

# Assignment 5 - Hartz

## 1. Repeated Substitution

$$T(n) = 2T(n/4) + 2n$$

$$= 2(2T(n/4) + 2n)(n/4) + 2n = (((4n+Tn)n)/4) + 2n$$

$$= (((4n+(2T(n/4) + 2n)n)/4) + 2n = (((8n+4n^2+(n^2)T)n)/8) + 2n$$

$$= (((16n+8n^2 + 4n^3+(n^3)T)n)/16) + 2n$$

$$= (((32n + 16n^2 + 8n^3 + 4n^4 +(n^4)T)n)/32) + 2n$$

$((j)n + (j-1)n^2 + (j-2)n^3 \dots (j-x)n^{(x-1)} + n^{(x-1)}T)/j) + 2n$ , where  $j$  is the series of  $2^2$ ;  $j(1) = 4$ ,  $j(2) = 8$ ,  $j(3) = 16$ , and so forth.

## 2. Master Theorem

a.  $T(n) = 2T(n/4) + n^3$

$a$  is 2,  $b$  is 4, and  $d$  is 3 ( $d$  being the growth rate of the function at the end,  $n^3$ ).  $b^d$  is  $4^3$ , which is 64, and  $2 < 64$ . Therefore, the first case of the master theorem applies: The time complexity is  $\theta(n^d)$ , or  $\theta(n^3)$ .

b.  $T(n) = 2T(n/2) + 6n^4$

$a$  is 2,  $b$  is 2, and  $d$  is 4.  $2^4$  is 16, which is greater than 2, so time comp is  $\theta(n^4)$ .

c.  $T(n) = 6T(n/7) + 23$

$a$  is 6,  $b$  is 7,  $d$  is 0.  $7^0 = 1$ , which is less than 6. Therefore, the third case of the master theorem applies: The time complexity is  $\theta(n \log n)$ , or  $\theta(n \log n)$ .

d.  $T(n) = 16T(n/4) + n^2$

$a$  is 16,  $b$  is 4, and  $d$  is 2.  $4^2$  is 16, which means  $a = b^d$ , leading to the second case. Therefore, time complexity is  $\theta(n^d \log n)$ , or is  $\theta(n^2 \log n)$

e.  $T(n) = 7T(n/9) + n^3$

$a$  is 7,  $b$  is 9, and  $d$  is 3.  $9^3$  is 729, which is much greater than 7. Therefore, time complexity is  $\theta(n^3)$ .

## 3. Double Hashing

[43, 22, 10, 8, 7, 4, 0, 11, 3, 28, 43, 36]

I'm not sure if I'm understanding the double hashing correctly. My interpretation that I used here is as follows: first, go through h1, of course. After that, take the key, multiply by which probe you're on, and then go through Reverse, and modulo that against 11.

0: 43

1: 22

2: 11

3: 8

4: 4

5: 43

6: 3

7: 7

8: 28

9: 10

10: 36

43 maps to 0. No probe sequence necessary, it goes to 0.

22 maps to 1. No probe sequence necessary, it goes to 1.

10 maps to 9. No probe sequence necessary, it goes to 9.

8 maps to 3. No probe sequence necessary, it goes to 3.

7 maps to 0. On probe one, 7 maps to 7.

4 maps to 3. On probe one, 4 maps to 4.

0 maps to 5. No probe sequence necessary, it goes to 5.

11 maps to 2. No probe sequence necessary, it goes to 2.

3 maps to 0. On probe one, 3 maps 3. On probe two, 3 maps to 6.

28 maps to 8. No probe sequence necessary, it goes to 8.

43 maps to 0. On probe one, 43 maps to 1. On probe two, 43 maps to 2. On probe three, 43 maps to 8. On probe four, 43 maps to 7. On probe 5, 43 maps to 6. On probe 6, 43 maps to 5.

36 maps to 10. No probe sequence necessary, it goes to 10.

#### 4. Radix Sort

CAT, SBX, LOG, SUN, MUG, ROW, JOB, COX, LAP, RAT, PER, DAD, CAR, FIG, PIG, VIA,  
LOW,

LOX, TEA, ATE, ARE, DOG, TSL

I solved #5 first and then used that code to generate the answer to this, hence why the output is lowercase. The preceding number is the bucket the string was sorted into.

Pass one, on last letter:

1 via  
1 tea  
2 job  
4 dad  
5 ate  
5 are  
7 log  
7 mug  
7 fig  
7 pig  
7 dog  
12 tsl  
14 sun  
16 lap  
18 per  
18 car  
20 cat  
20 rat  
23 row  
23 low  
24 sbx  
24 cox  
24 lox

Pass two, on middle letter:

1 dad  
1 lap  
1 car  
1 cat  
1 rat  
2 sbx  
5 tea

5 per  
9 via  
9 fig  
9 pig  
15 job  
15 log  
15 dog  
15 row  
15 low  
15 cox  
15 lox  
18 are  
19 tsl  
20 ate  
21 mug  
21 sun

Pass three, on first letter:

1 are  
1 ate  
3 car  
3 cat  
3 cox  
4 dad  
4 dog  
6 fig  
10 job  
12 lap  
12 log  
12 low  
12 lox  
13 mug  
16 per  
16 pig  
18 rat  
18 row  
19 sbx  
19 sun  
20 tea  
20 tsl  
22 via

## 7. Algo Analysis

4's algorithm is the same as 5's.

For #5, time complexity is  $O(n*d)$ , where  $n$  is equal to the length of the input array and  $d$  is equal to the maximum amount of digits present in the input. This will be a constant, ultimately, so it's  $O(n)$ , really. Space complexity for this method is  $O(n)$  as well, with  $n$  still being the size of the input array because I called `.toCharArray` within a for loop that runs  $n$  times, meaning a new char array is initialized  $n$  times.

For #6, the time complexity is  $O(n*k)$ , where  $n$  is equal to the size of the first input and  $k$  is equal to the size of the second input. Radixsort is called on both, which has an  $O(j)$  time complexity with  $j$  being its input, but more important, I have a nested for loop in my answer, which compares every element of  $n$  to every element of  $k$ . Space complexity for this method is  $O(n)$ , as the loop variable for the nested  $k$  loop is initialized  $n$  times.