Sieve Problems

Veteran Track

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Problem 1: Counting Prime Divisors (Easy)

- Create a program to determine the number of prime factors for each number in the range $[1, N]$. Your program must run in $O(n \log n)$ or $O(n \log \log n)$.
- You may check you program's output against https://oeis.org/A001221. Click the link titled "list" for a table of values to test your program against ^^

Problem 2: Squarefree Numbers (Medium)

- Create a program to determine whether each number in the range $[1, N]$ is squarefree. Your program must run in $O(n \log n)$ or $O(n \log \log n)$.
- Note: A **squarefree** number contains no perfect square factors greater than 1. Note that 1 is squarefree.
- You may check you program's output against https://oeis.org/A008966. Click the link titled "list" for a table of values to test your program against ^^

Problem 3: Divisor Listing (Medium)

- Create a program that computes all divisors of all numbers in the range $[1, N]$.
- Specifically, your program will handle T test cases. For each test case, you are given a number n satisfying $1 \leq n \leq N$. You must print all the divisors of n.
- Your target complexity is $O(N \log N + T\tau\left(N\right))$, where $\tau\left(N\right)$ is the maximum number of divisors of any number in $[1, N]$.

Problem 4: Mobius Function (Difficult)

- Create a program that computes the value of the mobius function for all numbers in the range $[1, N]$ in $O(N \log N)$ time.
- Hint: You may have to use your solutions from previous problems to do this ^^

Problem 5: Simplified Fractions (Very Difficult; Optional but Recommended)

- Given two integers $1 \leq i, j \leq N$ (not necessarily distinct), what is the probability that the fraction $\frac{i}{j}$ is simplified?
- Output the result $\mod 10^9 + 7$. Note that $\frac{p}{q} \mod 10^9 + 7 \equiv pq^{-1} \mod 10^9 + 7$, where q^{-1} is the modular multiplicative inverse of q.
- Your goal is to find something linear or loglinear in N
- As an added hint: is there a technique that we could use from this week that could help here? ^^

Problem 5: Simplified Fractions (Very Difficult; Optional but Recommended)

- Here are the answers for the first few values of N:

N	**Answer**
1	1
2	750000006
3	777777784
4	187500002
5	320000003

Problem 5: Simplified Fractions (Very Difficult; Optional but Recommended)

- Here are the answers for the first few values of N:

N	**Answer**
6	638888894
7	714285720
8	46875001
9	790123463
10	910000007

Problem 5: Simplified Fractions (Very Difficult; Optional but Recommended)

- Here are the answers for the first few values of N:

N	**Answer**
30	816666673
100	55900001
300	436833337
1000	770231006
100000	927438557