Stable Populations Assignment

European Doctoral School of Demography 2021

Liliana Patricia Calderón Bernal
Gonzalo Daniel Garcia
Ainhoa-Elena Leger
Ozge Elif Ozer

26/3/2021

Exercise 1: Everybody loves penguins

Here are two stage-classified matrices for the Emperor penguin. The life cycle diagram is given on the next-to-last page of the Jenouvrier et al. (2009) paper in the readings for the course. The first of these matrices was obtained under normal conditions, the other during an unusual warm event during which the penguin population declined by about 50%.

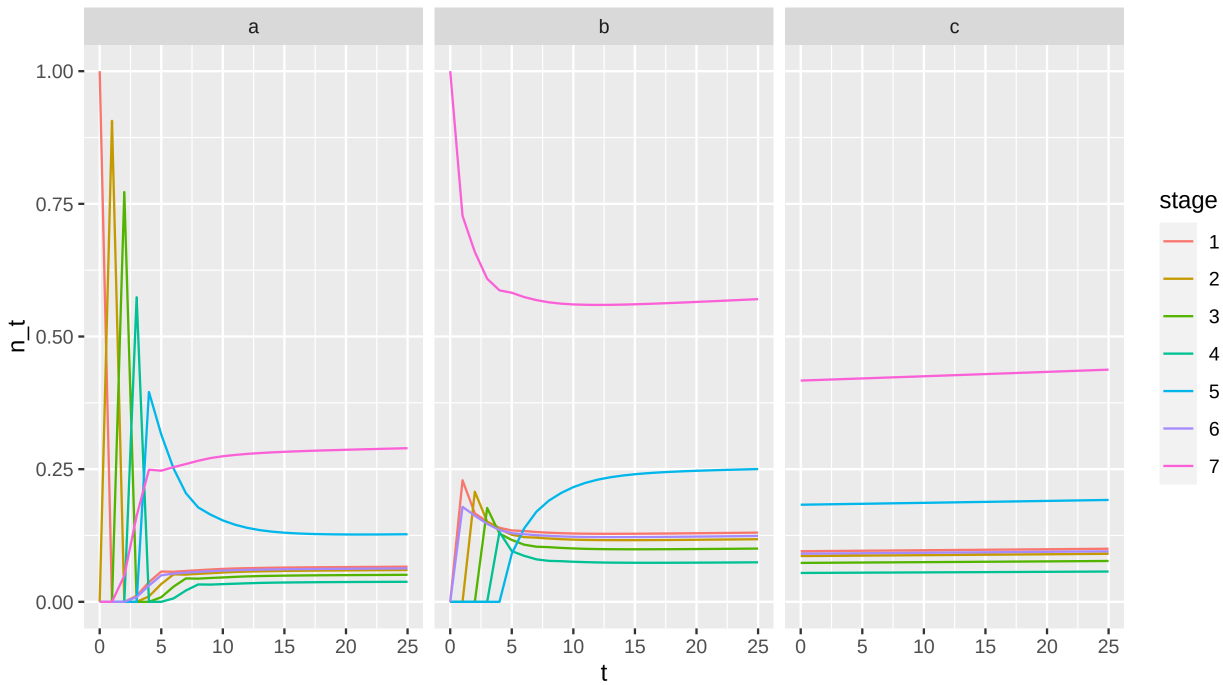
(a) Write a program to project the population (for what seems like an interesting length of time) starting from several different initial conditions:

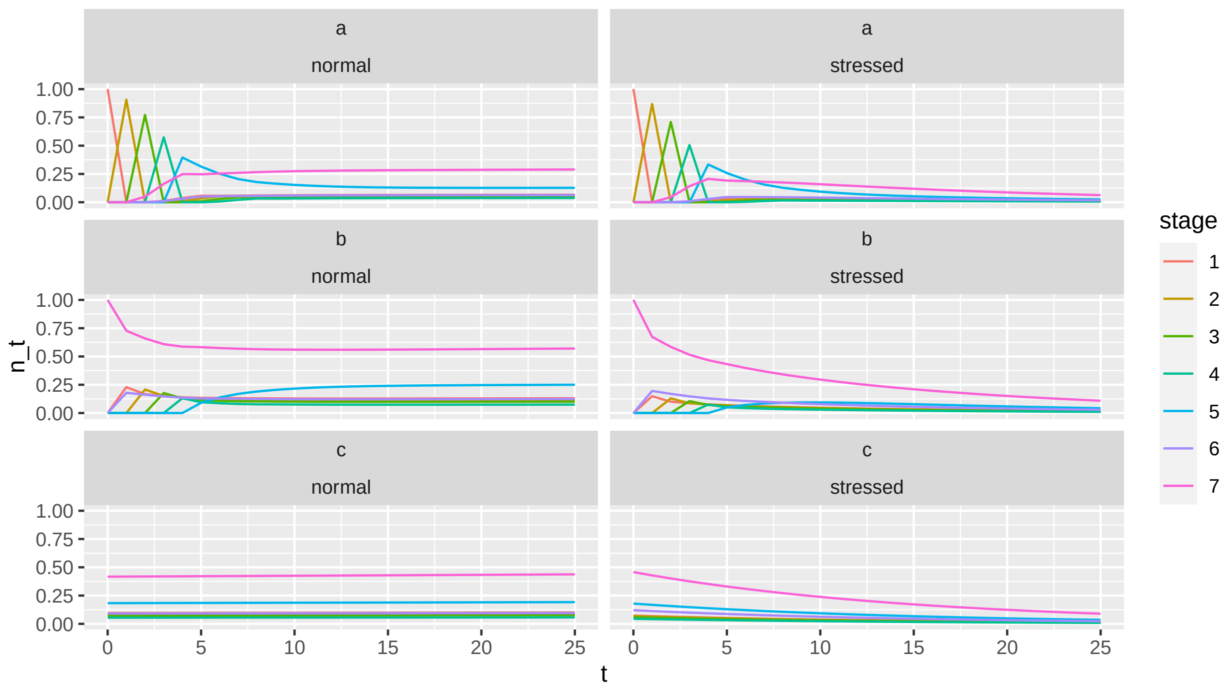
$\mathbf{n}(0)$ = one newborn baby penguin
$\mathbf{n}(0)$ = one breeding adult penguin
$\mathbf{n}(0)$ = a population with the stable stage distribution

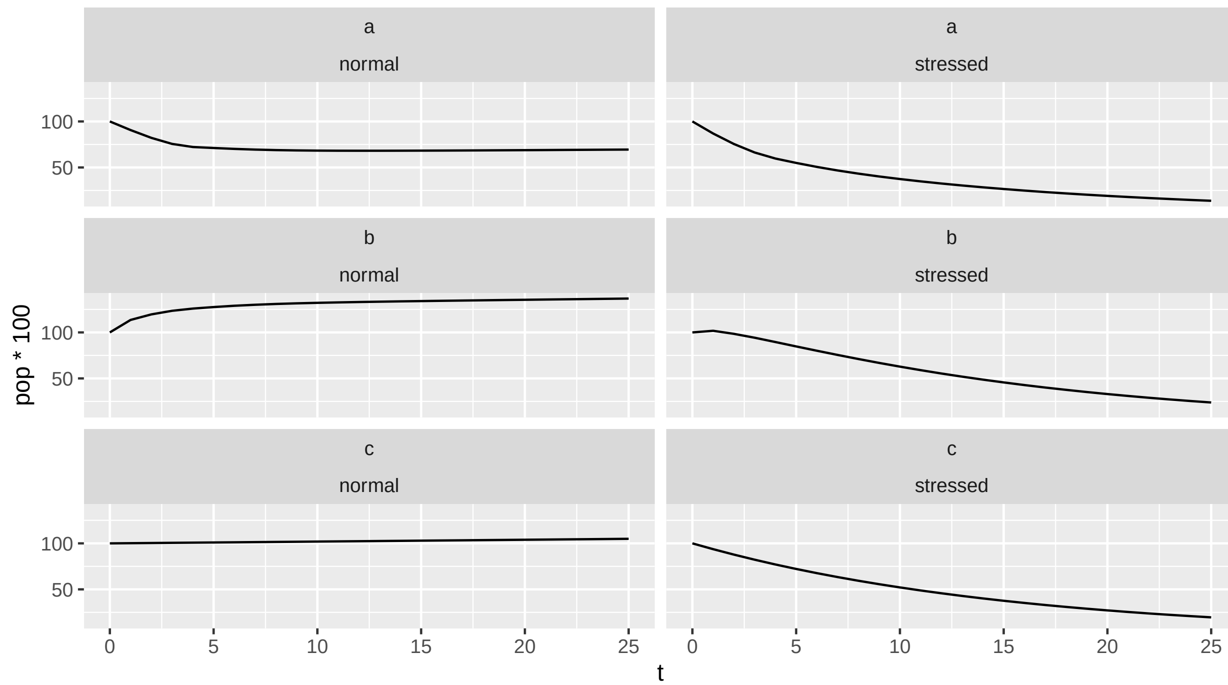
Plot the results (plotting on a log scale will probably be particularly informative). Compare the fates of the population under the two different environmental conditions.

[Sol.]

```
        [,1]      [,2]     [,3]     [,4]     [,5]     [,6]     [,7]
V1 1.0000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000
V2 0.9064000 1.0000000 0.000000 0.000000 0.000000 0.000000 0.000000
V3 0.7722528 0.8520000 1.000000 0.000000 0.000000 0.000000 0.000000
V4 0.5739383 0.6332064 0.743200 1.000000 0.000000 0.000000 0.000000
V5 1.9486746 2.1499058 2.523364 3.395269 4.928536 0.000000 0.000000
V6 0.9813666 1.0827081 1.148678 1.125640 1.030185 2.912393 1.912393
V7 4.5011284 4.9659404 5.268519 5.162852 4.725039 7.771368 8.771368
```


a
b
c
n_t
t
stage
1
2
3
4
5
6
7


a
normal
a
stressed
b
normal
b
stressed
c
normal
c
stressed
n_t
t
stage
1
2
3
4
5
6
7



(b) Find the population growth rate λ, the stable stage distribution $\mathbf{w}$, and the reproductive value distribution v for each matrix. Make some plots; make some comparisons.

[Sol.]

Exercise 2: The Irish tea-towel problem

In an airport in Belfast, I found a souveneir tea-towel, inscribed with a verse identified as "Irish philosophy". See the figure. It looks like an incidence-based health model to me.

(a) Create a life cycl graph for this system, based on your interpretation of the verse.

[Sol.]

(b) Identify the transient and absorbing states.

[Sol.]

(c) Make up some transition probabilities (your choice) and calculate mean occupancy times and the probabilities of ending up in Heaven or Hell.

[Sol.]

Exercise 3: An extra problem about Sweden

Sweden has an unusually long sequence of mortality and fertility data. There are two text files (`parray.txt` and `fertarray.txt`) in the Calculation Materials folder. One has survival probabilities as a function of age, the other has fertility as a function of age; one column for each year from 1891 to 2007.

(a) Write a program to use this information to create an age-classified projection matrix $\mathbf{A}$ for each year.

[Sol.]

(b) Compute the population growth rate λ_1 and the corresponding right and left eigenvectors $\mathbf{w1}$ and $\mathbf{v1}$ for each year.

[Sol.]

(c) This is a lot of information. To summarize the population structure, compute and plot the early-age dependency ratio and the old-age dependency ratio calculated from **w**.

[The dependency ratio is the ratio of population numbers during `dependent" ages (0-15 and older than 65) to the numbers in`productive" years (16-65). The early age and late age ratios just look at those portions of the dependent population.]

[Sol.]