The Examination Timetabling Problem

Project work

Ainur Almukambetova

Vendel Péter Kupás

University of Verona

2023.07.25.

Content

Problem

Data

- exams: $E = \{e_1, e_2, \ldots, e_n\}$
- students: $S = \{s_1, s_2, \ldots, s_m\}$. Each student is enrolled in a non-empty subset of exams.
- time slots: $Ts = \{t_1 \leq t_2 \leq \ldots \leq t_T\}$
- n_{e_1,e_2} = number of students enrolled both e_1 and e_2
  if $n_{e_1,e_2} > 0$ then e_1 and e_2 are conflicting.

Constraints

1. each exam is scheduled exactly once during the examination period
2. two conflicting exams are not scheduled in the same time-slot
3. given two exams $e_1, e_2 \in E$ scheduled at distance i of time-slots, with $1 \leq i \leq 5$, the relative penalty is

$$rp_{e_1,e_2} = 2^{(5-i)} \frac{n_{e_1,e_2}}{|S|}$$

4. we would like to minimize the sum of the penalties:

$$min \sum_{(e_1,e_2):\ e_1,e_2 \in E,\ e_1 \neq e_2} rp_{e_1,e_2}$$

Basic Model

Data initialization

- **exams**: List for the exams
- **time_slots**: List for the time slots from 1 to T.
- **enrollment_matrix**: a dictionary for the enrollments. The students are the keys and a list of exams belongs to a key.
- **students**: keys of the enrollment_matrix
- **conflicting exams**: A dictionary for the conflicting exams. For every pair of conflicting exams we indicated n_{e_1,e_2}, the number of conflicts.

Modelling Constraints

- We modeled the exercise as an integer (binary) programming problem.
- $$x_{t,e} = \begin{cases} 1, & \text{if the } e \in E \text{ exam is held in the } t \in Ts \text{ time-slot} \\ 0, & \text{otherwise} \end{cases}$$

Modelling Constraints

1. Each exam is scheduled exactly once during the examination period.

$$\sum_{k=1}^{T} x_{t_k,e} = 1, \quad \forall e \in E$$

2. Two conflicting exams are not scheduled in the same time-slot.

$$x_{t,e_1} + x_{t,e_2} \leq 1, \quad \forall t \in Ts, \ \forall (e_1, e_2) \in \text{conflicting\_exams}$$

Modelling Constraints

3

- For every conflicting pair of exams let u_{e_1,e_2} be the penalty if the distance between them is not more then 5 time slots.
- Then the objective function is

$$\min \sum_{(e_1,e_2) \text{ conflicting exams}} u_{e_1,e_2}$$

- For every conflicting pair $(e1, e2)$, for every $i \in \{1, 2, 3, 4, 5\}$ and for every $t \in \{t_1, \ldots, t_{T-i}\}$

$$u_{e_1,e_2} \geq 2^{5-i}(x_{t,e_1} + x_{t+i,e_2} - 1)\frac{n_{e_1,e_2}}{|S|}$$

$$u_{e_1,e_2} \geq 2^{5-i}(x_{t+i,e_1} + x_{t,e_2} - 1)\frac{n_{e_1,e_2}}{|S|}.$$

Advanced Model

Equity measures

Maximum distance

- Let u be smaller or equal with the number of time-slots between any two conflicting exams.

$$\max u$$

- For every (e_1, e_2) conflicting pair and for every $t_1 \leq t_2$ time slots:

$$u \leq (x_{t_1,e_1} + x_{t_2,e_2} - 1)(t_2 - t_1 - T) + T$$

$$u \leq (x_{t_2,e_1} + x_{t_1,e_2} - 1)(t_2 - t_1 - T) + T$$

Number of students with back-to-back exams

- Let u_s be the next binary variable for every student $s \in S$:

$$u_s = \begin{cases} 1, & \text{if student } s \text{ have two conflicting exams scheduled} \\ & \text{in consecutive time-slots} \\ 0, & \text{otherwise} \end{cases}$$

- Then we would like to minimize the number of students, where $u_s = 1$

$$\min \sum_{s \in S} u_s$$

- For every $t \in \{t_1, \ldots, t_{T-1}\}$, for every (e_1, e_2) conflicting pair, if $s \in e_1$ and $s \in e_2$ then

$$u_s \geq x_{t,e_1} + x_{t+1,e_2} - 1$$
$$u_s \geq x_{t+1,e_1} + x_{t,e_2} - 1$$

Additional restrictions

At most 3 consecutive time slots can have conflicting exams

- This means that two consecutive time slots can not have conflicting exams.
- For every $t \in Ts,\ (e_1, e_2)$ conflicting pair set the next constraint:

$$x_{t,e_1} + x_{t+1,e_2} \leq 1$$
$$x_{t+1,e_1} + x_{t,e_2} \leq 1$$

If two consecutive time slots contain conflicting exams, then no conflicting exam can be scheduled in the next 3 time slots

- Let $u2_t$ and $u3_t$ be the next binary variables:

$$u2_t = \begin{cases} 1, & \text{if there is a conflicting pair in time periods } \{t, t+1\} \\ 0, & \text{otherwise} \end{cases}$$

$$u3_t = \begin{cases} 1, & \text{if there is a conflicting pair in time periods } \{t, t+1, t+2\} \\ 0, & \text{otherwise} \end{cases}$$

- Then the additional constraint is equivalent with this expression:

$$u2_t + u3_{t+2} \leq 1 \quad \forall t \in \{t_1, \dots, t_{T-4}\}$$

If two consecutive time slots contain conflicting exams, then no conflicting exam can be scheduled in the next 3 time slots

- For every $t_1, t_2 \in \{t, t+1\}$ and (e_1, e_2) conflicting pair

$$u2_t \geq x_{t_1,e_1} + x_{t_2,e_2} - 1$$

and for every $t_1, t_2 \in \{t, t+1, t+2\}$ and (e_1, e_2) conflicting pair

$$u3_t \geq x_{t_1,e_1} + x_{t_2,e_2} - 1.$$

Bonus Profit

- For every $t_1, t_2 \in \{t, t+1, \ldots, t+5\}$ and (e_1, e_2) conflicting pair let $u6_t$ be a binary variable with the next constraints:

$$u6_t \geq x_{t_1,e_1} + x_{t_2,e_2} - 1,$$

- Then would like to minimize the next expression

$$\min \sum_{t:\ t \leq T-6} b \cdot u6_t$$

Changing the constraints: at most 3 conflicting pairs can be scheduled in the same time slot

- Let u_{t,e_1,e_2} be the next binary variable for every $t \in Ts$ and for every (e_1, e_2) conflicting pair:

$$u_{t,e_1,e_2} = \begin{cases} 1, & \text{if } e_1 \text{ and } e_2 \text{ is held in time slot } t \\ 0, & \text{otherwise} \end{cases}$$

- We can express u_{t,e_1,e_2} with the variables $x_{t,e}$ easily again:

$$u_{t,e_1,e_2} \geq x_{t,e_1} + x_{t,e_2} - 1$$

- Then the constraint for every $t \in Ts$:

$$\sum_{(e_1,e_2) \text{ conflicting pair}} u_{t,e_1,e_2} \leq 3.$$

Experiments

Experiments

- We implemented all of the variants of the model in python
- We couldn't try out the implementation on the instances
- We got proper results in small instances