# 1 Introduction

Intuitively, we like to think that catastrophic events have equally catastrophic causes. In physics terms, we might postulate some unusual circumstances that perturb the energy of the system proportionally to the resulting catastrophe. An easy example to think about is the extinction of the dinosaurs - it was a very large and high-energy meteorite that impacted the Earth and set off a catastrophic chain of events. Or in more recent times, we might look to the Great Depression, traced back to the Wall Street crash in October 1929. There are many examples of big events with big causes, but we should not be led to believe that this is the only way dramatic change can occur. Often we will be interested in large dissipative systems, usually with many interconnected components. In systems like this, frequently we will find that system-level change hinges on the drop of pin or, as we will see in this prac, on a single grain of sand.

The laws of physics are well defined and absolute. A naive physicist might end the story there. Just write down the Hamiltonian or Lagrangian and you're set! Unfortunately, for systems with huge numbers of degrees of freedom and many interacting components, the equations of motion become computationally intractable. Solving for the exact dynamics might take longer than the human race has left on earth. As a result, we need heuristic theories that allow us to study the dynamics of complex systems. One such theory is that of self-organizing criticality.

Self-organizing criticality was first proposed in 1987 by Bak, Tang and Wiesenfeld. The theory states that a large and complex system with short range interactions naturally evolves into a critical state. What does this mean? Well there are different definitions, but to put it simply, a small perturbation to an element of a system in a critical state will result in a response that can affect any number of elements in the system. To put it simply-er, the same mechanism that leads to a minor response can also lead to a catastrophic response. For a system to "naturally evolve" into this state, it must have some slow driving mechanism, and a means of dissipation. For example, the earth's crust consists of tectonic plates, slowly moving against each other. The microscopic components can be thought of as rigid particles, each subject to straining force. In this case the shearing movement of tectonic plates drives the system. When the strain on one particle exceeds a critical value, the particle 'slips' and transfers the strain to its neighbouring particles. Depending on the state of the surrounding particles, this dissipation of energy could result in further particles slipping. This process, as one might imagine, can become extremely large (as in an earthquake) or may just involve a single event that slightly lowers the energy of the system.

Since we cannot study exact system dynamics, we must rely on statistical approaches to get an insight into what is happening. In driven, critical systems like the ones we will be studying, catastrophic events are often not distributed, in a probabilistic sense, as we might expect. To really get a handle on what is happening we will need to enter the world of non-equilibrium statistical mechanics - goodbye nice, normal distributions, and hello power law behaviour and scale-free networks. Non-equilibrium statistics underlie how we think about systems that display self-organizing criticality. In terms of the theory itself, one important thing to understand is the sense in which it is holistic. When a system is critical, microscopic mechanisms (local increases in stress, for example) are not responsible for the macroscopic observables of the system (think earthquake size, duration, area etc). In particular the proportions of large and small events do not depend on the exact microscopic details of what is happening (where exactly local stress increases occur, for example). Consequentially, one cannot analyze the "inner workings" of a system at the smallest scale, and from there try to explain the effects occurring on a larger scale. In plain language, it is vain to hope that detailed understanding of a part of the system will translate into system-level understanding.

Self-organizing criticality as a framework has been used to study an extraordinarily diverse range of systems. It has been used to characterise, analyze and predict earthquakes, financial markets, evolution and extinction events, pulsar glitches, neuronal behaviour and much more.