

Declarative Programming

Workshop exercises set 11.

QUESTION 1

Write a function `fibs :: Int -> [Integer]` which returns a list containing the first `n` numbers in the Fibonacci sequence: `[0,1,1,2,3,5,8,...]`, where the third and subsequent numbers are the sum of the two preceding numbers ($0+1=1$, $1+1=2$, $1+2=3$, $2+3=5$, etc). We use `Integer` rather than `Int` because the numbers grow exponentially and therefore overflow native `Int`s quite quickly. Is the algorithmic complexity of your solution acceptable?

QUESTION 2

If we do pairwise addition of the elements of the Fibonacci sequence and its tail, we get the tail of the tail of the sequence:

0	1	1	2	3	5	8	...	<code>fibs</code>
+	1	1	2	3	5	8	...	<code>tail fibs</code>
=	1	2	3	5	8	...		<code>tail (tail fibs)</code>

Use this property to write a definition of `allfibs :: [Integer]` which is the (infinite) Fibonacci sequence (Hint: the `zipWith` Prelude function is useful). Define `fibs` in terms of `allfibs`. How efficient is this definition of `fibs` compared to your previous one?

QUESTION 3

Consider the bottom-up merge sort implementation from workshop 2.

```
>mergesort xs = repeat_merge_all (merge_consec (to_single_els xs))
>
>to_single_els [] = []
>to_single_els (x:xs) = [x] : to_single_els xs
>
>merge [] ys = ys
>merge (x:xs) [] = x:xs
>merge (x:xs) (y:ys)
>    | x <= y = x : merge xs (y:ys)
>    | x > y = y : merge (x:xs) ys
>
>merge_consec [] = []
>merge_consec [xs] = [xs]
>merge_consec (xs1:xs2:xss) = (merge xs1 xs2) : merge_consec xss
>
>repeat_merge_all [] = []
>repeat_merge_all [xs] = xs
>repeat_merge_all xss@(_:_:_) = repeat_merge_all (merge_consec xss)
```

With list `xs` of length `n`, what is the maximum additional space that is needed at any one time, assuming strict evaluation, for evaluating `merge_consec (to_single_els xs)`? What if lazy evaluation is used instead?

What is the maximum additional space is needed at any one time, assuming strict evaluation, for evaluating `mergesort xs`? Can we do significantly better than this?

QUESTION 4

Consider an interpreter for a language which produces a pair

containing the result of the computation plus some debugging information, which is a string containing information about all assignment statements and function calls. Compare the efficiency of the following:

- a) Execution of the interpreter using strict evaluation and printing the debugging string.
- b) Execution of the interpreter using lazy evaluation and printing the debugging string.
- c) Execution of the interpreter using strict evaluation but not printing the debugging string.
- d) Execution of the interpreter using lazy evaluation but not printing the debugging string.
- e) Execution of a similar interpreter which doesn't produce the debugging string at all.

QUESTION 5

Here are some students' answers to one of the questions on a sample mid-semester test, which were posted on the LMS (thanks to the authors). The question asked for a Haskell function to print out Mtrees with indentation showing the structure. Compare and contrast these solutions. Can you come up with something better than all three?

```
>data Mtree a = Mnode a [Mtree a]

>print_mtree :: Show a => Mtree a -> IO()
>print_mtree tree = indent_mtree 0 tree
>  where
>      indent_mtree :: Show a => Int -> Mtree a -> IO()
>      indent_mtree i (Mnode val children) = do
>          putStrLn $ (replicate i ' ') ++ (show val)
>          foldl (>>) (return ()) (map (indent_mtree (i+1)) children)

>type Line = String
>
>print_mtree' :: Show a => Mtree a -> IO ()
>print_mtree' t =
>  let
>      toLines :: Show a => Mtree a -> [Line]
>      toLines (Mnode val cs) = show val : map (' ':) (concatMap (toLines) cs)
>  in foldl (\acc str -> acc >> (putStrLn str)) (return ()) (toLines t)
>
>-- A clearer version
>print_mtree2 :: Show a => Mtree a -> IO ()
>print_mtree2 t =
>  let
>      toLines :: Show a => Mtree a -> [IO ()]
>      toLines (Mnode val cs) =
>          print val : map (putChar ' ' >>) (concatMap (toLines) cs)
>  in foldl1 (>>) (toLines t)
```