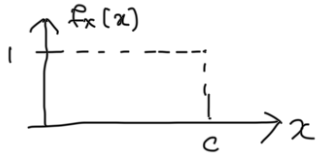
Degenerate Distribution :-

Definition : A random variable X is said to have Degenerate Distribution if $P[X=c]=1$. for some c .

pmf → $f_X(x) = \begin{cases} 1 & x=c \\ 0 & \text{otherwise} \end{cases}$



cdf → $F_X(x) = P(X \leq x) = \begin{cases} 0 & x < c \\ 1 & x \geq c \end{cases}$

Moments : $\mu'_r = E(X^r) = c^r \, P(X=c)$

$= c^r$

$\mu'_1 = E(X) = c$

$V(X) = \mu'_2 - \mu'^2_1$

$= c^2 - c^2$

$= 0$ (It has no variability)

Note that $V(X)=0$ iff $E(X-\mu)^2=0$, $\mu = E(X)$

iff $P(X-\mu=0)=1$

iff X is a degenerate random variable.

(Characterisation)