Standard & Canonical Forms

In this lesson, we learn

* Canonical form
  - minterms
  - maxterms
  - Sum of Minterms (SOM)
  - Product of Maxterms (POM)
    - Canonical form
  - Conversion SOM ⟷ POM
  - Operations

* Standard form
  - Sum of Products (SOP)
  - Product of Sums (POS)

* Two-Level Implementation

* Propagation Delay & Critical path

Minterms

* Consider: set of functions of $\underline{\underline{3}}$ vars. (A, B, C)

* <u>Minterm:</u> Product term s.t. all literals show up the product in true or complement form.

Ex1

ABC is a minterm

$A\bar{B}\bar{C}$ is a minterm

AB is NOT a minterm
{Missing C}

* Minterm Numbering:

Question: How many minterms can we have in $\underline{\underline{3}}$ variable system?

m_i is minterm # i

$\Rightarrow$ Binary representation of $\underline{\underline{i}}$ tells us the algebraic form of m_i

Ex 2

$$m_{2} = \bar{A}\,B\,\bar{C}$$

0 1 0

$\bar{A}$ B $\bar{C}$

$$m_{6} = A\,B\,\bar{C}$$

1 1 0

A B $\bar{C}$

MSB of index LSB of index

Assuming functions of (A, B, C)

Note: minterms are also Boolean functions.

Ex 3 Derive the truth table of m_3, m_5, and m_7, assuming functions of (A, B, C).

011 $\bar{A}BC$

101 $A\bar{B}C$

110 $AB\bar{C}$

A	B	C	m_3	m_5	m_6
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	0	0	1
1	1	1	0	0	0

Observation?

* A minterm will be 1 only once in the truth table (TT)

* m_i will be $\underline{\underline{1}}$ at Row i of TT and $\underline{\underline{0}}$ otherwise

Sum of Minterms

* All Boolean functions can be written in sum-of-minterms (SOM) form

Ex4

X	Y	F	m_1	m_2
0	0	0	0	0
0	1	1	1	0
1	0	1	0	1
1	1	0	0	0

$$F = m_1 + m_2$$

$$\Rightarrow F(x,y) = m_1 + m_2 = \underbrace{\bar{x}y + x\bar{y}}_{\text{Algebraic SOM form}}$$

m_1: 01 → $\bar{x}$ y

m_2: 10 → x $\bar{y}$

Notation

$$F(x,y) = \sum m(1,2)$$

* What did we just do?

we went from TT to gate implementation

$$\text{Verbal description} \xrightarrow{\text{Design step}} TT \xrightarrow{\text{Implementation step}} \text{Logic Circuit}$$

Ex 5 Express $G(a,b,c)$ given in TT below in SOM canonical form using Σ notation

a	b	c	G	
0	0	0	1	m_0
0	0	1	0	
0	1	0	1	m_2
0	1	1	0	
1	0	0	0	
1	0	1	1	m_5
1	1	0	0	
1	1	1	0	

$$\Rightarrow \quad G(a,b,c) = \sum m(0,2,5)$$

if the question says express G algebraically in SOM form

$$G(a,b,c) = \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + a\bar{b}c$$

Maxterms

* Consider: set of functions of $\underline{\underline{3}}$ vars. (A, B, C)

* Maxterm: Sum term s.t. all literals show up the sum in true or complement form.

Ex 6

$A + B + \bar{C} \longrightarrow$ maxterm

$\bar{A} + \bar{B} + \bar{C} \longrightarrow$ maxterm

$A + C \longrightarrow$ Not maxterm (B missing)

* Similarly, we number Maxterms

* Maxterm #i, denoted M_i

* $M_i = 0$ at row $\underline{\underline{i}}$ of truth table otherwise 1 (complement of m_i)

* $M_i = \overline{m_i}$

Ex 7 $M_3 = \overline{m_3} = (\bar{A}BC)' = A + \bar{B} + \bar{C}$

* We can also compute M_i directly from i

Ex 8

$$M_{2} = A + \bar{B} + C$$

010

$A \quad \bar{B} \quad C$

* Functions can be represented in Product-of-Maxterms (POM) canonical form

Ex9

A	B	C	F	M_1	M_4	M_7
0	0	0	1	1	1	1
0	0	1	0	0	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	0

$$F(A,B,C) = M_1 \cdot M_4 \cdot M_7 = \prod M(1,4,7)$$

* Algebraically

$$F(A,B,C) = (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$$

* Use DeMorgan's law to convert between the two canonical forms

<u>Ex 10</u>

$G(A,B,C) = \sum m(1,3,4)$

Express G in POM using Π notation

$G(A,B,C) = m_1 + m_3 + m_4$

$\overline{G}(A,B,C) = (m_1 + m_3 + m_4)'$

$\overline{\overline{G}} = G \qquad = M_1 \cdot M_3 \cdot M_4$

$= \Pi M(1,3,4)$

This tells us that the $\underline{\underline{0}}$s of $\overline{G}$ are at Rows 1, 3, 4

$\Rightarrow$ The $\underline{\underline{0}}$s of G are in the other rows

$\Rightarrow G(A,B,C) = \Pi M(0,2,5,6,7)$

Shortcut

To find G in the other canonical form, just list the indices not appearing in the given canonical form.

Ex 11

$$F(A,B,C,D) = \prod M(0,1,2,3,4,5,6,8,10,11,12,13,14)$$

* Express F in SOM

$$\Rightarrow F(A,B,C,D) = \sum m(7,9,15)$$

* Express $\overline{F}$ in POM and SOM.

$$\overline{F}(A,B,C,D) = \prod M(7,9,15)$$

$$\overline{F}(A,B,C,D) = \sum m(0,1,2,3,4,5,6,8,10,11,12,13,14)$$

Operations on Canonical Forms

* ANDing two SOMs
  $\Rightarrow$ take intersection of indecies
* ORing two SOMs
  $\Rightarrow$ Union indecies
* ANDing two POMs
  $\Rightarrow$ Union
* ORing two POMs
  $\Rightarrow$ intersection

Ex 12

Let

$$F(A,B,C) = \sum m(0,1,3,5)$$
$$G(A,B,C) = \prod M(1,2,3,4)$$

- $F \cdot G = \left(\sum m(0,1,3,5)\right) \cdot \left(\prod M(1,2,3,4)\right)$
  $= \prod M(2,4,6,7) \cdot \prod M(1,2,3,4)$
  $= \prod M(1,2,3,4,6,7)$

Let $F(A,B,C) = \sum m(0,1,3,5)$
$G(A,B,C) = \prod M(1,2,3,4)$

- $F+G = (\sum m(0,1,3,5)) + (\sum m(0,5,6,7))$
  $= \sum m(0,1,3,5,6,7)$

- How about $F+G$ in POM?
  $F+G = \prod M(2,4)$

Standard Forms

* A product term is an ANDing of one or more literals

Ex

$$\left.\begin{array}{l} AB \\ ABC \\ \bar{A}B\bar{C} \end{array}\right\} \longrightarrow \text{Product terms}$$

A minterm is a special case of a product term

* Similarly, we define sum terms as the ORing of one or more literals

Ex

$$\left.\begin{array}{l} \bar{A}+B \\ A+B+C \\ \bar{A}+\bar{C} \end{array}\right\} \longrightarrow \text{All are sum terms}$$

A maxterm is a special case sum term.

* We can express Boolean functions as sum-of-products (SOP) or product-of-sums (POS) form.

* These two forms are called the standard forms

Ex

Indicate which of the following functions is in SOP or POS standard form.

- $ab + cd$ SOP
- $(a+b)(\bar{c}+a)$ POS
- $ab + (a+b)(c+\bar{d})$ Not in standard form
- $a + cd$ SOP
- $a(b+c)$ POS

Two-Level Implementations

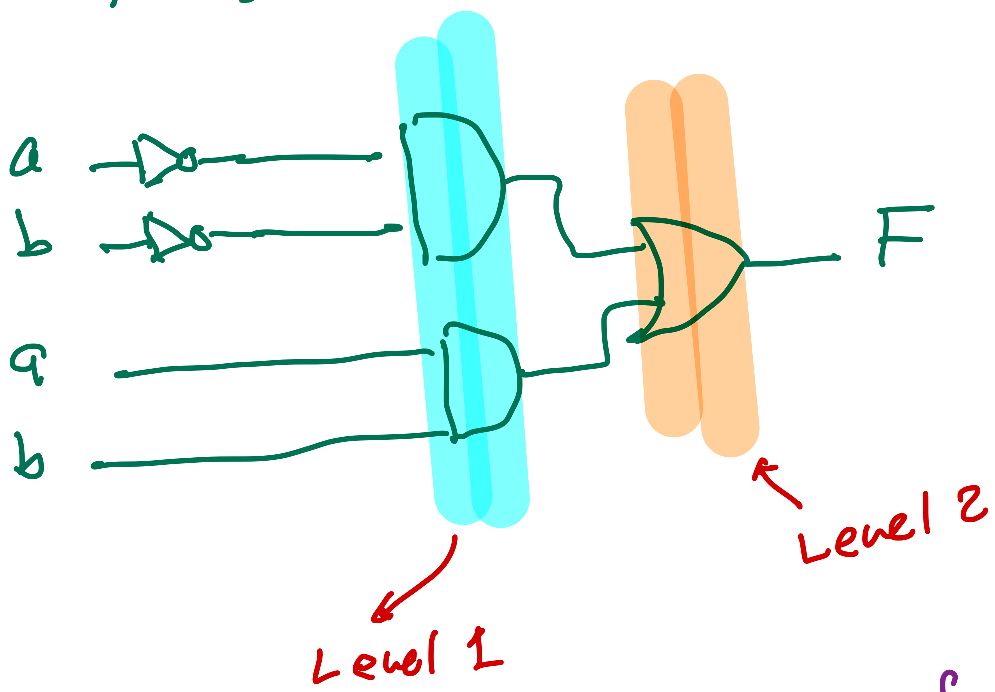
* Both standard and canonical forms result in 2-level logic circuits

* <u>SOP/SOM :</u> One level of AND gates followed by one OR gate

  → total two levels

<u>Ex</u>

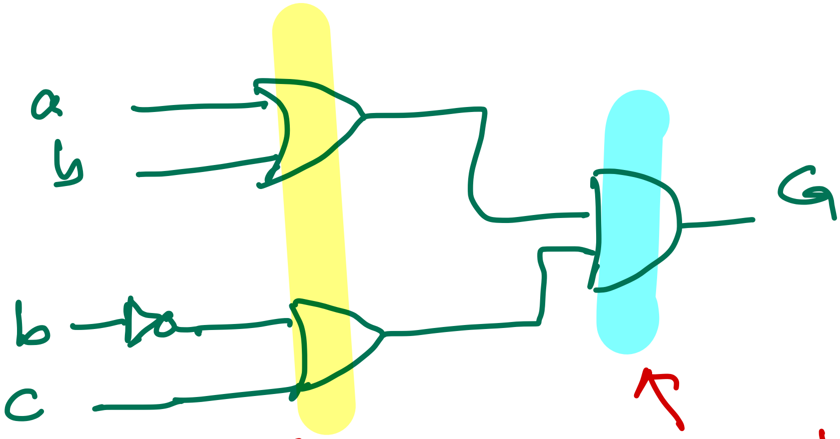
$$F(a,b) = \sum m(0,3) = \bar{a}\bar{b} + ab$$



For obvious reasons, SOP/SOM forms are also referred to as AND-OR implementations.

Ex

$G = (a+b)(\bar{b}+c)$ ← POS



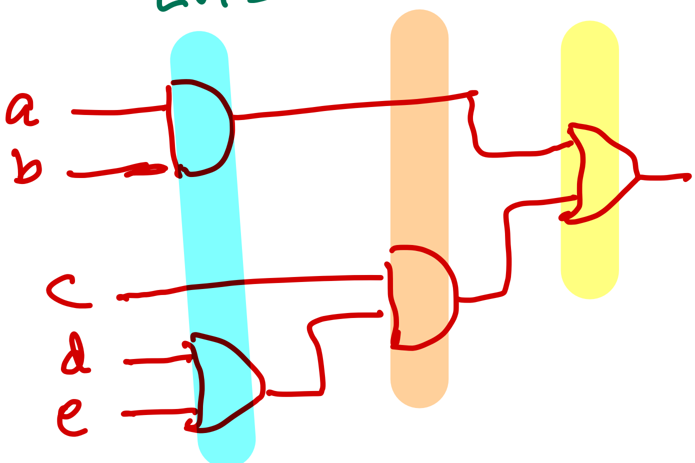
Level 2

Level 1

Similarly, POS / POM forms are OR-AND implementations

* if your implementation has > 2 levels, it is not in standard form

Lvl 1 2 3



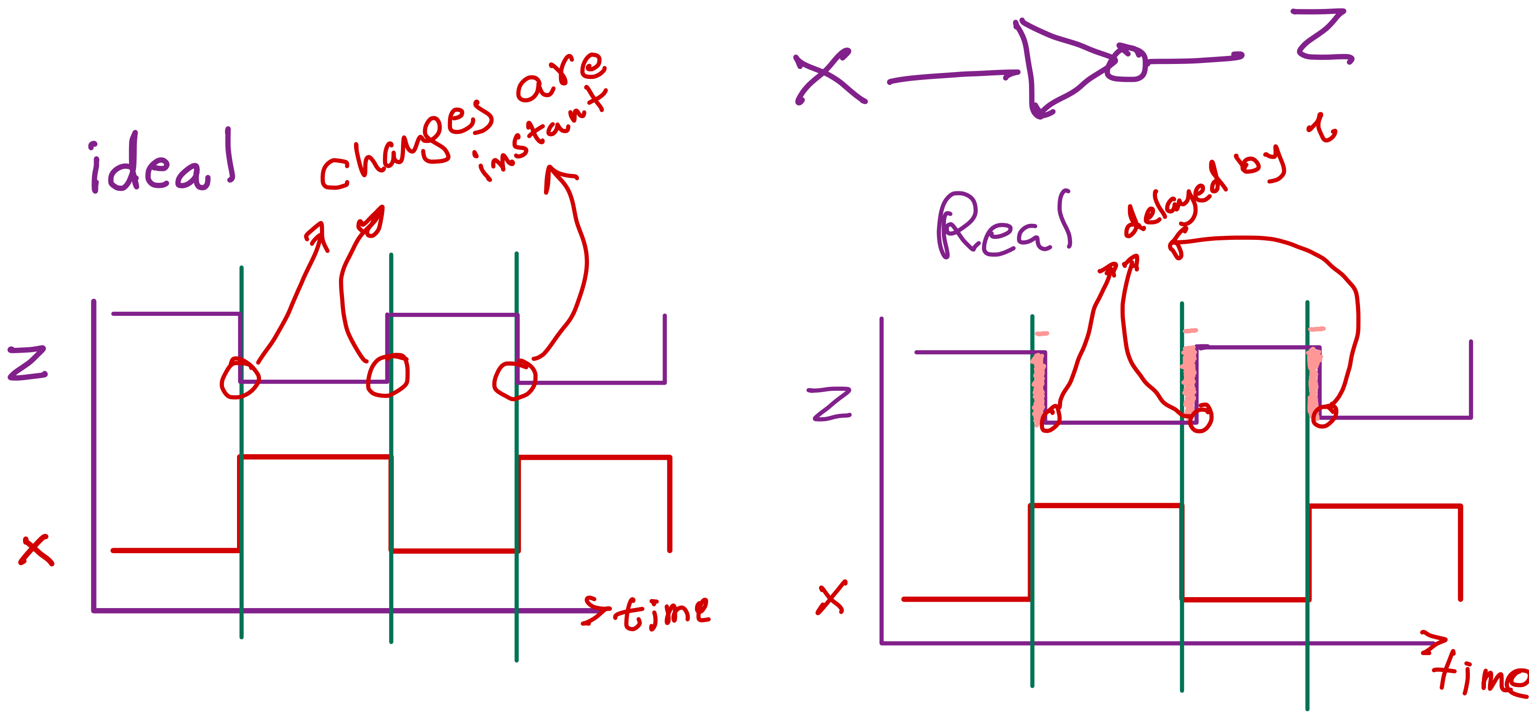
$h = ab + c(d+e)$

Not in standard form !!

Propagation Delay

* So far, we ignored any delay from input to output

* In practice, there is a delay, and the changes in output due to changes in input are not instantaneous



* This delay in output change due to input change is "propagation delay"

* These plots are referred to as timing diagram or waveform.

Procedure For Finding The Longest Propagation Delay In Logic Ciruits

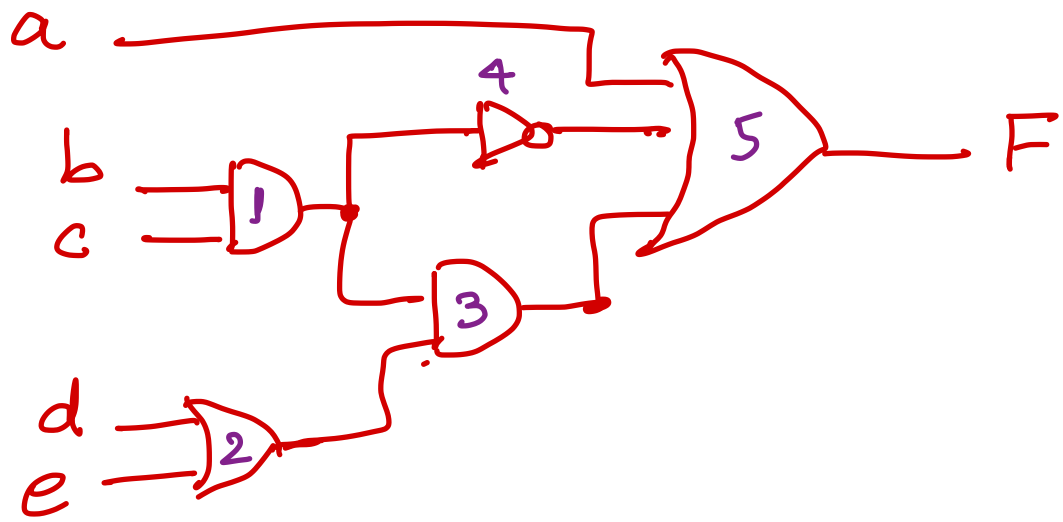
1. Every gate has its own propagation delay (Given)

2. Start from inputs, compute the delay at output of each gate = delay of the gate + max. delay of its inputs

3. The maximum propagation delay from inputs to outputs is called the

CRITICAL PATH

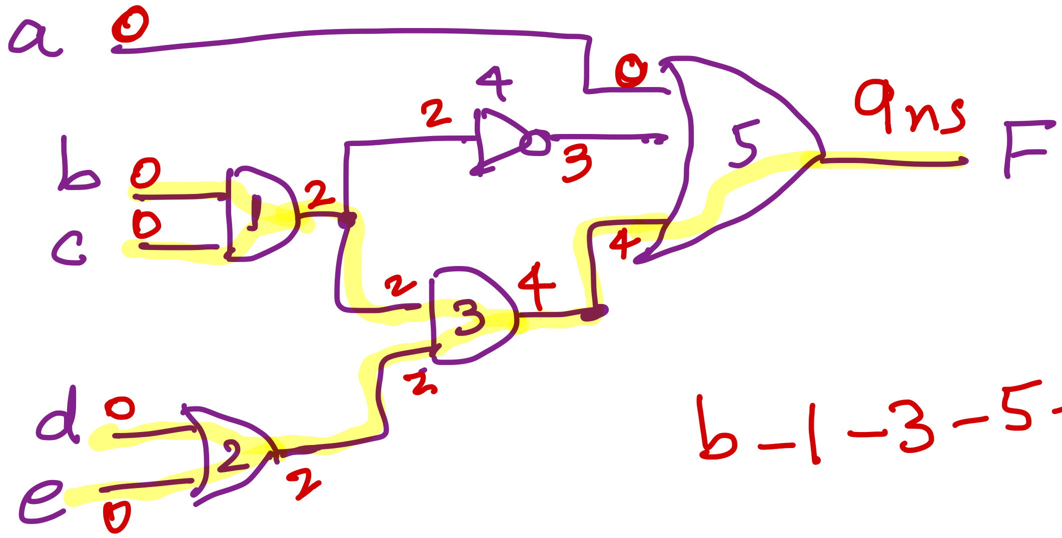
Ex



Assume the gates propagation delays are

Gate	Prop. delay
2-input AND/OR	2ns
Inverter	1ns
3-input OR	5 ns

Find the delay at the output of every gate, and determine the critical path of this circuit.



b – 1 – 3 – 5 – F

Exam Questions

3. Let $F(A,B,C) = \overline{A}B + A\overline{C}$. Express $F(A,B,C)$ as a sum of minterms using the $\sum$ notation.
(2 Point)

* We need to convert every product to a minterm
* Use algebraic manipulation
  - AND identity + Complement properties

$$\overline{A}B = \overline{A}B \cdot 1 = \overline{A}B(C+\overline{C}) = \overline{A}BC + \overline{A}B\overline{C}$$

$\overline{A}BC \to 011 \to 3$, $\quad \overline{A}B\overline{C} \to 010 \to 2$

$$A\overline{C} = A(B+\overline{B})\overline{C} = AB\overline{C} + A\overline{B}\overline{C}$$

$AB\overline{C} \to 6$, $\quad A\overline{B}\overline{C} \to 4$

$$\Rightarrow F(A,B,C) = \sum m(2,3,4,6)$$

Shortcut

$\overline{A}B-$: $010 \to 2$, $\quad 011 \to 3$

$A-\overline{C}$: $100 \to 4$, $\quad 110 \to 6$

$$\Rightarrow F(A,B,C) = \sum m(2,3,4,6)$$

6. Let $G = AB + \overline{C}D$. Express G in a product of sums standard form. **(3 Points)**

Using dist. law

$$G = (AB+\overline{C})(AB+D)$$
$$= (A+\overline{C})(B+\overline{C})(A+D)(B+D)$$

OR you can use duality to solve this problem

Question 3: **(14 points)**

a) **(1 point)** The function F, where F (A,B,C,D)=∑(2,3,6), can be expressed algebraically in canonical form as:

a. *A'BC' + A'BC + ABC'*
b. *A'B'C + A'CD'*
c. *(A + B + C)(A + B + C')(A + B' + C)(A' + B + C')(A' + B' + C')*
d. Answers (a) and (c)
e. None of the Above.

b) **(1 point)** Refer to the following statements:

Statement 1: All canonical forms for representing a function are standard forms.
Statement 2: All standard forms for representing a function are canonical forms.
Statement 3: The canonical forms and the standard forms are unique for each function

Which of these statements is/are correct?

a. All statements.
b. Statement 1 only.
c. Statement 2 only.
d. Statement 3 only.
e. None.

c) **(5 points)** Given $G(x,y,z) = x'y + xz + yz$.

(i) **(2 points)** Derive the truth table for function G(x,y,z).

(ii) **(1 point)** List all the Minterms of function G(x,y,z) using the ∑ notation.

(iii) **(2 points)** Write function G(x,y,z) as a product of Maxterms using algebraic form.

G

d) **(4 points)** Given the Boolean functions F(x,y,z) and G(x,y,z) as:

F (x,y,z) =$\sum(0,2,4,5)$

G (x,y,z) = (x + y + z') (x + y' + z) (x + y' + z') (x' + y + z') :

(i) **(2 points)** List the minterms of (*F.G'*) using the $\sum$ notation.

(ii) **(2 points)** List the maxterms of (*F'*+ *G*) using the $\prod$ notation.

e) **(3 points)** Given the following implementation of function F. Calculate the propagation delay of F and determine the critical path. Assume the delay of each gate is equal to the number of inputs (i.e. the delay of an inverter is 1ns, the delay of a 2-input AND/OR gate is 2ns)