Data Representation

COE 202

Digital Logic Design

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Outline

- Introduction
- Numbering Systems
- Binary & Hexadecimal Numbers
- Base Conversions
- Binary Addition, Subtraction, Multiplication
- Hexadecimal Addition
- Binary Codes for Decimal Digits
- Character Storage

Introduction

- Computers only deal with binary data (0s and 1s), hence all data manipulated by computers must be represented in binary format.
- Machine instructions manipulate many different forms of data:
  - Numbers:
    - Integers: 33, +128, -2827
    - Real numbers: 1.33, +9.55609, -6.76E12, +4.33E-03
  - Alphanumeric characters (letters, numbers, signs, control characters): examples: A, a, c, 1 ,3, ", +, Ctrl, Shift, etc.
  - Images (still or moving): Usually represented by numbers representing the Red, Green and Blue (RGB) colors of each pixel in an image,
  - Sounds: Numbers representing sound amplitudes sampled at a certain rate (usually 20kHz).
- So in general we have two major data types that need to be represented in computers; numbers and characters.

Numbering Systems

- Numbering systems are characterized by their base number.
- In general a numbering system with a base *r* will have *r* different digits (including the 0) in its number set. These digits will range from *0 to r-1.*
- The most widely used numbering systems are listed in the table below:

Numbering System	Base	Digits Set
Binary	2	1 0
Octal	8	7 6 5 4 3 2 1 0
Decimal	10	9 8 7 6 5 4 3 2 1 0
Hexadecimal	16	F E D C B A 9 8 7 6 5 4 3 2 1 0

Weighted Number Systems

- A number D consists of *n* digits with each digit having a particular *position*.

$$D = d_{n-1} \; d_{n-2} \; \ldots\ldots \; d_2 \; d_1 \; d_0$$

Position n-1 | Position 2 | Position 1 | Position 0

- Every digit *position* is associated with a *fixed weight*.
- If the weight associated with the *ith* position is w_i, then the value of D is given by:

$$D = d_{n-1} w_{n-1} + d_{n-2} w_{n-2} + \ldots + d_2 w_2 + d_1 w_1 + d_0 w_0$$

Example of Weighted Number Systems

- The Decimal number system (النظام العشري) is a weighted system.
- For integer decimal numbers, the weight of the rightmost digit (*at position 0*) *is* 1, the weight of *position 1* digit is 10, that of *position 2* digit is 100, *position 3* is 1000, etc.
- Thus, $w_0 = 1$, $w_1 = 10$, $w_2=100$, $w_3 = 1000$, etc.
- Example:
- Show how the value of the number 9375 is

First Position Index

Position	3	2	1	0
Number	9	3	7	5
Weight	1000	100	10	1
Value	9 x 1000	3x100	7x10	5x1
Value	9000 +	300 +	70 +	5

First Position Index (0)

The Radix (Base)

- For *digit position i*, most weighted number systems use weights (w_i) that are powers of some constant value called the radix (*r*) or the base such that $w_i = r^i$.
- A number system of radix *r*, typically has a set of r allowed digits $\in \{0,1, \ldots,(r-1)\}$.
- The leftmost digit has the highest weight → Most Significant Digit (MSD).
- The rightmost digit has the lowest weight → Least Significant Digit (LSD).

The Radix (Base)

- **Example: Decimal Number System**
- 1. Radix (Base) = *Ten*
- 2. Since $w_i = r^i$, *then*
  - $w_0 = 10^0 = 1,$
  - $w_1 = 10^1 = 10,$
  - $w_2 = 10^2 = 100,$
  - $w_3 = 10^3 = 1000$, *etc.*
- 3. Number of Allowed Digits is Ten:
  - {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

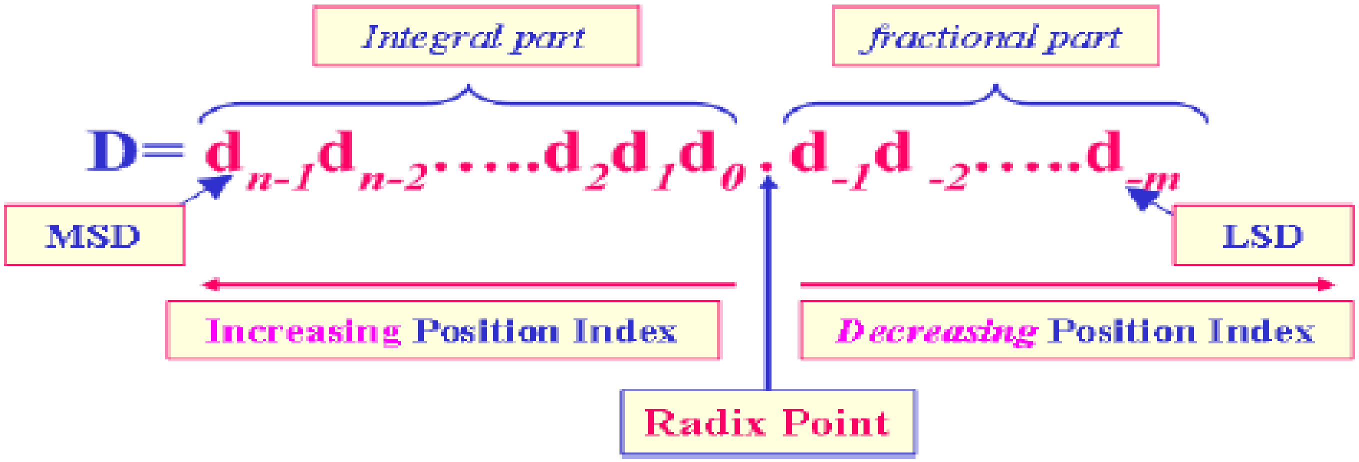
MSD LSD

$$9375 = 5x10^0 + 7x10^1 + 3x10^2 + 9x10^3$$
$$= 5x1 + 7x10 + 3x100 + 9x\ 1000$$

Position	3	2	1	0
Weight	1000 $= 10^3$	100 $= 10^2$	10 $= 10^1$	1 $= 10^0$

The Radix Point

- A number D of *n* integral digits and *m* fractional digits is represented as shown:



- Digits to the left of the radix point (*integral digits*) *have positive* position indices, while digits to the right of the radix point (*fractional digits*) *have negative position indices.*

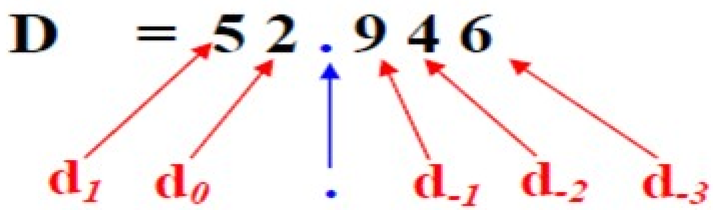
The Radix Point

- Position indices of digits to the left of the radix point (the integral part of D) start with a 0 and are incremented as we move left ($d_{n-1}d_{n-2}.....d_2d_1d_0$).
- Position indices of digits to the right of the radix point (the fractional part of D) start with a -1 and are decremented as we move right($d_{-1}d_{-2}.....d_{-m}$).
- The weight associated with digit *position i* is given by $w_i = r^i$, where *i* is the position index $\forall i= -m, -m+1, \ldots, -2, -1, 0, 1, \ldots\ldots, n-1.$
- The Value of D is Computed as:

$$D = \sum_{i=-m}^{n-1} d_i \, r^i$$

The Radix Point

- Example: Show how the value of the decimal number 52.946 is estimated.



Number	5	2	.	9	4	6
Position	1	0	.	-1	-2	-3
Weight	10^1 = 10	10^0 = 1	.	10^{-1} = 0.1	10^{-2} = 0.01	10^{-3} = 0.001
Value	5 x 10	2 x 1	.	9 x 0.1	4 x 0.01	6 x 0.001
Value	50 + 2 + 0.9 +0.04 +0.006					

$$D = 5x10^1 + 2x10^0 + 9x10^{-1} + 4x10^{-2} + 6x10^{-3}$$

Notation

- Let $(D)_r$ denote a number D expressed in a number system of radix r.
- In this notation, r will be expressed in decimal.
- Examples:
- $(29)_{10}$ Represents a decimal value of 29. The radix “10” here means ten.
- $(100)_{16}$ is a Hexadecimal number since r = “16” here means sixteen. This number is equivalent to a decimal value of $16^2=256$.
- $(100)_2$ is a Binary number (radix =2, i.e. two) which is equivalent to a decimal value of $2^2 = 4$.

Binary System

- r=2
- Each digit (bit) is either 1 or 0
- Each bit represents a power of 2
- Every binary number is a sum of powers of 2

1	1	1	1	1	1	1	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Table 1-3 Binary Bit Position Values.

2^n	Decimal Value	2^n	Decimal Value
2^0	1	2^8	256
2^1	2	2^9	512
2^2	4	2^{10}	1024
2^3	8	2^{11}	2048
2^4	16	2^{12}	4096
2^5	32	2^{13}	8192
2^6	64	2^{14}	16384
2^7	128	2^{15}	32768

Binary System

- Examples: Find the decimal value of the two Binary numbers $(101)_2$ and $(1.101)_2$

MSB LSB

$$(1\ 0\ 1)_2 = 1\text{x}2^0 + 0\text{x}2^1 + 1\text{x}2^2$$
$$= 1\text{x}1 + 0\text{x}2 + 1\text{x}\,4$$
$$= (5)_{10}$$

MSB LSB

$$(1\,.\,1\ 0\ 1)_2 = 1\text{x}2^0 + 1\text{x}2^{-1} + 0\text{x}2^{-2} + 1\text{x}2^{-3}$$
$$= 1 + 0.5 + 0 + 0.125$$
$$= (1\,.\,6\ 2\ 5)_{10}$$

Octal System

- r = 8 (Eight = 2^3)
- Eight allowed digits {0, 1, 2, 3, 4, 5, 6, 7}
- Examples: Find the decimal value of the two Octal numbers $(375)_8$ and $(2.746)_8$

MSD LSD

$$
\begin{aligned}
(375)_8 &= 5\text{x}8^0 + 7\text{x}8^1 + 3\text{x}8^2 \\
&= 5\text{x}1 + 7\text{x}8 + 3\text{x}64 \\
&= (253)_{10}
\end{aligned}
$$

MSD LSD

$$
\begin{aligned}
(2.746)_8 &= 2\text{x}8^0 + 7\text{x}8^{-1} + 4\text{x}8^{-2} + 6\text{x}8^{-3} \\
&= (2.94921875)_{10}
\end{aligned}
$$

Hexadecimal System

- r = 16 (Sixteen = 2^4)
- Sixteen allowed digits {0-to-9 and A, B, C, D, E, F}
- Where: A = Ten, B = Eleven, C = Twelve, D = Thirteen, E = Fourteen & F = Fifteen.
- Examples: Find the decimal value of the two Hexadecimal numbers $(9E1)_{16}$ and $(3B.C)_{16}$

MSD LSD

$$(9E1)_{16} = 1\text{x}16^0 + \text{E}\text{x}16^1 + 9\text{x}16^2$$
$$= 1\text{x}1 + 14\text{x}16 + 9\text{x}256$$
$$= (2529)_{10}$$

MSD LSD

$$(3B.C)_{16} = \text{C}\text{x}16^{-1} + \text{B}\text{x}16^0 + 3\text{x}16^1$$
$$= 12\text{x}16^{-1} + 11\text{x}16^0 + 3\text{x}16$$
$$= (59.75)_{10}$$

Hexadecimal Integers

- Binary values are represented in hexadecimal.

Table 1-5 Binary, Decimal, and Hexadecimal Equivalents.

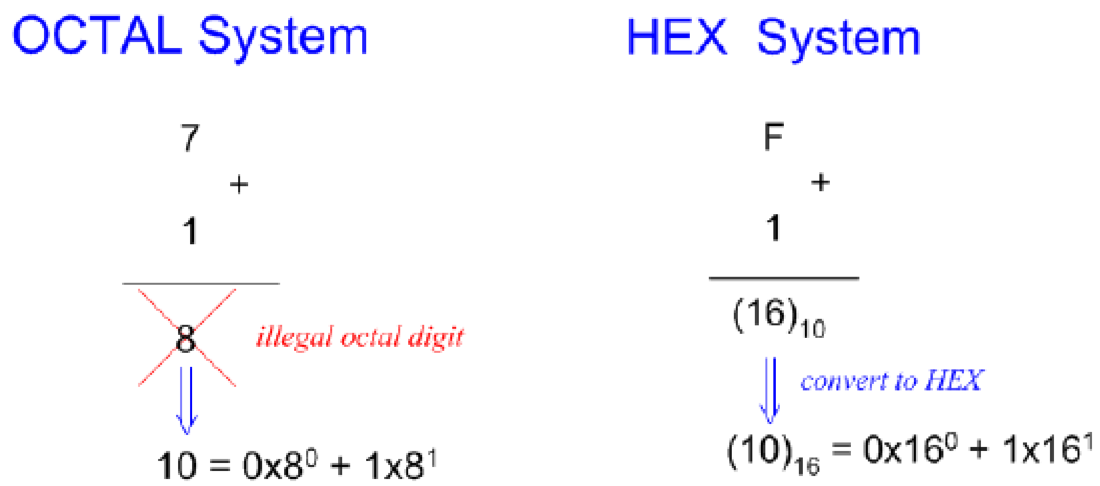
Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	10	A
0011	3	3	1011	11	B
0100	4	4	1100	12	C
0101	5	5	1101	13	D
0110	6	6	1110	14	E
0111	7	7	1111	15	F

Important Properties

- The Largest value that can be expressed in n integral digits is (r^n-1).
- The Largest value that can be expressed in m fractional digits is ($1-r^{-m}$).
- The Largest value that can be expressed in n integral digits and m fractional digits is ($r^n - r^{-m}$)
- Total number of values (patterns) representable in n digits is r^n.

Important Properties

- Q. What is the result of adding 1 to the largest digit of some number system??
  - For the decimal number system, $(1)_{10} + (9)_{10} = (10)_{10}$
  - For the binary number system, $(1)_2 + (1)_2 = (10)_2 = (2)_{10}$
  - For the octal number system, $(1)_8 + (7)_8 = (10)_8 = (8)_{10}$
  - For the hexadecimal system, $(1)_{16} + (F)_{16} = (10)_{16} = (16)_{10}$

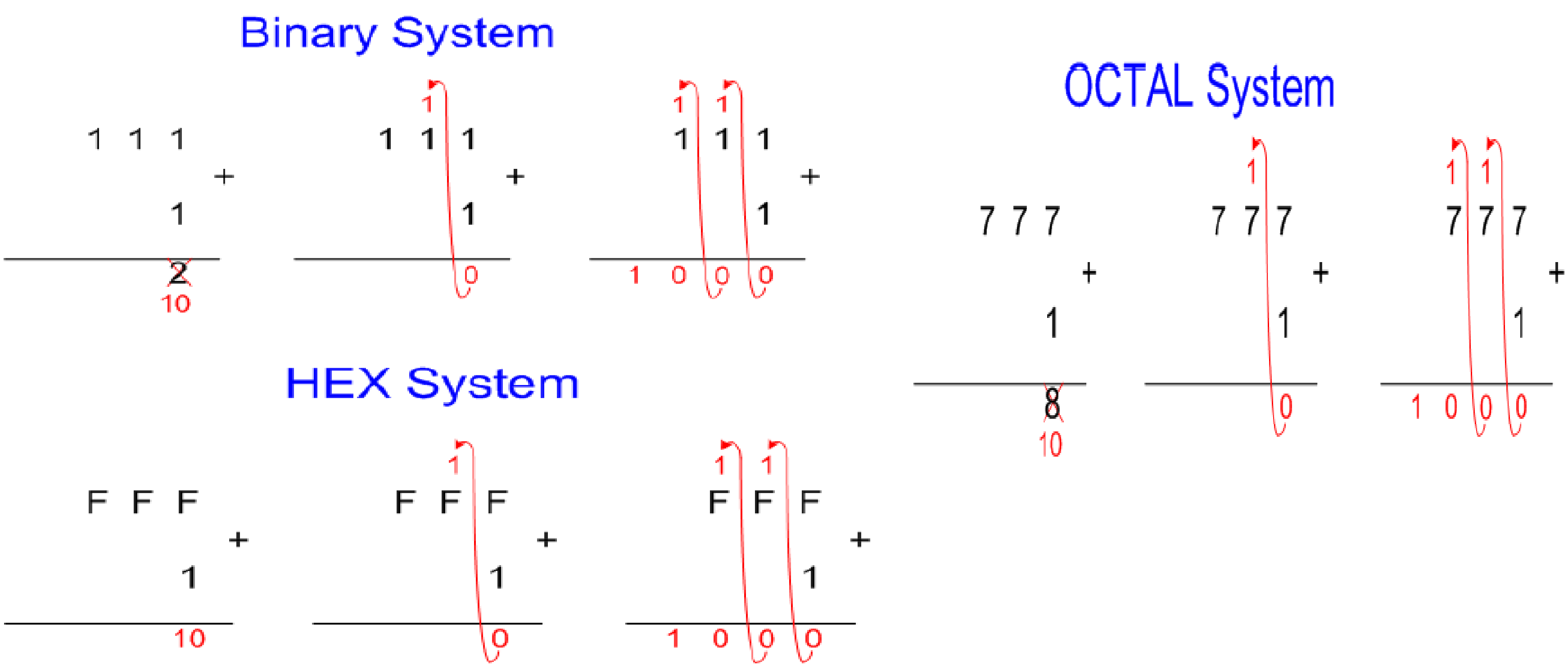


Important Properties

- Q. What is the largest value representable in 3-integral digits?
- A. The largest value results when all 3 positions are filled with the largest digit in the number system.
  - For the decimal system, it is $(999)_{10}$
  - For the octal system, it is $(777)_8$
  - For the hex system, it is $(FFF)_{16}$
  - For the binary system, it is $(111)_2$
- Q. What is the result of adding 1 to the largest 3-digit number?
  - For the decimal system, $(1)_{10} + (999)_{10} = (1000)_{10} = (10^3)_{10}$
  - For the octal system, $(1)_8 + (777)_8 = (1000)_8 = (8^3)_{10}$

Important Properties

- In general, for a number system of radix r, adding 1 to the largest *n-digit* number = r^n.
- Accordingly, the value of largest *n-digit number* = $r^n - 1$.



Number Base Conversion

- Given the representation of some number (X_B) in a number system of radix B, we need to obtain the representation of the same number in another number system of radix A, i.e. (X_A).
- For a number that has both integral and fractional parts, conversion is done separately for both parts, and then the result is put together with a system point in between both parts.
- Converting Whole (Integer) Numbers
  - Assume that X_B has n digits $(b_{n-1} \ldots\ldots\ldots b_2\ b_1\ b_0)_B$, where b_i is a digit in radix B system, i.e. $b_i \in \{0, 1, \ldots.., \text{"B-1"}\}$.
  - Assume that X_A has m digits $(a_{m-1} \ldots\ldots\ldots a_2\ a_1\ a_0)_A$, where a_i is a digit in radix A system, i.e. $a_i \in \{0, 1, \ldots.., \text{"A-1"}\}$.

Converting Whole (Integer) Numbers

- Dividing X_B by A, the remainder will be a_0.

$$X_B = \overbrace{(b_{n-1}\ldots\ldots\ldots b_2\, b_1\, b_0)_B}^{\text{Knowns}} \Rightarrow \underbrace{(a_{m-1}\ldots\ldots\ldots a_2\, a_1\, a_0)_A}_{\text{Unknowns}}$$

$$X_B = \underbrace{a_{m-1}*A^{m-1}+\ldots\ldots+a_2*A^{2}+a_1*A^{1}}_{\text{Divisible by A}} + \underbrace{a_0*A^{0}}_{\text{Not Divisible by A}}$$

- In other words, we can write $X_B = Q_0.A+a_0$

Converting Whole (Integer) Numbers

Where, $Q_0 = \underbrace{a_{m-1}*A^{m-2} + \ldots + a_2*A^1}_{\text{Divisible by A}} + \underbrace{a_1*A^0}_{\text{Not Divisible by A}}$

$Q_0 = Q_1A+a_1$

$Q_1 = Q_2A+a_2$

....

$Q_{m-3}=Q_{m-2}A+a_{m-2}$

$Q_{m-2}=a_{m-1} < A$ (not divisible by A)

$=Q_{m-1}A+a_{m-1}$

Where $Q_{m-1} = 0$

Converting Whole (Integer) Numbers

- This division procedure can be used to convert an integer value from some radix number system to any other radix number system.
- The first digit we get using the division process is a_0, then a_1, then a_2, till a_{m-1}
- Example: Convert $(53)_{10}$ to $(?)_2$

Division Step	Quotient	Remainder	
53 ÷ 2	$Q_0 = 26$	$1 = a_0$	LSB
26 ÷ 2	$Q_1 = 13$	$0 = a_1$	
13 ÷ 2	$Q_2 = 6$	$1 = a_2$	
6 ÷ 2	$Q_3 = 3$	$0 = a_3$	
3 ÷ 2	$Q_4 = 1$	$1 = a_4$	
1 ÷ 2	0	$1 = a_5$	MSB

Stopping Point

Thus $(53)_{10} = (110101.)_2$

Converting Whole (Integer) Numbers

- Since we always divide by the radix, and the quotient is re-divided again by the radix, the solution table may be compacted into 2 columns only as shown:

Binary Point

51	●		53	●	
25	1	a_0	26	1	← **LSB**
12	1		13	0	
6	0		6	1	
3	0		3	0	
1	1		1	1	
0	1	a_m	0	1	← **MSB**

$(51)_{10} = (110011.)_2$

$(53)_{10} = (110101.)_2$

Converting Whole (Integer) Numbers

- Example: Convert $(755)_{10}$ to $(?)_8$

Division Step	Quotient	Remainder	
755 ÷ 8	$Q_0 = 94$	$3 = a_0$	LSB
94 ÷ 8	$Q_1 = 11$	$6 = a_1$	
11 ÷ 8	$Q_2 = 1$	$3 = a_2$	
1 ÷ 8	0	$1 = a_3$	MSB

755	●
94	3
11	6
1	3
0	1

$(755)_{10} \Rightarrow (1363.)_8$

-

1606 ÷12	●		
133 ÷12	10 = A	LSB
11 ÷12	1	
0	11 = B	MSB

For radix twelve, the allowed digit set is:

{0-9, A, B}

$(1606)_{10} \Rightarrow (B1A.)_{12}$

Converting Binary to Decimal

- Weighted positional notation shows how to calculate the decimal value of each binary bit:

$Decimal = (d_{n-1} \times 2^{n-1}) + (d_{n-2} \times 2^{n-2}) + ... + (d_1 \times 2^1) + (d_0 \times 2^0)$

d = binary digit

- binary 10101001 = decimal 169:

$(1 \times 2^7) + (1 \times 2^5) + (1 \times 2^3) + (1 \times 2^0) =$
$128+32+8+1=169$

Convert Unsigned Decimal to Binary

- Repeatedly divide the decimal integer by 2. Each remainder is a binary digit in the translated value:

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

least significant bit

most significant bit

stop when quotient is zero

37 = 100101

Another Procedure for Converting from Decimal to Binary

- Start with a binary representation of all 0's
- Determine the highest possible power of two that is less or equal to the number.
- Put a 1 in the bit position corresponding to the highest power of two found above.
- Subtract the highest power of two found above from the number.
- Repeat the process for the remaining number

Another Procedure for Converting from Decimal to Binary

❖ Example: Converting $(76)_{10}$ to Binary

- The highest power of 2 less or equal to 76 is 64, hence the seventh (MSB) bit is 1

1	.	.	.	.	.	.

- Subtracting 64 from 76 we get 12.
- The highest power of 2 less or equal to 12 is 8, hence the fourth bit position is 1

1	0	0	1	.	.	.

- We subtract 8 from 12 and get 4.
- The highest power of 2 less or equal to 4 is 4, hence the third bit position is 1

1	0	0	1	1	.	.

- Subtracting 4 from 4 yield a zero, hence all the left bits are set to 0 to yield the final answer

1	0	0	1	1	0	0

Binary to Octal Conversion

- Each octal digit corresponds to 3 binary bits.

$$(b_n \ldots\ldots b_5\ b_4\ b_3\ b_2\ b_1\ b_0\ .\ b_{-1}\ b_{-2}\ b_{-3}\ b_{-4}\ b_{-5} \ldots\ldots)_2 \Rightarrow (\ ?\ )_8$$

$$(b_n \ldots\ \underbrace{b_5\ b_4\ b_3}_{\text{3-bits}}\ \underbrace{b_2\ b_1\ b_0}_{\text{3-bits}}\ .\ \underbrace{b_{-1}\ b_{-2}\ b_{-3}}_{\text{3-bits}}\ \underbrace{b_{-4}\ b_{-5} \ldots}_{\text{3-bits}})_2$$

Starting Point

- Example: Convert $(1110010101.1011011)_2$ into Octal.

$$\underbrace{001}_{1}\_\_\underbrace{110}_{6}\_\_\underbrace{010}_{2}\_\_\underbrace{101}_{5}\_.\_\underbrace{101}_{5}\_\_\underbrace{101}_{5}\_\_\underbrace{100}_{4} = (1625.554)_8$$

Binary to Hexadecimal Conversion

- Each hexadecimal digit corresponds to 4 binary bits.

$$(b_n\ldots\ldots\, b_5\, b_4\, b_3\, b_2\, b_1\, b_0\,.\, b_{-1}\, b_{-2}\, b_{-3}\, b_{-4}\, b_{-5}\ldots\ldots)_2 \Rightarrow (\ ?\ )_{16}$$

Starting Point

$$(b_n\overbrace{\ldots\ldots\, b_5\, b_4}^{4\text{-bits}}\,\overbrace{b_3\, b_2\, b_1\, b_0}^{4\text{-bits}}\,.\,\overbrace{b_{-1}\, b_{-2}\, b_{-3}\, b_{-4}}^{4\text{-bits}}\, b_{-5})_2$$

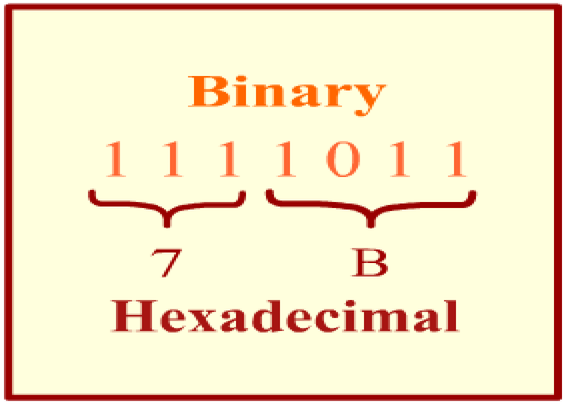
- Example: Convert $(1110010101.1011011)_2$ into hex.

$$\underbrace{0011}_{3}\_\_\underbrace{1001}_{9}\_\_\underbrace{0101}_{5}\_.\_\underbrace{1011}_{B}\_\_\underbrace{0110}_{6} = (395.B6)_{16}$$

Binary to Hexadecimal Conversion

- Example: Translate the binary integer 000101101010011110010100 to hexadecimal

1	6	A	7	9	4
0001	0110	1010	0111	1001	0100



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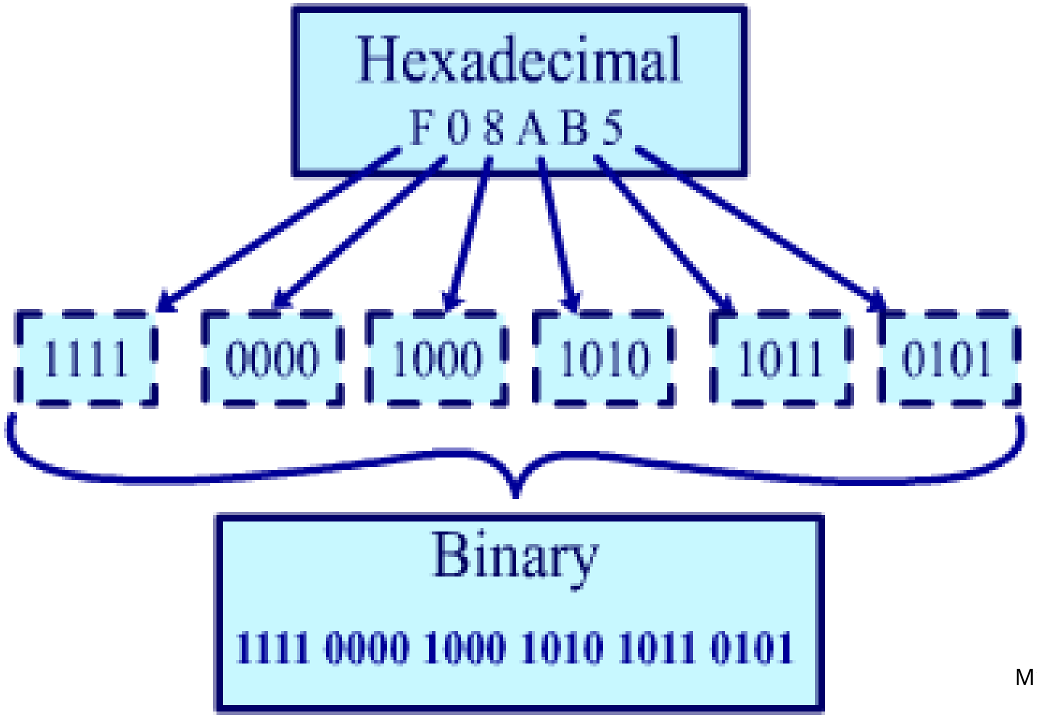
Converting Hexadecimal to Binary

- Each Hexadecimal digit can be replaced by its 4-bit binary number to form the binary equivalent.



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Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16:

  $\text{Decimal} = (d3 \times 16^3) + (d2 \times 16^2) + (d1 \times 16^1) + (d0 \times 16^0)$

  d = hexadecimal digit

- Examples:
  - $(1234)_{16} = (1 \times 16^3) + (2 \times 16^2) + (3 \times 16^1) + (4 \times 16^0) =$

    $(4{,}660)_{10}$
  - $(3BA4)_{16} = (3 \times 16^3) + (11 * 16^2) + (10 \times 16^1) + (4 \times 16^0) =$

    $(15{,}268)_{10}$

Converting Decimal to Hexadecimal

❖ Repeatedly divide the decimal integer by 16. Each remainder is a hex digit in the translated value:

Division	Quotient	Remainder
422 / 16	26	6
26 / 16	1	A
1 / 16	0	1

least significant digit

most significant digit

stop when quotient is zero

$(422)_{10} = (1A6)_{16}$

Converting Fractions

- Assume that X_B has n digits, $X_B = (0.b_{-1}\ b_{-2}\ b_{-3} \ldots\ldots b_{-n})_B$
- Assume that X_A has m digits, $X_A = (0.a_{-1}\ a_{-2}\ a_{-3} \ldots\ldots a_{-m})_A$

$$\text{Thus, } X_B = \overbrace{(0.b_{-1}\ b_{-2}\ b_{-3} \ldots\ldots b_{-n})_B}^{\text{Knowns}} \Rightarrow \underbrace{(0.a_{-1}\ a_{-2}\ a_{-3} \ldots \ldots a_{-m})_A}_{\text{Unknowns}}$$

$$X_B = a_{-1} * A^{-1} + a_{-2} * A^{-2} + \ldots\ldots\ldots a_{-m} * A^{-m}$$

$$X_B * A = \underbrace{a_{-1}}_{\text{Integer}} + \underbrace{X_{B1}}_{\text{Fraction}}$$

$$X_{B1} * A = a_{-2} + X_{B2}$$

$$X_{Bm-2} * A = a_{-m-1} + X_{Bm-1}$$

$$X_{Bm-1} * A = a_{-m}$$

Converting Fractions

❖ Example: Convert $(0.731)_{10}$ to $(?)_2$

Binary Point

0.731*2=**1**.462

0.462*2=**0**.924

0.924*2=**1**.848

0.848*2=**1**.696

0.696*2=**1**.392

0.392*2=**0**.784

0.784*2=**1**.568

$$(0.731)_{10} = (.1011101)_2$$

Converting Fractions

- Example: Convert $(0.731)_{10}$ to $(?)_8$

Octal Point

$8*0.731 = 5.848$

$8*0.848 = 6.784$

$8*0.784 = 6.272$

$8*0.272 = 2.176$

$(0.731)_{10} = (0.5662)_8$

- Example: Convert $(0.357)_{10}$ to $(?)_{12}$

System Point

$12*0.357 = 4.284$

$12*0.284 = 3.408$

$12*0.408 = 4.896$

$12*0.896 = 10.752 \Rightarrow$ A=10

=A

$(0.357)_{10} \Rightarrow (0.434A)_{12}$

Binary Addition

- 1 + 1 = 2, but 2 is not allowed digit in binary
- Thus, adding 1 + 1 in the binary system results in a Sum bit of 0 and a Carry bit

Example

Carries

+		1	1	1	1		
	1	0	1	1	0	1	
	1	0	0	1	1	1	+
1	0	1	0	1	0	0	

Binary Addition Table

	Carry	Sum
Weight	2^1	2^0
0 + 0	0	0
0 + 1	0	1
1 + 0	0	1
1 + 1	1	0

$\equiv 1 \times 2^1$ $\equiv 0 \times 2^0$

$\equiv +2$

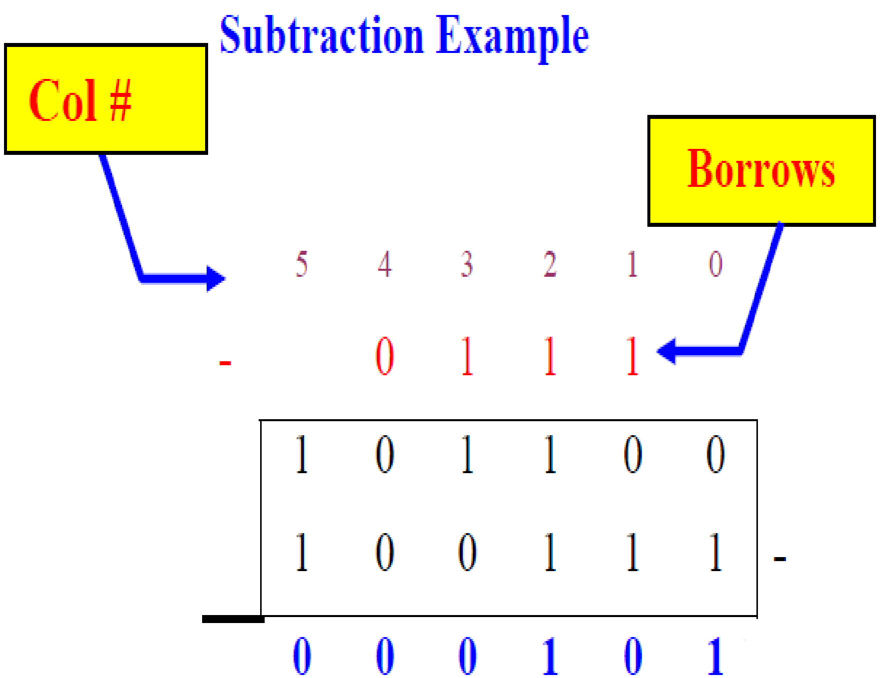
Binary Addition

- Start with the least significant bit (rightmost bit)
- Add each pair of bits
- Include the carry in the addition, if present

carry:					1				
	0	0	0	0	0	1	0	0	(4)
+	0	0	0	0	0	1	1	1	(7)
	0	0	0	0	1	0	1	1	(11)
bit position:	7	6	5	4	3	2	1	0	

Binary Subtraction

- The borrow digit is negative and has the weight of the next higher digit.



	Borrow	Difference
Weight	-2^1	$+2^0$
0 - 0	0	0
1 - 1	0	0
1 - 0	0	1
0 - 1	1	1

$\equiv 1\text{x}(-2^1)$ $\quad \equiv +1\text{x}2^0$

$\equiv -1$

Binary Multiplication

- Binary multiplication is performed similar to decimal multiplication.
- Example: 11 * 5 = 55

Multiplicand			1	0	1	1	
Multiplier				1	0	1	x
			1	0	1	1	
		0	0	0	0		+
	1	0	1	1			+
	1	1	0	1	1	1	

Hexadecimal Addition

- Divide the sum of two digits by the number base (16). The quotient becomes the carry value, and the remainder is the sum digit.

		1	1
36	28	28	6A
42	45	58	4B
78	6D	80	B5

21 / 16 = 1, remainder 5

Binary Codes for Decimal Digits

- Internally, digital computers operate on binary numbers.
- When interfacing to humans, digital processors, e.g. pocket calculators, communication is decimal-based.
- Input is done in decimal then converted to binary for internal processing.
- For output, the result has to be converted from its internal binary representation to a decimal form.
- To be handled by digital processors, the decimal input (output) must be coded in binary in a digit by digit manner.

Binary Codes for Decimal Digits

- For example, to input the decimal number 957, each digit of the number is individually coded and the number is stored as 1001_0101_0111.
- Thus, we need a specific code for each of the 10 decimal digits. There is a variety of such decimal binary codes.
- One commonly used code is the Binary Coded Decimal (BCD) code which corresponds to the first 10 binary representations of the decimal digits 0-9.
  - The BCD code requires 4 bits to represent the 10 decimal digits.
  - Since 4 bits may have up to 16 different binary combinations, a total of 6 combinations will be unused.
  - The position weights of the BCD code are 8, 4, 2, 1.

Binary Codes for Decimal Digits

- Other codes use position weights of
  - 8, 4, -2, -1
  - 2, 4, 2, 1.
- An example of a non-weighted code is the excess-3 code
  - digit codes are obtained from their binary equivalent after adding 3.
  - Thus the code of a decimal 0 is 0011, that of 6 is 1001, etc.

Binary Codes for Decimal Digits

Decimal Digit	BCD 8	4	2	1	8	4	-2	-1	2	4	2	1	Excess-3			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	1	1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	1	0	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	0	1	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	1	1	1	0	1	1	1	0	0	0
6	0	1	1	0	1	0	1	0	1	1	0	0	1	0	0	1
7	0	1	1	1	1	0	0	1	1	1	0	1	1	0	1	0
8	1	0	0	0	1	0	0	0	1	1	1	0	1	0	1	1
9	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0
U	1	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0
N	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1
U	1	1	0	0	0	0	1	1	0	1	1	1	0	0	1	0
S	1	1	0	1	1	1	0	0	1	0	0	0	1	1	0	1
E	1	1	1	0	1	1	0	1	1	0	0	1	1	1	1	0
D	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	1

Number Conversion versus Coding

- Converting a decimal number into binary is done by repeated division (multiplication) by 2
- Coding a decimal number into its BCD code is done by replacing each decimal digit of the number by its equivalent 4 bit BCD code.
- Example: Converting $(13)_{10}$ into binary, we get 1101, coding the same number into BCD, we obtain 00010011.
- Exercise: Convert $(95)_{10}$ into its binary equivalent value and give its BCD code as well.
- Answer: $(1011111)_2$, and 10010101.

Character Storage

- Character sets
  - Standard ASCII: 7-bit character codes (0 – 127)
  - Extended ASCII: 8-bit character codes (0 – 255)
  - Unicode: 16-bit character codes (0 – 65,535)
  - Unicode standard represents a universal character set
    - Defines codes for characters used in all major languages
    - Used in Windows-XP: each character is encoded as 16 bits
    - Arabic codes: from 0600 to 06FF (hex)
  - UTF-8: variable-length encoding used in HTML
    - Encodes all Unicode characters
    - Uses 1 byte for ASCII, but multiple bytes for other characters

ASCII Codes

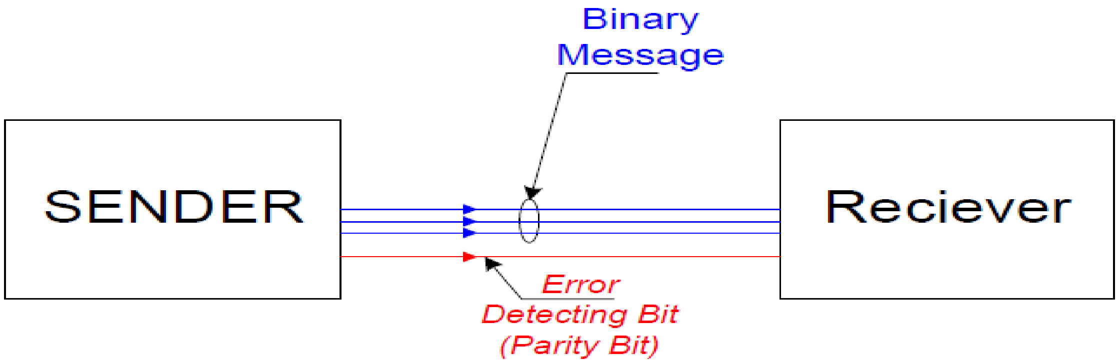
The Charcter set of the ASCII Code

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

- Examples:
  - ASCII code for space character = 20 (hex) = 32 (decimal)
  - ASCII code for ‘A' = 41 (hex) = 65 (decimal)
  - ASCII code for 'a' = 61 (hex) = 97 (decimal)

Error Detection

- Binary information may be transmitted through some communication medium, e.g. using wires or wireless media.
- A corrupted bit will have its value changed from 0 to 1 or vice versa.
- To be able to detect errors at the receiver end, the sender sends an extra bit (parity bit) with the original binary message.



Parity Bit

- A parity bit is an extra bit included with the n-bit binary message to make the total number of 1's in this message (including the parity bit) either odd or even.
- The 8th bit in the ASCII code is used as a parity bit.
- There are two ways for error checking:
  - Even Parity: Where the 8th bit is set such that the total number of 1s in the 8-bit code word is even.

P

0	1	0	0	0	0	0	1

  - Odd Parity: The 8th bit is set such that the total number of 1s in the 8-bit code word is odd.

P

1	1	0	0	0	0	0	1