Lesson's Objectives

1. Convert Numbers from <u>any base</u> to <u>decimal</u>

2. Convert Numbers from <u>decimal</u> to <u>any base</u>
   - a. Integers
   - b. Fractions

3. Convert from <u>binary</u> to <u>octal/hexadecimal</u> and <u>vice versa</u>

4. Important properties

$\boxed{5}$ Addition and Subtraction

of unsigned binary numbers

(And hexadecimal numbers)

$\boxed{6}$ Binary codes, Characters,

ASCII codes, parity codes

1 Convert Numbers from any base to decimal

Concepts

a) Radix or base

b) Radix point

c) Most/Least significant digit

d) Bit (Binary digit)

Conversion Algorithm

The decimal equivalent of

$$(d_{n-1}\, d_{n-2} \cdots d_1 d_0 . d_{-1} d_{-2} \cdots d_{-m})_r$$

is

$$\sum_{i=-m}^{n-1} d_i \times r^i$$

Ex 1 Convert $(1011.01)_2$ to decimal.

$m =$ ____ $n-1 =$ ____ $r =$ ____

Ex 2 Convert $(1B0C.F)_{16}$ to decimal.

$m =$ ____ $n-1 =$ ____ $r =$ ____

Ex 3

Find the radix r that satisfy

$\boxed{a}$ $(54)_r \div (4)_r = (13)_r$

$\boxed{b}$ $(24)_r + (17)_r = (40)_r$

Ex 4

The solutions of the equation

$$x^2 - (11)_{r_1} x + (22)_{r_2} = 0$$

are $x = 3$ and $x = 6$.

Find r_1, r_2 that satisfy this equation.

2 Convert Numbers from decimal to any base

a) Integers case

Algorithm

Keep dividing by target base and collecting remainders until division qoutiont=0

Ex 5

Convert $(156)_{10}$ to binary

Ex 6

Convert $(175)_{10}$ to octal.

Ex 7

Convert $(715)_{10}$ to hexadecimal

2 Convert Numbers from decimal to any base

b Fractions

Algorithm

Keep multiplying the fraction by the target base and collect the integer parts of answers. Stop when fraction = 0.

Ex 8

Convert $(0.625)_{10}$ to binary

$0.625 \times 2 = 1.25$

$0.25 \times 2 = 0.5$

$0.5 \times 2 = 1.0$

stop

collect integer parts in this order

$\Rightarrow (0.625)_{10} = (0.101)_2$

Ex 9

convert $(0.5)_{10}$ to base 7.

$0.5 \times 7 = 3.5$

$0.5 \times 7 = 3.5$

$\vdots$

Repeats forever

$\Rightarrow (0.5)_{10} \cong (0.333)_7$ upto 3 digits accurate

Ex 10

Convert $(253.1)_{10}$ to hexadecimal.

Integer:

	Q	R
$253 \div 16 =$	15	13
$15 \div 16 =$	0	15

$$\begin{array}{r} 15 \\ 16 \overline{)\,253} \\ \underline{16} \\ 93 \\ \underline{80} \\ 13 \end{array}$$

$(253)_{10} = (FD)_{16}$

Fraction

$0.1 \times 16 = 1.6$

$0.6 \times 16 = 9.6$

$0.6 \times 16 = 9.6$

$\vdots$

Repeats

$\Rightarrow (0.1)_{10} = (0.1\overline{9})_{16}$

$\Rightarrow (253.1)_{10} = (FD.1\overline{9})_{16}$

Ex 11

Given d_0, d_1, d_2 integers between 0 and 15, find their values that satisfy this equation

$$d_0 + 16d_1 + 256d_2 = 2049$$

$\boxed{3}$ Convert from binary to octal/hexadecimal and vice versa

* Every octal digit corresponds to exactly $\underline{\underline{3}}$ bits

* Every hex digit corresponds to exactly $\underline{\underline{4}}$ bits

Ex12

convert $(10010100110.01)_2$ to octal

Octal $\Rightarrow$ group bits by 3 starting from binary point

$$\underbrace{010}_{2}\ \underbrace{010}_{2}\ \underbrace{100}_{4}\ \underbrace{110}_{6}.\underbrace{010}_{2} \Rightarrow (2246.2)_8$$

Ex 13

convert $(715.34)_8$ to binary

$$\underbrace{7}_{111}\ \underbrace{1}_{001}\ \underbrace{5}_{101}\ .\ \underbrace{3}_{011}\ \underbrace{4}_{100}$$

$\Rightarrow (715.34)_8 = (111001101.011100)_2$

Ex 14 convert $(82.1C)_{16}$ to octal.

convert to binary then to octal

$$\underbrace{8}_{1000}\ \underbrace{2}_{0010}\ .\ \underbrace{1}_{0001}\ \underbrace{C}_{1100}$$

Regroup to 3 bits each

$$\underbrace{010}_{2}\ \underbrace{000}_{0}\ \underbrace{010}_{2}\ .\ \underbrace{000}_{0}\ \underbrace{111}_{7}\ \underbrace{000}_{0}$$

$\Rightarrow (82.1C)_{16} = (202.070)_8$

Tip

* Memorize the conversion between binary, decimal, Octal, and hexadecimal for integers 0 → 15
* Construct a table and make sure you master it

$\boxed{4}$ Important properties

* Largest integer of $\underline{\underline{n}}$ digits in base $r = r^n - 1$ in decimal
* " fraction of $\underline{\underline{m}}$ " " $= 1 - r^{-m}$
* n integer digits, m fractional digits
  max in base r $= r^n - r^{-m}$

* Result of adding 1 to max integer of
  n digits?

$$(9)_{10} + 1 = (10)_{10}$$

$$(7)_8 + 1 = (8)_{10} = (10)_8$$

$$(F)_{16} + 1 = (10)_{16}$$

$$(FFFF)_{16} + 1 = (10000)_{16}$$

$$\parallel$$

16^4 in decimal

$$(FFFF)_{16} = 16^4 - 1$$

add one to both sides

$$(FFFF)_{16} + 1 = 16^4$$

in general

$$(\underbrace{(r-1)(r-1)\cdots(r-1)}_{n \text{ times}})_r + 1 = r^n$$

* How many integers are there using $\underline{\underline{n}}$ digits in base $\underline{\underline{r}}$?

$$0,\ 1,\ 2,\ \ldots\ [\underbrace{(r-1)(r-1)\cdots(r-1)}_{n}]_r$$

$\Rightarrow$ Since we started from $\underline{\underline{0}}$, the answer is = largest number + 1

$$= r^n - 1 + 1 = r^n$$

* Given an integer Z, what is the representation of $Z \cdot r^k$ in base r?

Assume $Z = (d_{n-1} d_{n-2} \cdots d_1 d_0)_r$

Objective → Find $Z \cdot r^k$ in base r

should be easy ⇒ Just use the right conversion algorithm

	Q	R
$Z \cdot r^k$	$Z \cdot r^{k-1}$	0
$Z r^{k-1}$	$Z r^{k-2}$	0
$\vdots$	$\vdots$	$\vdots$
$Z r$	Z	0
Z	$\lfloor Z/r \rfloor$	d_0
$\lfloor Z/r \rfloor$	$\lfloor Z/r^2 \rfloor$	d_1
$\vdots$	$\vdots$	$\vdots$
$\lfloor Z/r^{n-1} \rfloor$	$\lfloor Z/r^n \rfloor$	d_{n-1}

zeros

$$\frac{Z \cdot r^{k-1}}{\cancel{r}} = Z \cdot r^{k-1}$$

$$Z \cdot r^k = (Z, \underbrace{00 \cdots 0}_{k \text{ zeros}})_r$$

Ex

* what is $(1FA)_{16} \times 16^4$?

$\Rightarrow (1FA)_{16} \times 16^4 = (1FA0000)_{16}$

* $(1011)_2 \times 2^2 = (101100)_2$

* $(7312)_8 \times 8 = (73120)_8$

* $(273)_{10} \times 10^2 = (27300)_{10}$

* Because the number moves to left when multiplied by r^K we call this operation $\boxed{\text{shift left}}$

* Similarly, Z/r^K causes the the number to move to the right by K positions

$\Rightarrow$ $\boxed{\text{Shift Right}}$ operation

Ex $(10CB0)_{16} \div 16 = (10CB)_{16}$

$(1011000)_2 \div 2^2 = (10110)_2$

$(10900)_{10} \div 10^2 = (109)_{10}$

5 Addition and Subtraction of unsigned binary numbers (And hexadecimal numbers)

Addition in binary is similar to decimal, we do it bit-by-bit, and propagate carry of stage n to stage $n+1$

Addition table in binary

x	y	$(x+y)_{10}$	$(x+y)_2$	c	s
0	0	0	0 0	0	0
0	1	1	0 1	0	1
1	0	1	0 1	0	1
1	1	2	$\boxed{1\,0}$	1	0

Question: what is $1+1+1$ in binary?

Decimal

$$\begin{array}{r} 197 \\ 235 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} 111 \\ +\,101 \\ \hline \end{array}$$

Ex15

Compute $(0111\_1111)_2 + (1101\_1111)_2$ and indicate if there is a carry out or not.

$\boxed{\text{Subtraction}}$ in binary is also similar to the decimal subtraction algorithm, but we optimize it to get the result faster.

Subtraction table in binary

x	y	$(x-y)_{10}$	$(x-y)_2$	B	D
0	0	0	0	0	0
0	1	-1		1	1
1	0	1	1	0	1
1	1	0	0	0	0

***Note that borrowing 1 from stage n is equivalent to borrowing 2 from stage n-1.**

{Compare this to decimal case and see if it makes sense}

$$\begin{array}{r} 173 \\ -\,185 \\ \hline \end{array} \qquad \begin{array}{r} 101 \\ -\,111 \\ \hline \end{array}$$

Ex 16

Compute $\overset{A}{(0011\_0110)_2} - \overset{B}{(1110\_1101)_2}$ and indicate if there is a final borrow or not.

Hexadecimal Addition is the same as decimal addition, but you need to convert the sum of two digits to hex.

Ex 17

Compute $(32A)_{16} + (2E8)_{16}$

$$
\begin{array}{r}
3\ 2\ A \\
+\ 2\ E\ 8 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
10 \\
+\ 8 \\
\hline
(18)_{10}
\end{array}
$$

	Q	R
$18 \div 16 =$	1	2
$1 \div 16 =$	0	1

$\Rightarrow (18)_{10} = (12)_{16}$

ONLY applicable when adding two integers

Shortcut

instead of converting $(18)_{10}$ to hex, the carry is 1 and the result is $18 - 16 = 2$

Hexadecimal Subtraction

$$
\begin{array}{r}
1\ D\ 3 \\
-\ 1\ B\ 5 \\
\hline
\end{array}
$$

Multiplication in Binary

$$
\begin{array}{r}
1101 \\
\times\ 110 \\
\hline
0000 \\
11010 \\
110100 \\
\hline
1001110
\end{array}
$$

6 Binary codes, Characters, ASCII codes, Parity codes

Binary Coded Decimals {BCDs}

Since digital systems understands only binary data, use binary codes to represent decimal digits 0-9. BCDs can use weighted codes or non-weighted codes.

Weighted BCDs:

Every bit is associated with a weight:

BCD8421, BCD84-2-1, BCD2421

To represent digit '6' in BCD8421 → 6=2+4

C	0	1	1	0
W	8	4	2	1

BCD84-2-1
6 = 8 − 2

1	0	1	0
8	4	-2	-1

BCD2421
6 = 2+4
↑
which 2?

c	1	1	0	0
w	2	4	2	1

BCD84-2-1, BCD2421 are self-complementing codes

Non-weighted BCDs

Includes excess-3 (XS-3) codes.

obtained by adding 3 to BCD8421 codes.

Ex 18

The decimal number 103 is to be stored in memory. Specify how is it represented using:

a) BCD5421:

$1 = 1$, $0 = 0$, $3 = 2+1$

$(0001 \quad 0000 \quad 0011)_{BCD5421} = (103)_{10}$

b) BCD XS-3:

$1 \rightarrow 1+3 = 4$, $0 \rightarrow 0+3 = 3$, $3 \rightarrow 3+3 = 6$

$(0100 \quad 0011 \quad 0110)_{XS3} = (103)_{10}$

Ex Decode $(0110\ 1100\ 1010)_{XS-3}$ and tell which decimal number it represents?

$$(0110 \quad 1100 \quad 1010)_{XS-3}$$

$$\begin{array}{ccc} 0110 & 1100 & 1010 \\ \downarrow & \downarrow & \downarrow \\ 6-3 & 12-3 & 10-3 \\ 3 & 9 & 7 \end{array}$$

$$(397)_{10}$$

Warning!!

DO NOT CONFUSE CODING WITH BASE CONVERSION

EX 19

Consider $(54)_{10}$

(a) Convert to binary

Note that $54 = 2^6 - 1 - 9 = 63 - 9$

$$\begin{array}{r} 111111 \\ -\quad 1001 \\ \hline 110110 \end{array} \Rightarrow (54)_{10} = (110110)_2$$

(b) Encode it using BCD 84-2-1

coding is performed for every digit alone

$5 = 8 - 2 - 1 \rightarrow 1011$

$4 = 4 \rightarrow 0100$

$\Rightarrow (54)_{10} = (1011 \quad 0100)_{BCD\,84-2-1}$

What if we want to encode letters and symbols in addition to digits?

→ Use ASCII characters

* Standard ASCII: 7-bit chars
* Extended ASCII: 8-bit chars.

EX20 Given that ASCII code of 'A' is $(41)_{16}$ and '0' is $(30)_{16}$, encode "COE202" in ASCII. Show the code as a sequence of hexadecimal numbers representing the string.

We need to know the code of every letter in the string

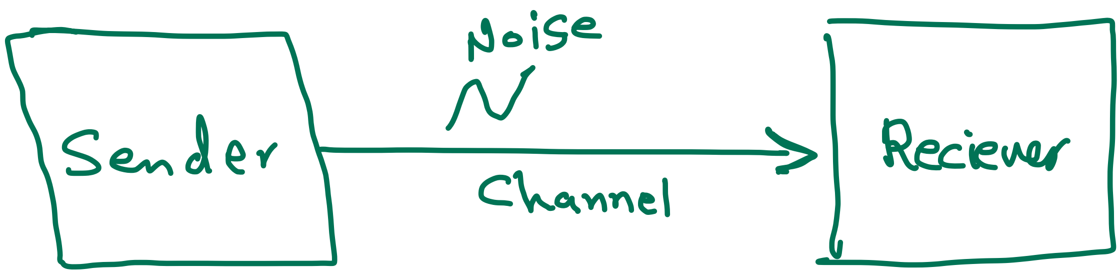
'A' $= (41)_{16}$, 'B' $= (42)_{16}$ ... 'E' $= (45)_{16}$

... 'O' $= (4F)_{16}$

And every number '0' $= (30)_{16}$, '2' $= (32)_{16}$

⇒ "COE202" → $(43)_{16}$ $(4F)_{16}$ $(45)_{16}$ $(32)_{16}$ $(30)_{16}$ $(32)_{16}$

ERROR DETECTION



* Sender tries to send data to reciever
* During transmission, data may get corrupted due to noise
* Reciever can check data against errors, if we add extra bit called 'parity bit'
* Even parity: number of 1s is even
* Odd parity: number of 1s is odd

Ex 21 A sender wants to transmit 1101011. Indicate the value of parity bit if we use even and odd parity system.

Even → $\boxed{1}$1101011

Odd → $\boxed{0}$1101011

d) The 8-bit binary code for character “C” is **01000011**. Using **even parity**, the sender inserts an extra parity bit equal to __________. The receiver receives a 9-bit binary code equal to **001000011**, would the receiver detect an error (Yes/No)? ______________ (**2 points**)