Uniform Distribution :-

Definition : A random variable X is said to have Uniform distribution over the set $\{x_1, x_2, \ldots x_N\}$ if

$P[X = x_i] = c \;\forall\; i = 1, 2, \ldots N$

Here,
$$1 = P(\Omega)$$
$$= P\left(\bigcup_{i=1}^{N} \{X = x_i\}\right)$$
$$= \sum_{i=1}^{N} P(X = x_i)$$
$$= cN$$
$$\Rightarrow c = \frac{1}{N} \qquad [\text{Uniform} = \text{No bias}]$$

pmf $\Rightarrow f_X(x) = \begin{cases} \frac{1}{N} & \text{if } x = x_i,\ i = 1(1)N \\ 0 & \text{, ow} \end{cases}$

Moments :

$$\mu_r' = E(X^r) = \frac{1}{N} \sum_{i=1}^{N} x_i^r$$
$$E(X) = \frac{1}{N} \sum_{i=1}^{N} x_i = \bar{x} \text{ (say)}$$
$$V(X) = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

If particular if $X \sim \text{Uniform}\{1, 2, \ldots N\}$
$$E(X) = \frac{N(N+1)}{2}$$
$$V(X) = \frac{N^2 - 1}{12}$$

1) An urn contains N balls no $1, 2, \ldots, N$ if n balls are randomly withdrawn in sequence each time replacing the ball selected previously. Find $P(X=k)$ and $P(Y=k)$, $k = 1, 2, \ldots N$ where X and Y is the max and min of n choosen no. Also, find $E(X)$. Show that for large N, $E(X) = \frac{nN}{n+1}$. Find $E(Y)$ as show that for large N, $E(Y) \simeq \frac{N}{n+1}$

let X_i be the ball number drawn in the ith draw.
$$P(X_i = x) = \frac{1}{N}, \quad x = 1, 2, \ldots N$$

Clearly, X_i's are iid Uniform$\{1, 2, \ldots N\}$ as drawings are made with replacement.

$X = \max\{X_i\}$

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$$F_X(x) = P(X \leq x) = P(\max_{i=1(1)n}\{X_i\} \leq x)$$

$$= P(X_1 \leq x, X_2 \leq x, \ldots X_n \leq x)$$

$$= [P(X_1 \leq x)]^n \quad (\text{iid})$$

$$= (F_X(x))^n$$

$$= \left(\sum_{i=1}^{x} P(X \leq i)\right)^n$$

$$= \left(\sum_{i=1}^{x} \frac{1}{N}\right)^n$$

$$= \left(\frac{x}{N}\right)^n, \quad x = 1, 2, \ldots N.$$

$$\therefore F_X(x) = \begin{cases} 0 & , x < 1 \\ \left(\frac{[x]}{N}\right)^n & , 1 \leq x < N \\ 1 & x \geq N \end{cases}$$

For the non-negative X,

$$E(X) = \sum_{x=0}^{\infty} \{1 - F_X(x)\}$$

$$= \sum_{x=0}^{N-1} \left\{1 - \left(\frac{x}{N}\right)^n\right\} + \sum_{x=N}^{\infty} (1-1)$$

$$= N - \sum_{x=0}^{N-1} \left(\frac{x}{N}\right)^n$$

For large N, $\frac{1}{N} \sum_{x=0}^{N-1} \left(\frac{x}{N}\right)^n \simeq \int_0^1 u^n \, du = \frac{1}{n+1}$

$$\Rightarrow \sum_{x=0}^{N-1} \left(\frac{x}{N}\right)^n \simeq \frac{N}{n+1}$$

and $E(X) \simeq N - \frac{N}{n+1} = \frac{nN}{n+1}$

Here, $Y = \min_{i=1(1)n} \{x_i\}$

$$F_Y(y) = P(Y \leq y) = 1 - P(Y > y)$$

$$= 1 - P(\min_{i=1(1)n} \{x_i\} > y)$$

$$= 1 - P(X_1 > y, X_2 > y, \ldots X_n > y)$$

$$= 1 - (P(X_1 > y))^n$$

$$= 1 - (1 - F_{X_1}(y))^n$$

$$= 1 - (1 - y)^n$$

$$\left\{ \quad \frac{\quad}{N} \right\}$$

For the non-negative Y,

$$E(Y) = \sum_{y=0}^{\infty} \{1 - F_Y(y)\}$$

$$= \sum_{y=0}^{N-1} \left(1 - \frac{y}{N}\right)^n + \sum_{y=N}^{\infty} (1-1)$$

$$= \sum_{x=1}^{N} \left(\frac{x}{N}\right)^n \qquad x = N - y$$

$$\simeq \frac{N}{n+1}, \quad \text{for large } N$$

Repeated Trails:

A trail is a single performance of a random experiment.

Suppose that we are interested in an event. A defined occurrence of A as success and non-occurrence of A as failure. When the outcome of a trail is classified as success or failure then the trail is known as Bernoulli trail.

In a sequence of trail (Bernoulli) the trails may be dependent or independent and the probability of success may remain constant or may vary from one trail to another.

A sequence of Bernoulli trail is called a repeated trails if the trails are independent and probability of success remains the constant from one trail to another.