$$T = \frac{1}{2} m \left(\dot{\theta}_1 \frac{l}{2}\right)^2 + \frac{1}{2} J \dot{\theta}_1^2 + \frac{1}{2} m \left(\dot{\theta}_2 \frac{l}{2}\right)^2 + \frac{1}{2} J \dot{\theta}_2^2 +$$
$$+ \frac{1}{2} m V_{CM3}^2 + \frac{1}{2} J \dot{\theta}_3^2 + \frac{1}{2} m V_{CM4}^2 + \frac{1}{2} J \dot{\theta}_4^2 +$$
$$+ \frac{1}{2} m_E V_{CE}^2 + \frac{1}{2} J_V \dot{\theta}_V^2$$

$$V_{CM3}^2 = V_{CM3X}^2 + V_{CM3Y}^2$$

$$P_{CM3X} = -d + l\cos\theta_1 + \frac{l}{2}\cos\theta_3$$

$$V_{CM3X} = -l\sin\theta_1 \dot{\theta}_1 - \frac{l}{2}\sin\theta_3 \dot{\theta}_3 = \dot{\theta}_1\left(-l\sin\theta_1 - \frac{l}{2}\sin\theta_3 \theta_3'\right)$$

$$P_{CM3Y} = l\sin\theta_1 + \frac{l}{2}\sin\theta_3$$

$$V_{CM3Y} = l\cos\theta_1 \dot{\theta}_1 + \frac{l}{2}\cos\theta_3 \dot{\theta}_3 = \dot{\theta}_1\left(l\cos\theta_1 + \frac{l}{2}\cos\theta_3 \theta_3'\right)$$

$$\dot{\theta}_3 = \frac{\partial\theta_3}{\partial\theta_1}\dot{\theta}_1 = \theta_3'\dot{\theta}_1 \qquad \dot{\theta}_4 = \frac{\partial\theta_4}{\partial\theta_2}\dot{\theta}_2 = \theta_4'\dot{\theta}_2$$

$$V_{CM3}^2 = \dot{\theta}_1^2\left(l^2\sin^2\theta_1 + l^2\sin\theta_1\sin\theta_3\theta_3' + \frac{l^2}{4}\sin^2\theta_3\theta_3'^2 + \right.$$
$$\left. + l^2\cos^2\theta_1 + l^2\cos\theta_1\cos\theta_3\theta_3' + \frac{l^2}{4}\cos^2\theta_3\theta_3'^2\right)$$
$$= \dot{\theta}_1^2\left(l^2 + \frac{l^2}{4}\theta_3'^2 + l^2\theta_3'\cos(\theta_1 - \theta_3)\right)$$

$$V_{CM4}^2 = \dot{\theta}_2^2\left(l^2 + \frac{l^2}{4}\theta_4'^2 + l^2\theta_4'\cos(\theta_2 - \theta_4)\right)$$

$$V_{CE}^2 = V_{CEX}^2 + V_{CEY}^2 + V_{CEZ}^2$$

$$V_{CEX}^2 = \dot{\theta}_1^2\left(l^2\sin^2\theta_1 + 2l^2\sin\theta_1\sin\theta_3\theta_3' + l^2\sin^2\theta_3\theta_3'^2\right)$$
$$V_{CEY}^2 = \dot{\theta}_1^2\left(l^2\cos^2\theta_1 + 2l^2\cos\theta_1\cos\theta_3\theta_3' + l^2\cos^2\theta_3\theta_3'^2\right)$$
$$V_{CEZ}^2 = \frac{p_V^2}{4\pi^2}\dot{\theta}_V^2$$

$$V_{CE}^2 = \dot{\theta}_1^2 l^2\left(1 + 2\cos(\theta_1 - \theta_3)\theta_3' + \theta_3'^2\right) + \dot{\theta}_V^2\frac{p_V^2}{4\pi^2}$$

$$T = \frac{1}{8} m l^2 \dot{\theta}_1^2 + \frac{1}{2} J \dot{\theta}_1^2 + \frac{1}{8} m l^2 \dot{\theta}_2^2 + \frac{1}{2} J \dot{\theta}_2^2 +$$
$$+ \frac{1}{2} m l^2 \dot{\theta}_1^2\left(1 + \theta_3'\cos(\theta_1 - \theta_3) + \frac{1}{4}\theta_3'^2\right) + \frac{1}{2} J \dot{\theta}_1^2 \theta_3'^2 +$$
$$+ \frac{1}{2} m l^2 \dot{\theta}_2^2\left(1 + \theta_4'\cos(\theta_2 - \theta_4) + \frac{1}{4}\theta_3'^2\right) + \frac{1}{2} J \dot{\theta}_2^2 \theta_4'^2 +$$
$$+ \frac{1}{2} m_E\left(\dot{\theta}_1^2 l^2\left(1 + 2\cos(\theta_1 - \theta_3)\theta_3' + \theta_3'^2\right) + \dot{\theta}_V^2\frac{p_V^2}{4\pi^2}\right) +$$
$$+ \frac{1}{2} J_V \dot{\theta}_V^2$$

$$T = \dot{\theta}_1^2\Big(\frac{1}{8}ml^2 + \frac{1}{2}J + \frac{1}{2}ml^2\big(1 + \theta_3'\cos(\theta_1-\theta_3) + \frac{1}{4}\theta_3'^2\big) +$$
$$+ \frac{1}{2}m_E l^2\big(1 + 2\cos(\theta_1-\theta_3)\theta_3' + \theta_3'^2\big) + \frac{1}{2}J\theta_3'^2\Big) +$$
$$+ \dot{\theta}_2^2\Big(\frac{1}{8}ml^2 + \frac{1}{2}J + \frac{1}{2}ml^2\big(1 + \theta_4'\cos(\theta_2-\theta_4) + \frac{1}{4}\theta_4'^2\big)$$
$$+ \frac{1}{2}J\theta_4'^2 + \dot{\theta}_V^2\Big(\frac{\rho_V^2}{4\pi^2}\frac{1}{2}m_E + \frac{1}{2}J_V\Big)$$

$$\frac{\partial T}{\partial \dot{\theta}_1} = 2\dot{\theta}_1\Big(\frac{1}{8}ml^2 + \frac{1}{2}J + \frac{1}{2}ml^2\big(1 + \theta_3'\cos(\theta_1-\theta_3) + \frac{1}{4}\theta_3'^2\big) +$$
$$+ \frac{1}{2}m_E l^2\big(1 + 2\cos(\theta_1-\theta_3)\theta_3' + \theta_3'^2\big) + \frac{1}{2}J\theta_3'^2\Big)$$

M1

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = 2\ddot{\theta}_1\Big[\frac{1}{8}ml^2 + \frac{1}{2}J + \frac{1}{2}ml^2\big(1 + \theta_3'\cos(\theta_1-\theta_3) + \frac{1}{4}\theta_3'^2\big)$$
$$+ \frac{1}{2}m_E l^2\big(1 + 2\cos(\theta_1-\theta_3)\theta_3' + \theta_3'^2\big) + \frac{1}{2}J\theta_3'^2\Big] +$$
$$+ 2\dot{\theta}_1\Big[\frac{1}{2}ml^2\big(\theta_3''\dot{\theta}_1\cos(\theta_1-\theta_3) + \theta_3'(-\sin(\theta_1-\theta_3)(\dot{\theta}_1-\dot{\theta}_3\dot{\theta}_1)$$

K1

$$+ \frac{1}{4}2\theta_3'\theta_3''\dot{\theta}_1\big) + \frac{1}{2}m_E l^2\big(2\theta_3''\dot{\theta}_1\cos(\theta_1-\theta_3) +$$
$$+ 2\theta_3'(-\sin(\theta_1-\theta_3)\dot{\theta}_1(1-\theta_3') + 2\theta_3'\theta_3''\dot{\theta}_1\big) + \frac{1}{2}J\,2\theta_3'\theta_3''\dot{\theta}_1\Big]$$

K2

$$\frac{\partial T}{\partial \theta_1} = \dot{\theta}_1^2\Big[\frac{1}{2}ml^2\big(\theta_3''\cos(\theta_1-\theta_3) + \theta_3'(-\sin(\theta_1-\theta_3)(1-\theta_3')) +$$
$$+ \frac{1}{4}2\theta_3'\theta_3''\big) + \frac{1}{2}m_E l^2\big(2\theta_3''\cos(\theta_1-\theta_3) + 2\theta_3'(-\sin(\theta_1-\theta_3)\cdot$$
$$\cdot(1-\theta_3')) + 2\theta_3'\theta_3''\big) + \frac{1}{2}J\,2\theta_3'\theta_3''\Big]$$
$$+ \dot{\theta}_2^2\Big[\frac{1}{2}ml^2\Big(\frac{d\theta_4'}{d\theta_1}\cos(\theta_2-\theta_4) + \theta_4'(-\sin(\theta_2-\theta_4)\Big(-\frac{d\theta_4}{d\theta_1}\Big)$$
$$+ \frac{1}{4}2\theta_4'\frac{d\theta_4'}{d\theta_1}\Big) + \frac{1}{2}J\,2\theta_4'\frac{d\theta_4'}{d\theta_1}\Big]$$ K3

M2

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = 2\ddot{\theta}_2\Big[\frac{1}{8}ml^2 + \frac{1}{2}J + \frac{1}{2}ml^2\big(1 + \theta_4'\cos(\theta_2-\theta_4) +$$
$$+ \frac{1}{4}\theta_4'^2\big) + \frac{1}{2}J\theta_4'^2\Big] +$$
$$+ 2\dot{\theta}_2\Big[\frac{1}{2}ml^2\big(\theta_4''\dot{\theta}_2\cos(\theta_2-\theta_4) + \theta_4'(-\sin(\theta_2-\theta_4)\cdot$$
$$\cdot\dot{\theta}_2(1-\theta_4') + \frac{1}{4}2\theta_4'\theta_4''\dot{\theta}_2\big) + \frac{1}{2}J\,2\theta_4'\theta_4''\dot{\theta}_2\Big]$$

h1

$$
\begin{aligned}
\frac{\partial T}{\partial \theta_2} = \dot{\theta}_1^2 \underbrace{\Bigg[\frac{1}{2} m e^2 \Big(\frac{d\theta_3'}{d\theta_2} \cos(\theta_1-\theta_3) + \theta_3' (-\sin(\theta_1-\theta_3)) \Big(-\frac{d\theta_3}{d\theta_2}\Big) + \\
+ \frac{1}{4} 2 \theta_3' \frac{d\theta_3'}{d\theta_2} \Big) + \frac{1}{2} m_E e^2 \Big(2 \frac{d\theta_3'}{d\theta_2} \cos(\theta_1-\theta_3) + \\
+ 2\theta_3' (-\sin(\theta_1-\theta_3)) \Big(- \frac{d\theta_3}{d\theta_2} \Big) + 2 \theta_3' \frac{d\theta_3'}{d\theta_2} \Big) + \\
+ \frac{1}{2} J \, 2 \theta_3' \frac{d\theta_3'}{d\theta_2} \Bigg]}_{h_2} + \\
+ \dot{\theta}_2^2 \underbrace{\Bigg[\frac{1}{2} m e^2 \Big(\theta_4'' \cos(\theta_2-\theta_4) + \theta_4' (-\sin(\theta_2-\theta_4))(1-\theta_4') \\
+ \frac{1}{4} 2 \theta_4' \theta_4'' \Big) + \frac{1}{2} J \, 2 \theta_4' \theta_4'' \Bigg]}_{h_3}
\end{aligned}
$$

$$
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_v} \right) = 2 \ddot{\theta}_v \overbrace{\left(\frac{p_v^2}{4\pi^2} \frac{1}{2} m_E + \frac{1}{2} J_v \right)}^{M_3}
$$

$$
\frac{\partial T}{\partial \theta_v} = 0
$$

$$
U = m_E g (h + z) = m_E g \overbrace{\left(h + \frac{p_v}{2\pi} \theta_v \right)}^{U_3}
$$

$$
\frac{\partial U}{\partial \theta_v} = m_E g \frac{p_v}{2\pi}
$$

$$
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1} \right) - \frac{\partial T}{\partial \theta_1} + \frac{\partial U}{\partial \theta_1} = Q_{\theta_1}
$$

$$
\begin{aligned}
2 M_{11} \ddot{\theta}_1 + 2 K_1 \dot{\theta}_1^2 - K_2 \dot{\theta}_1^2 - K_3 \dot{\theta}_2^2 &= Q_{\theta_1} = \text{Coppia link 1} \\
2 M_2 \ddot{\theta}_2 + 2 h_1 \dot{\theta}_2^2 - h_2 \dot{\theta}_1^2 - h_3 \dot{\theta}_2^2 &= Q_{\theta_2} = \text{Coppia link 2} \\
2 M_3 \ddot{\theta}_v + U_3 &= Q_{\theta v} = \text{coppia vite}
\end{aligned}
$$