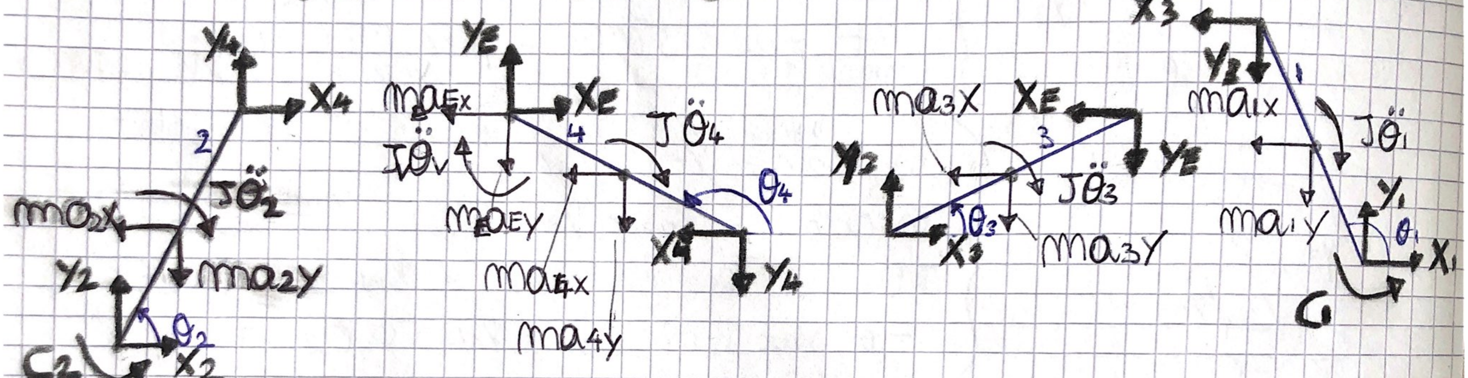
Metodo azioni interne



$\sum F_x^2 = 0 \rightarrow$ 1) $X_2 + X_4 - m\,a_{2x} = 0$

$\sum F_y^2 = 0 \rightarrow$ 2) $Y_2 + Y_4 - m a_{2y} = 0$

$\sum M_{G2}^2 = 0 \rightarrow$ 3) $C_2 - J\ddot{\theta}_2 + X_2 \frac{l}{2} \sin\theta_2 - Y_2 \frac{l}{2} \cos\theta_2 +$
$- X_4 \frac{l}{2} \sin\theta_2 + Y_4 \frac{l}{2} \cos\theta_2 = 0$

$\sum F_x^1 = 0 \rightarrow$ 4) $X_1 - X_3 - m a_{1x} = 0$

$\sum F_y^1 = 0 \rightarrow$ 5) $Y_1 - Y_3 - m a_{1y} = 0$

$\Sigma M_{G1} = 0 \rightarrow 6)\ C_1 - J\ddot{\theta}_1 + X_1 \frac{l}{2}\sin\theta_1 + X_3 \frac{l}{2}\sin\theta_1 +$
$\quad - Y_1 \frac{l}{2}\cos\theta_1 - Y_3 \frac{l}{2}\cos\theta_1 = 0$

$\Sigma F_x^3 = 0 \rightarrow 7)\ X_3 - X_E - m a_{3x} = 0$

$\Sigma F_y^3 = 0 \rightarrow 8)\ Y_3 - Y_E - m a_{3y} = 0$

$\Sigma M_{G3}^3 = 0 \rightarrow 9)\ - J\ddot{\theta}_3 + X_3 \frac{l}{2}\sin\theta_3 - Y_3 \frac{l}{2}\cos\theta_3 +$
$\quad + X_E \frac{l}{2}\sin\theta_3 - Y_E \frac{l}{2}\cos\theta_3 = 0$

$\Sigma F_x^4 = 0 \rightarrow 10)\ X_E - X_4 - m a_{4x} - m_E a_{Ex} = 0$

$\Sigma F_y^4 = 0 \rightarrow 11)\ Y_E - Y_4 - m a_{4y} - m_E a_{Ey} = 0$

$\Sigma M_{G4}^4 = 0 \rightarrow 12)\ - J\ddot{\theta}_4 - X_4 \frac{l}{2}\sin\theta_4 - X_E \frac{l}{2}\sin\theta_4 +$
$\quad + m_E a_{Ex} \frac{l}{2}\sin\theta_4 + Y_4 \frac{l}{2}\cos\theta_4 + Y_E \frac{l}{2}\cos\theta_4 +$
$\quad - m_E a_{Ey} \frac{l}{2}\cos\theta_4 - J_v \ddot{\theta}_v = 0$

12 equazioni $\leftrightarrow$ 12 incognite

sostituisco nella 9) e nella 12) rispettivamente X_3, Y_3 e X_4, Y_4, poi ricavo X_E, Y_E

$9)\ - J\ddot{\theta}_3 + X_E \frac{l}{2}\sin\theta_3 - Y_E \frac{l}{2}\cos\theta_3 + (X_E + m a_{3x}) \frac{l}{2}\sin\theta_3 +$
$\quad - (Y_E + m a_{3y}) \frac{l}{2}\cos\theta_3 = 0$

$\rightarrow - J\ddot{\theta}_3 + (2X_E + m a_{3x}) \frac{l}{2}\sin\theta_3 - (2Y_E + m a_{3y}) \frac{l}{2}\cos\theta_3 = 0$

$$\rightarrow Y_E = \frac{- J\ddot{\theta}_3 + (2X_E + m a_{3x}) \frac{l}{2}\sin\theta_3 - m a_{3y} \frac{l}{2}\cos\theta_3}{l\cos\theta_3}$$

$$= - \frac{J\ddot{\theta}_3}{l\cos\theta_3} + (2X_E + m a_{3x}) \frac{1}{2}\tan\theta_3 - \frac{1}{2} m a_{3y}$$

$12)\ - J\ddot{\theta}_4 - (X_E - m a_{4x} - m_E a_{Ex}) \frac{l}{2}\sin\theta_4 - X_E \frac{l}{2}\sin\theta_4 +$
$\quad + m_E a_{Ex} \frac{l}{2}\sin\theta_4 + (Y_E - m a_{4y} - m_E a_{Ey}) \frac{l}{2}\cos\theta_4 +$
$\quad + \left(- \frac{J\ddot{\theta}_3}{l\cos\theta_3} + (2X_E + m a_{3x}) \frac{1}{2}\tan\theta_3 - \frac{1}{2} m a_{3y} \right) \frac{l}{2}\cos\theta_4 +$
$\quad - m_E a_{Ey} \frac{l}{2}\cos\theta_4 - J_v \ddot{\theta}_v = 0$

$\rightarrow X_E \left(- l\sin\theta_4 + \tan\theta_3\, l\cos\theta_4 \right) = J\ddot{\theta}_4 - (m a_{4x} + m_E a_{Ex})$
$\quad \cdot \frac{l}{2}\sin\theta_4 - m_E a_{Ex} \frac{l}{2}\sin\theta_4 + (m a_{4y} + m_E a_{Ey}) \frac{l}{2}\cos\theta_4 +$
$\quad - \left(- \frac{J\ddot{\theta}_3}{l\cos\theta_3} + m a_{3x} \frac{1}{2}\tan\theta_3 - \frac{1}{2} m a_{3y} \right) l\cos\theta_4 +$
$\quad + m_E a_{Ey} \frac{l}{2}\cos\theta_4 + J_v \ddot{\theta}_v$

$\to X_E \left(-l\sin\theta_4 + l\tan\theta_3\cos\theta_4\right) = J\ddot{\theta}_4 + J_V\ddot{\theta}_V +$
$-\left(\frac{1}{2}ma_{4x} + m_E a_{EX}\right) l\sin\theta_4 + \left(\frac{1}{2}ma_{4y} + m_E a_{EY}\right.$
$\left. + \frac{J\ddot{\theta}_3}{l\cos\theta_3} - ma_{3x}\frac{1}{2}\tan\theta_3 + \frac{1}{2}ma_{3y}\right) l\cos\theta_4$

$\to X_E \to Y_E$

Noti X_E e Y_E posso ricavare X_1, X_2, X_3, X_4, Y_1, Y_2, Y_3, Y_4 da cui posso ricavare C_1 e C_2 da 3) e 6)

11) $Y_4 = Y_E - ma_{4y} - m_E a_{EY}$
10) $X_4 = X_E - ma_{4x} - m_E a_{EX}$
8) $Y_3 = Y_E + ma_{3y}$
7) $X_3 = X_E + ma_{3x}$
5) $Y_1 = Y_3 + ma_{1y}$
4) $X_1 = X_3 + ma_{1x}$
2) $Y_2 = -Y_4 + ma_{2y}$
1) $X_2 = -X_4 + ma_{2x}$

6) $C_1 = J\ddot{\theta}_1 - (X_1 + X_3)\frac{l}{2}\sin\theta_1 + (Y_1 + Y_3)\frac{l}{2}\cos\theta_1$

3) $C_2 = J\ddot{\theta}_2 + (X_4 - X_2)\frac{l}{2}\sin\theta_2 + (Y_2 - Y_4)\frac{l}{2}\cos\theta_2$

Per ottenere la coppia devo definire le accelerazioni baricentriche 1, 2, 3, 4, E

$p_{1x} = \frac{l}{2}\cos\theta_1 \to v_{1x} = -\frac{l}{2}\sin\theta_1\dot{\theta}_1$
$\to a_{1x} = -\frac{l}{2}\cos\theta_1\dot{\theta}_1^2 - \frac{l}{2}\sin\theta_1\ddot{\theta}_1$

$p_{1y} = \frac{l}{2}\sin\theta_1 \to v_{1y} = \frac{l}{2}\cos\theta_1\dot{\theta}_1$
$\to a_{1y} = -\frac{l}{2}\sin\theta_1\dot{\theta}_1^2 + \frac{l}{2}\cos\theta_1\ddot{\theta}_1$

$\to a_{2x} = -\frac{l}{2}\cos\theta_2\dot{\theta}_2^2 - \frac{l}{2}\sin\theta_2\ddot{\theta}_2$

$\to a_{2y} = -\frac{l}{2}\sin\theta_2\dot{\theta}_2^2 + \frac{l}{2}\cos\theta_2\ddot{\theta}_2$

$p_{3x} = l\cos\theta_1 + \frac{l}{2}\cos\theta_3 \to v_{3x} = -l\sin\theta_1\dot{\theta}_1 - \frac{l}{2}\sin\theta_3\dot{\theta}_3$
$\to a_{3x} = -l\cos\theta_1\dot{\theta}_1^2 - l\sin\theta_1\ddot{\theta}_1 - \frac{l}{2}\cos\theta_3\dot{\theta}_3^2 - \frac{l}{2}\sin\theta_3\ddot{\theta}_3$

$p_{3y} = l\sin\theta_1 + \frac{l}{2}\sin\theta_3 \to v_{3y} = l\cos\theta_1\dot{\theta}_1 + \frac{l}{2}\cos\theta_3\dot{\theta}_3$
$\to a_{3y} = -l\sin\theta_1\dot{\theta}_1^2 + l\cos\theta_1\ddot{\theta}_1 - \frac{l}{2}\sin\theta_3\dot{\theta}_3^2 + \frac{l}{2}\cos\theta_3\ddot{\theta}_3$
$= 2a_{1y} - \frac{l}{2}\sin\theta_3\dot{\theta}_3^2 + \frac{l}{2}\cos\theta_3\ddot{\theta}_3$

$$\rightarrow a_{4x} = 2a_{2x} - \frac{l}{2}\cos\theta_4\,\dot{\theta}_4^2 - \frac{l}{2}\sin\theta_4\,\ddot{\theta}_4$$

$$\rightarrow a_{4y} = 2a_{2y} - \frac{l}{2}\sin\theta_4\,\dot{\theta}_4^2 + \frac{l}{2}\cos\theta_4\,\ddot{\theta}_4$$

$$p_{Ex} = l\cos\theta_1 + l\cos\theta_3 \rightarrow v_{Ex} = -l\sin\theta_1\,\dot{\theta}_1 - l\sin\theta_3\,\dot{\theta}_3$$

$$\rightarrow a_{Ex} = 2a_{1x} - l\cos\theta_3\,\dot{\theta}_3^2 - l\sin\theta_3\,\ddot{\theta}_3$$

$$\rightarrow a_{Ey} = 2a_{1y} - l\sin\theta_3\,\dot{\theta}_3^2 + l\cos\theta_3\,\ddot{\theta}_3$$