

CS 5012: Foundations of Computer Science

Asymptotic Complexity Exercise

Given the following code snippets, provide the worst case time complexity in the form of Big-O notation. Justify your response and state any assumptions made. Treat these functions as constant runtime: print(), append()

```
► def measure(inputList):  
    int n = len(inputList)  $O(1)$   
    int sum = 0;  $O(1)$   
    for i in range(0, n):  $O(n)$   
        for j in range(0, 5):  $O(1)$   
            sum += j * inputList[i]  $O(1)$   
            for k in range(0, n):  $O(n)$   
                sum -= inputList[k]  $O(1)$ 
```

$\rightarrow n \cdot O(5) = n \cdot O(1) = O(n)$
 \rightarrow from nested for loop
 $\rightarrow n \cdot O(n) = O(n^2)$

The asymptotic complexity of this algorithm is: $O(n^2)$

$$T(n) = O(1) + O(1) + O(n) + O(n) + O(n) + O(n^2) + O(n) = O(n^2)$$

```
► def addElement(ele):  
    myList = []  $O(1)$   
    myList.append(666)  $O(1)$   
    print myList  $O(1)$ 
```

*print and append are $O(1)$
(constant run time)

The asymptotic complexity of this algorithm is: $O(1)$

$$T(n) = O(1) + O(1) + O(1) = O(1)$$

► num = 10 $O(1)$

def addOnesToTestList(num):

testList = [] $O(1)$

for i in range(0, num): $O(1)^*$ → $O(\text{num}) = O(1)$

testList.append(1) $O(1)$

print(testList) $O(1)$

return testList $O(1)$

→ returns are $O(1)$

The asymptotic complexity of this algorithm is: $O(1)$

$$T(n) = O(1) + O(1) + O(1) + O(1) + O(1) + O(1) = O(1)$$

► testList = [1, 43, 31, 21, 6, 96, 48, 13, 25, 5] $O(1)$

← consider list changeable (not fixed, will be modified)

def someMethod(testList):

for i in range(len(testList)): $O(n)$

for j in range(i+1, len(testList)): $O(n - (i+1)) = O(n) \rightarrow n \cdot O(n) = O(n^2)$

if testList[j] < testList[i]: $O(1)$ → $O(1) \cdot n^2 = O(n^2)$

testList[j], testList[i] = testList[i], testList[j] $O(1)$

print(testList) $O(1)$

The asymptotic complexity of this algorithm is: $O(n^2)$

$$T(n) = O(1) + O(n) + O(n^2) + O(n^2) + O(n^2) + O(n^2) = O(n^2)$$

► def searchTarget(target_word):

Assume range variables are unrelated to size of aList

for (i in range1): $O(1)$

for (j in range2): $O(1)$

for (k in range3): $O(1)$

if (aList[k] == target_word): $O(1)$

return 1 $O(1)$

return -1 $O(1)$

return -1 $O(1)$

The asymptotic complexity of this algorithm is: $O(1)$

$$T(n) = O(1) + O(1) + O(1) + O(1) + O(1) + O(1) + O(1) = O(1)$$

► def someSearch(sortedList, target):

left = 0 $O(1)$

right = len(sortedList) - 1 $O(1)$

while (left <= right): $O(\log n)$

mid = (left + right) / 2 $O(1)$

if (sortedList[mid] == target): $O(1)$

return mid $O(1)$

elif (sortedList[mid] < target): $O(1)$

left = mid + 1 $O(1)$

else:

right = mid - 1 $O(1)$

return -1 $O(1)$

$$\rightarrow O(1) \cdot \log n = O(\log n)$$

The asymptotic complexity of this algorithm is: $O(\log n)$

$$T(n) = O(1) + O(1) + O(\log n) + O(\log n) + O(\log n) + O(\log n) + \dots + O(\log n) = O(\log n)$$

► #Assume data is a list of size n

total = 0 $O(1)$

for j in range(n): $O(n)$

total += data[j] $O(1)$

big = data[0] $O(1)$

for k in range(1, n): $O(n)$

big = max(big, data[k]) $O(1)$

$$\rightarrow * O(1) \cdot n = O(n)$$

The asymptotic complexity of this algorithm is: $O(n)$

$$T(n) = O(1) + O(n) + O(n) + O(1) + O(n) + O(1) = O(n)$$

► powers = 0 $O(1)$

k = 1 $O(1)$

while k < n: $O(\log n)$

k = 2*k

powers += 1 $O(1)$

$$\rightarrow * O(1) \cdot \log n = O(\log n)$$

The asymptotic complexity of this algorithm is: $O(\log n)$

$$T(n) = O(1) + O(1) + O(\log n) + O(\log n) + O(\log n)$$

*Don't understand this one

► k = 1 $O(1)$

while k < n: $O(n)$

for j in range(k): $O(n)$

steps += 1

k = 2*k

$O(1)$

not $O(\log n)$ since k does not get to that limit

$$O(n) \cdot (n) = O(n^2)$$

$$O(1) \cdot n^2 = O(n^2)$$

The asymptotic complexity of this algorithm is: $O(n^2)$

$$T(n) = O(1) + O(n) + \underline{O(n^2)} + O(n^2) + O(n^2)$$

```
► for k in range(1, n):  $O(n)$ 
    j = 1  $O(1)$ 
    while j < k:  $O(\log n) \times$ 
        total += 1  $O(1) \times$ 
        j = 2 * j  $O(1)$ 
```

The asymptotic complexity of this algorithm is: $O(n \log n)$

$$T(n) = O(n) + O(n) + O(n \log n) + O(n \log n) + O(n \log n)$$

$\downarrow \qquad \qquad \downarrow$

$$\begin{matrix} * n \cdot O(\log n) & * O(1) \cdot n \log n = O(n \log n) \end{matrix}$$

For second to last :

while $\rightarrow O(n)$

for $\rightarrow \sum_{k=1}^{\log(n)} O(k) = O(2^{\log_2(n)}) = O(n)$