATIONS

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- More complex models can guarantee all input words are aligned but ....

# Бегемотика укусила собака A dog bit the little hippopotamus

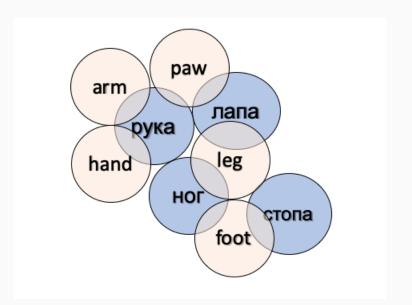


# null Бегемотика укусила собака A dog bit the little hippopotamus





# **SEMANTIC DRIFT**



The expected log-likelihood for f given e under IBM Model 1 is

$$\mathbb{E}[log(f|e,\theta)] = \sum_{j=1}^{J} \sum_{i=1}^{I} \Pr(a_j = i|f,e,\theta) \log \Pr(f_j,a_j = i|e_i,\theta)$$

$$= \sum_{i=1}^{J} \sum_{j=1}^{I} \Pr(a_j = i|f,e,\theta) \log t(f_j|e_i) + C.$$

To apply EM we need to compute  $\Pr(a_j = i | f, e, \theta)$  for each source and target pair and then maximize this term w.r.t. our parameters  $\theta = t(f|e)$ .

The posterior alignment probabilities,  $Pr(a_j = i | f, e, \theta)$  can be computed as follows

$$Pr(\mathbf{a}|\mathbf{f}, \mathbf{e}, \theta) = \frac{Pr(\mathbf{f}, \mathbf{a}|\mathbf{e}, \theta)}{\sum_{k} Pr(\mathbf{f}, \mathbf{a}' = k|\mathbf{e}, \theta)}$$
(1)  
$$Pr(a_j = i|\mathbf{e}, \theta) Pr(f_j|a_j = i, \mathbf{e}, \theta)$$
(2)

$$= \frac{\Pr(a_j = i | \mathbf{e}, \theta) \Pr(f_j | a_j = i, \mathbf{e}, \theta)}{\sum_{k=1}^{I} \Pr(a_j = k | \mathbf{e}, \theta) \Pr(f_j | a_j = k, \mathbf{e}, \theta)}$$
(2)

$$=\frac{\epsilon t(f_j|e_i)}{\sum_{k=1}^{I} \epsilon t(f_j|e_k)}$$
 (3)

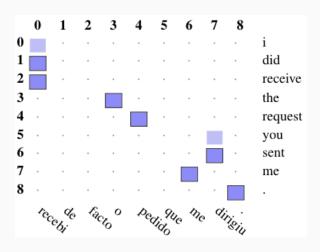
$$= \frac{t(f_j|e_i)}{\sum_{k=1}^{I} t(f_j|e_k)}.$$
 (4)

# MEASURING ALIGNMENT QUALITY

Given a golden set of manually created *M* consisting of probable *P* and sure *S* alignments. We can measure the error rate of an automatic alignment *A*:

$$\begin{split} \textit{Precision}(A;P) &= \frac{|P \cap A|}{|A|} \\ \textit{Recall}(A;S) &= \frac{|S \cap A|}{|S|} \\ \textit{AlignmentErrorRate}(A;S,P) &= 1 - \frac{|P \cap A| + |S \cap A|}{|S| + |A|}. \end{split}$$

### **WORD ALIGNMENT MATRIX**



Natural way to visualize an alignment.

# **NOISY CHANNEL MODEL**

'The Mathematics of Machine Translation', Brown et al. (1993).

$$e^* = \operatorname{argmax} \Pr(e) \Pr(f|e)$$

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Why is modelling Pr(f|e) easier than modelling Pr(e|f) if we want to translate from f to e?

- Attention scores

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$$Pr(a_j = i | e_1, ..., e_m, s_j) = \frac{e^{A_{\theta}(e_i, s_j)}}{\sum_{k} e^{A_{\theta}(e_k, s_j)}}$$

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- Soft attention averages contexts

$$c_j = \sum_i \Pr(a_j = i | e_1, ..., e_m, s_j) e_i$$

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- So something like

$$\Pr(f_j|s_j,c_j) = \Pr(f_j|s_j, \sum_i \Pr(a_j = i|e_1,...,e_m,s_i)e_i)$$

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- Hard attention is closer to a probabilistic alignment

$$Pr(f_j|e_1,...,e_m,s_j) = \sum_i Pr(a_j = i|e_1,...,e_m,s_t)Pr(f_j|e_j,s_j)$$