SYS5160 Systems Integration
Assignment 1
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Question 1

We first convert the given differential equation into state space form-

$\dot{x}$ = Ax + Bu

y = Cx + Du

Where x is the state vector,

$\dot{x}$ is the State vector derivative

u is the input vector,

y is the output vector,

A,B,C,D are matrices

For the given differential equation,

$\frac{d^3 y}{dt^3} + 5\frac{d^2 y}{dt^2} + 17\frac{dy}{dt} + 11y = 13\frac{du}{dt} + 9u$

So the state variables will be given as $x_1 = y$, $x_2 = \frac{dy}{dt}$, $x_3 = \frac{d^2 y}{dt^2}$, $\dot{x} = \frac{d^3 y}{dt^2}$

and $\dot{x}_1 = x_2$ and $\dot{x}_2 = x_3$

therefore we can write the equation as,

$\dot{x} + 5x_3 + 17x_2 + 11x_1 = 13\frac{du}{dt} + 9u$

Now to get $\dot{x}$ we can rewrite the equation as :

$\dot{x} = -11x_1 + -17x_2 - 5x_3 + 13\frac{du}{dt} + 9u$

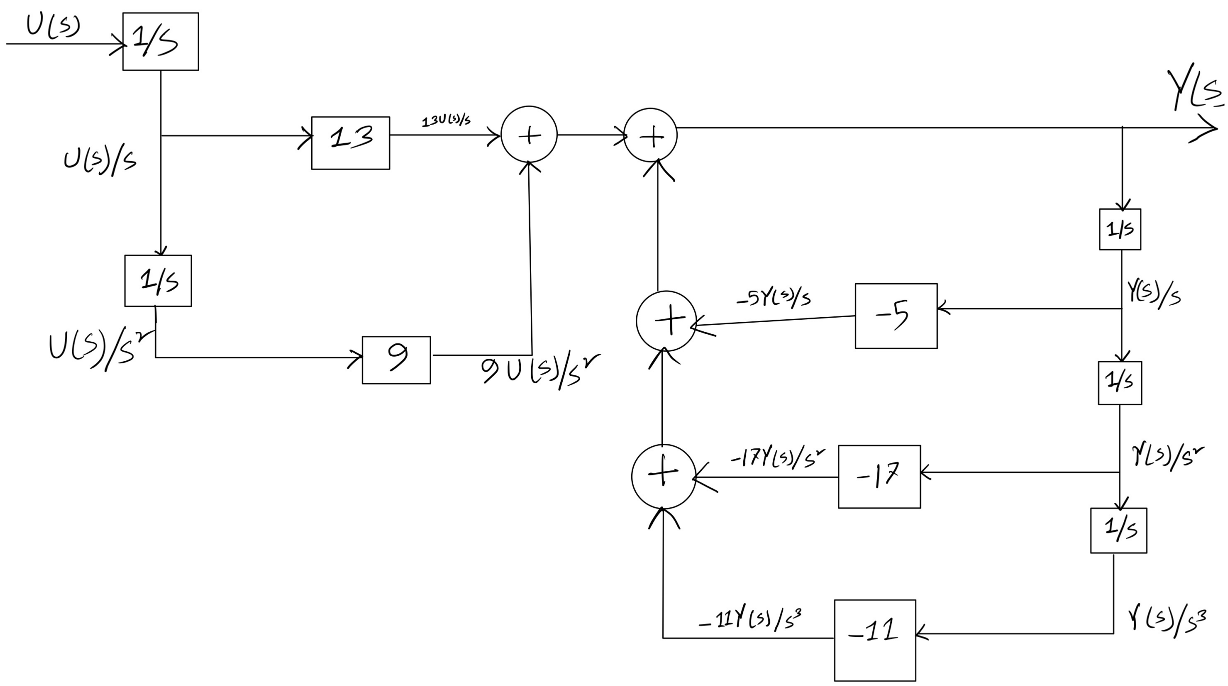
The Matrix form of Above equation is

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -11 & -17 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 13 \end{pmatrix} \frac{du}{dt} + \begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix} u$$

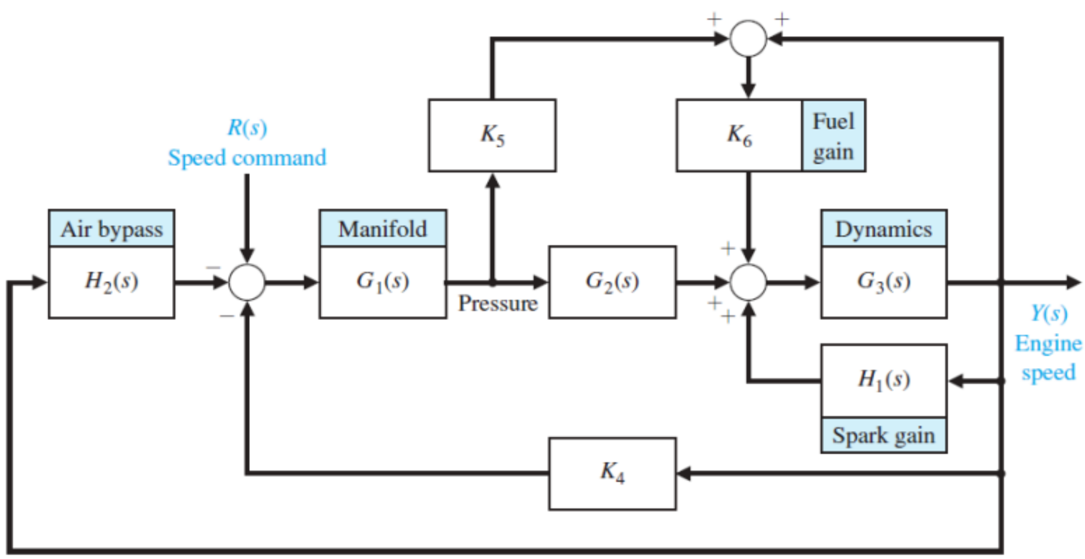
And the output equation is

$$y \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

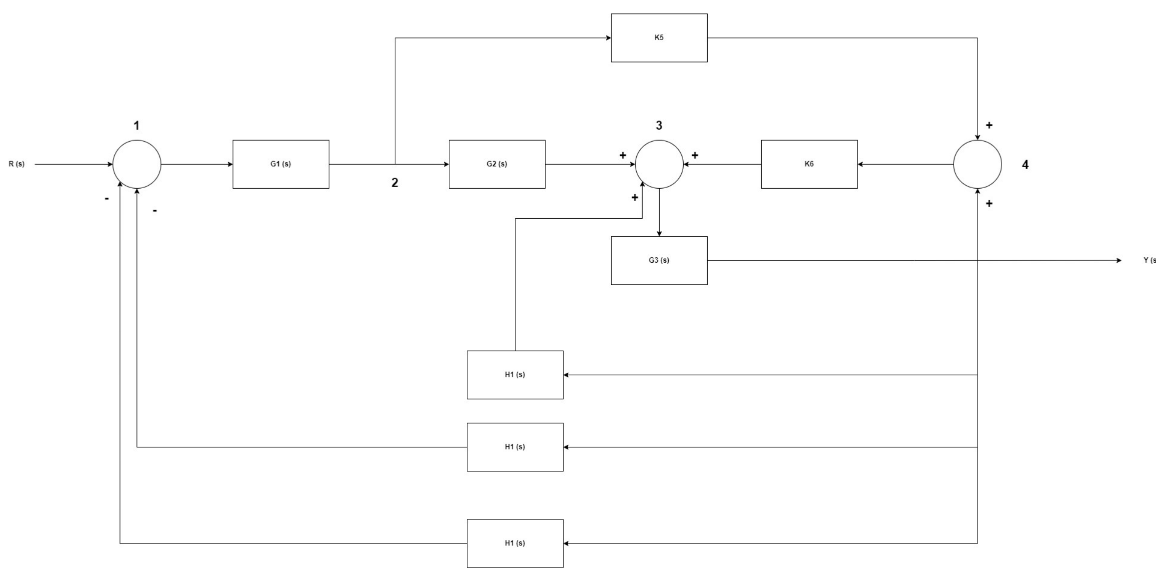
The final Output is :



Question 2



Simplified the diagram in following manner:



From above diagram we can calculate the gain of the forward path as follows:

$\text{Gain}_1 = G_1\,(s)\,G_2\,(s)\,G_3(s)$

$\text{Gain}_2 = G_1\,(s)\,K_5\,K_6\,G_3\,(s)$

Calculating the Individual loop gains as follows:

$\text{Loop}_1 = -G_1\,(s)\,G_2\,(s)\,G_3(s)\,K_4$

$\text{Loop}_2 = -G_1\,(s)\,G_2\,(s)\,G_3(s)\,H_2(s)$

$\text{Loop}_3 = G_3\,(s)\,H_1\,(s)$

$\text{Loop}_4 = G_3\,(s)\,K_6$

$\text{Loop}_5 = -G_1\,(s)\,K_5\,K_6\,G_3\,(s)\,K_4$

$\text{Loop}_6 = -G_1(s) K_5 K_6 G_3(s) H_2(s)$

Here, as previously demonstrated, we can observe that it excludes any touching loops.

In[∘]:= $\Delta = 1 - (\text{Loop}_1 + \text{Loop}_2 + \text{Loop}_3 + \text{Loop}_4 + \text{Loop}_5 + \text{Loop}_6)$

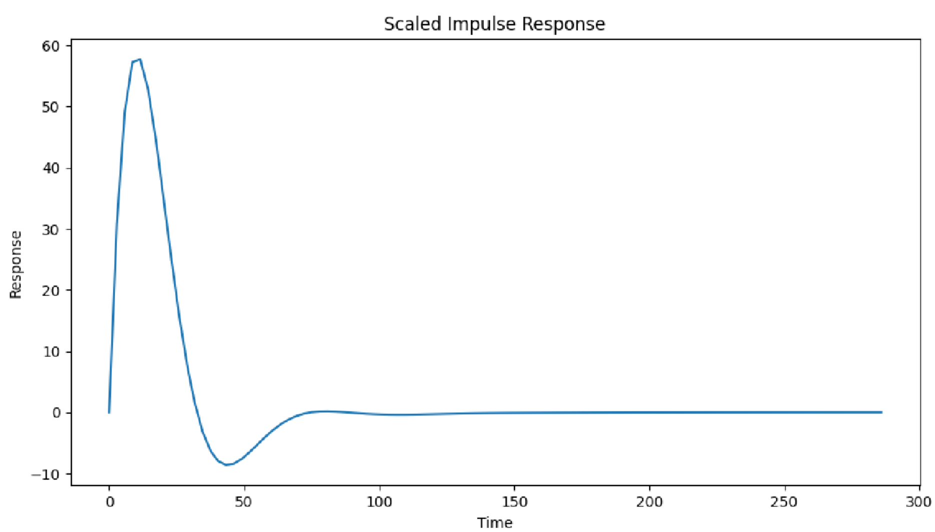
$\Delta_1 = 1$ for Gain_1 & $\Delta_2 = 1$ for Gain_2

$$\frac{Y(s)}{R(s)} = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta} = \frac{G_1(s) G_2(s) G_3(s) + G_1(s) K_5 K_6 G_3(s)}{1 + G_1(s) G_2(s) G_3(s) K_4 + G_1(s) G_2(s) G_3(s) H_2(s) - G_3(s) H_1(s) - G_3(s) K_6 + G_1(s) K_5 K_6 G_3(s) K_4 + G_1(s) K_5 K_6 G_3(s) H_2(s)}$$

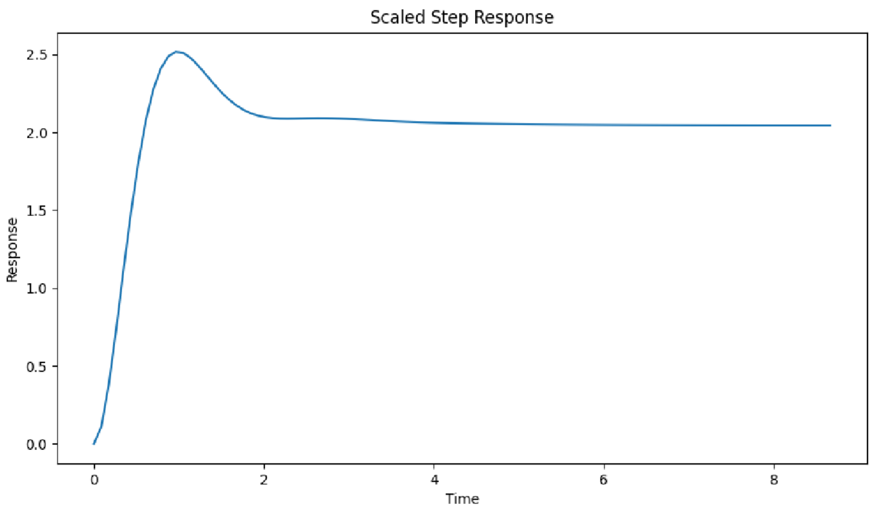
Question 3

The following responses were required -

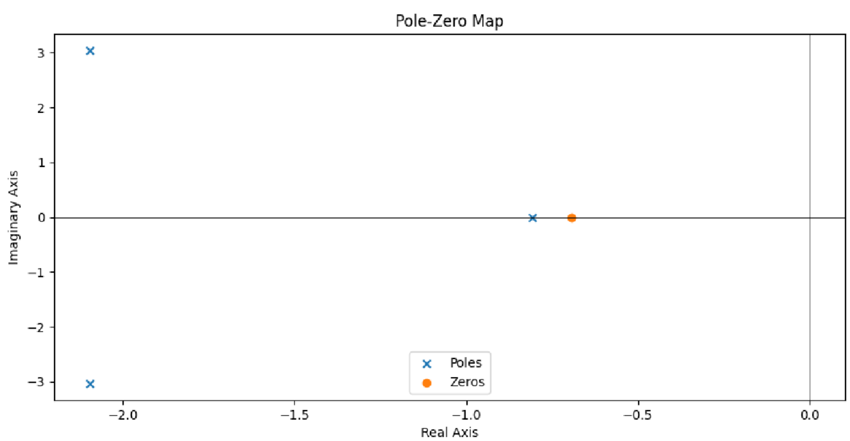
● **Response of the system when the system is subjected to an impulse of 33 units:-**



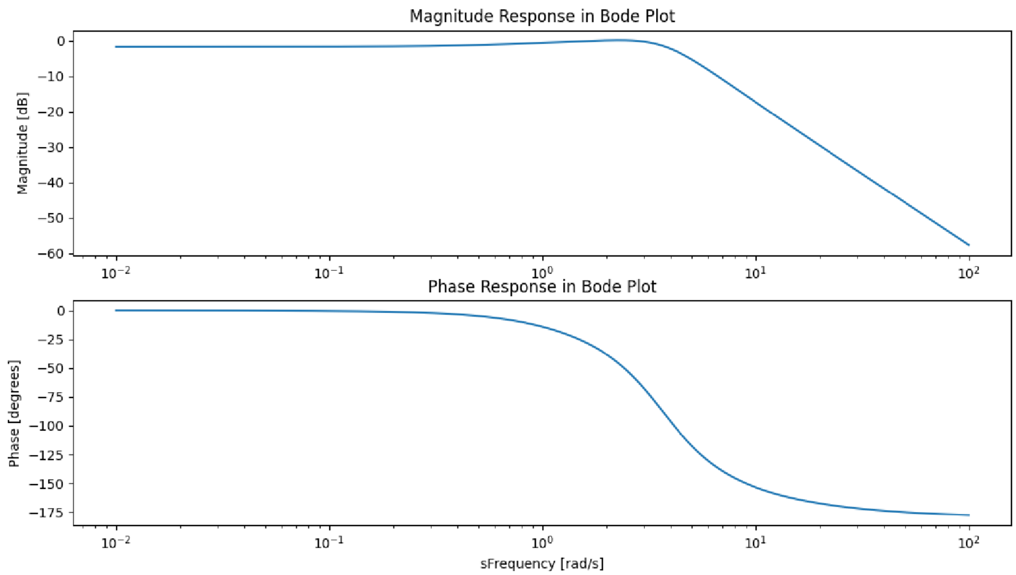
● **Step Response of the system when the system is subjected to step input of 2.5 units:-**



- **A pole-zero plot**



- **A Bode plot:**



Stopping search: maximum iterations reached --> 100

Optimized Parameters: [8.36953895 -8.4344072 -6.05079775 -6.83768027 -4.85971778 5.44292186]]

Final Objective Function Value: 0.0

The code is Available here on My **Github**