Delaunay Triangulation Algorithm for Fingerprint Matching

Conference Paper · December 2006	
DOI: 10.1109/ISVD.2006.19 · Source: DBLP	
CITATIONS	READS
38	1,429

Delaunay Triangulation Algorithm for Fingerprint Matching

Chengfeng Wang and Marina L. Gavrilova

Department of Computer Science, University of Calgary, Calgary, AB, Canada

{marina, cwang}@cpsc.ucalgary.ca

Abstract. Fingerprint matching is one of the most important steps in fingerprint identification. This paper presents new results on fingerprint alignment and matching scheme based on the Delaunay Triangulation approach. In order to compare results with other matching techniques, we develop an efficient global matching scheme based on the comparisons of minutiae sets and singular points in the fingerprints. Alternative approaches for fingerprint matching were implemented to showcase proposed algorithm advantages. New experimental results provide a compelling evidence of a better performance and higher accuracy rates achieved by the Delaunay Triangulation based algorithm.

Keywords. Voronoi diagram, Delaunay triangulation, biometrics, fingerprint matching, minutiae set, ridge geometry, distance transform.

1. Voronoi Diagrams in Biometric

Voronoi diagram and Delaunay triangulation methods continue to receive compelling attention in the various areas of research, and most recently, in the area of biometrics. Within two years since the article "Computational Geometry and Biometrics: on the Path to Convergence" has appeared as part of International Workshop on Biometrics Technologies 2004 [9], the number of attempts to extract geometric information and apply topology to solve biometric problems has increased significantly. There has been research on application of topological methods, including Voronoi diagrams, for hand geometry detection, iris synthesis, signature recognition, face modeling and in fingerprint recognition [8, 16, 18, 19]. While the methodology is new, many questions on what is the best way to utilize this data structure, which topological information to use and in which context, how to make implementation decisions and whether the performance is comparable with other techniques used to solve these problems.

This paper attempts to address above questions on the example of Delaunay triangulation method for minutiae matching and singular-point comparison in the fingerprints. The method was implemented as part of the global fingerprint recognition system [14] and is sustainable under presence of elastic deformations [15]. The minutiae-matching algorithm, in which context the Delaunay triangulation approach is utilized, is based on classic fingerprint matching technique described in

[10]. Using Delaunay triangulation brings unique challenges and advantages. First of all, Delaunay edges rather than minutiae or whole minutiae triangles are selected as matching index which provides an easier way to compare two fingerprints. Secondly, the method is combined with deformation model, which helps to preserve consistency of the results under elastic finger deformations. Third, to improve matching performance, we introduce feature based on spatial relationship and geometric attribute of ridges, and combine it with information from both singular point sets and minutiae sets to increase matching precision.

2. Voronoi Diagram and Delaunay Triangulation of Minutiae Set

The use of geometric information in application to biometrics is a new area of research. The Voronoi diagram and Delaunay triangulation are two fundamental geometric structures that can be used to describe the topological structure of the fingerprint, which is considered to be the most consistent information for fingerprint matching purposes.

A Voronoi region associated with a feature is a set of points that are closer to that feature than to any other feature. Given a set S of points $p_1, p_2,...,p_n$, the Voronoi diagram decomposes the space into regions around each point such that all points in the region around p_i are closer to p_i than to any other point. Let V(S) be a Voronoi diagram of a planar point set S. Consider the straight-line dual D(S) of V(S), that is, the graph obtained by adding an edge between each pair of points in S whose Voronoi regions share an edge. The dual of an edge in V(S) is an edge in D(S). D(S) is a triangulation of the original point set, and is called the Delaunay triangulation. The Delaunay triangulation for a set of minutiae is depicted in Figure 2.

There are some compelling reasons why the Delaunay triangulation is a most naturally suitable data structure for the alignment of minutiae set. The Delaunay triangulation is uniquely identified by the set of points. Inserting or removing a new point typically has only local affect on the triangulation, which means the algorithm is tolerant to some imprecision of minutiae extraction technique. The Delaunay edges are on average shorter than edges connecting two randomly chosen minutiae, as the Delaunay triangulation is regular triangulation. Considering the elastic deformation of fingerprint, when the distance between two points in template image is increases, it becomes more challenging to match point pairs in the corresponding images. Introducing deformation tolerant method based on topological information seems as a logical and promising way to deal with the problem. Finally, use of additional topological information extracted from ridge geometry or based on singular points in addition to minutia set allows to significantly improve matching accuracy at the negligibly small computational overhead.

The outlined above ideas are described in the subsequent sections and were fully implemented in our fingerprint recognition software. The experimental results based on comparison of our system with the other traditional as well as recent methods are provided in experimental section and clearly demonstrate the advantages of the Delaunay-based techniques.

3. Relevant Research

The large number of approaches to fingerprint matching can be coarsely classified into three classes:

- Correlation-based matching.
- Minutiae-based matching.
- Ridge-feature-based matching.

In correlation-based matching, two fingerprint images are superimposed and the correlation between corresponding pixels is computed for different alignments [10]. During minutiae-based matching, the set of minutiae are extracted from the two fingerprints and stored as sets of points in the two dimensional plane [5, 6, 16]. Ridge-feature-based matching is based on such features as orientation map, ridge lines and ridge geometry [6] using the above techniques, minutiae-matching is most widely used approach that yields best matching results.

On the perpetual quest for perfection, a number of techniques devised for reducing FAR (False Acceptance Rate) and FRR (False rejection Rate) were developed; computational geometry being one of such techniques. Thus, Voronoi diagrams were utilized for face partitioning onto segments and facial feature extraction in [18]. Recently, method for binary fingerprint image denoising based on Distance Transform realized through Voronoi method was introduced in [8].

Bebis et. al. [1] used the Delaunay triangle as the comparing index in fingerprint matching. The method works under assumption that at least one corresponding triangle pair can be found between the input and template fingerprint images. Unfortunately, in real world situation this assumption might not hold due to low quality of fingerprint images, unsatisfactory performance of the feature extraction algorithm or distorted image. Another problem in fingerprint matching is noticeable when the distance between two points in template image is increases. It becomes much more challenging to match point pairs in the corresponding images. For instance, Kovacs-Vajna [7] have shown that local deformation of less than 10% can cause global deformation reaching 45% in edge length. In this paper, we discuss how to deal with shortcoming of the above methods using Delaunay triangulation technique and applying radial functions for deformation modeling.

4. Fingerprint Matching

The purpose of fingerprint identification is to determine whether two fingerprints are from the same finger or not. In order to do this, the input fingerprint needs to be aligned with the template fingerprint represented by its minutia pattern [5]. The following rigid transformation can be performed:

$$F_{s,\Delta\theta,\Delta x,\Delta y} \begin{pmatrix} x_{templ} \\ y_{templ} \end{pmatrix} = s \begin{pmatrix} \cos \Delta\theta & -\sin \Delta\theta \\ \sin \Delta\theta & \cos \Delta\theta \end{pmatrix} \begin{pmatrix} x_{input} \\ y_{input} \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$
(1)

where $(s, \Delta\theta, \Delta x, \Delta y)$ represent a set of rigid transformation parameters: (scale, rotation, translation). Under a simple affine transformation, a point can be transformed to its corresponding point after rotating $\Delta\theta$ and translating $(\Delta x, \Delta y)$. In our research, we assume that the scaling factor between input and template images is identical since both images are captured with the same device.

We divide our algorithm into three stages. During the first stage, identification of feature patterns utilizing Delaunay triangulation takes place. During the second step, Radial Basis Function (RBF) is applied to model the finger deformation and align images. Finally, global matching is employed to compute the combined matching score, using additional topological information extracted from ridge geometry (see Figure 1).

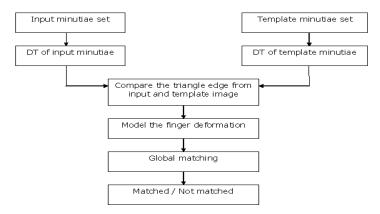


Figure 1. Flow chart of generic fingerprint identification system.

4.1 Delaunay Triangulation for Minutiae Set Alignment

Let $Q = ((x_1^Q, y_1^Q, \theta_1^Q, t_1^Q)...(x_n^Q, y_n^Q, \theta_n^Q, t_n^Q))$ denote the set of n minutiae in the input image ((x,y): location of minutiae; θ : orientation field of minutiae; t: minutiae type, end or bifurcation;) and $P = ((x_1^P, y_1^P, \theta_1^P, t_1^P)...(x_m^P, y_m^P, \theta_m^P, t_m^P))$ denote the set of m minutiae in template image.

Feature	Fields					
Minutiae Point	x (Y)	y (Y)	θ (Y)	Type (N)		
Triangle Edge	Length (N)	θ_{I} (N)	θ_2 (N)	<i>Type</i> ₁ (N)	$Type_2$ (N)	Ridge count (N)

Table 1. DT data structure used for comparing fingerprint images: Y: Dependent on fingerprint transformation; N: Independent of it.

Table 1 shows features that we can use in local and global matching of fingerprints. In the above table, *Length* is the length of edge; θ_1 is the angle between the edge and the orientation field at the first minutiae point; $Type_1$ denotes minutiae type of the first minutiae; $Ridge\ count$ is the number of ridges that these two minutiae points cross.

Using triangle edge as comparing index has many advantages. For local matching, we first compute the Delaunay triangulation of minutiae sets Q and P. Second, we use triangle edge as our comparing index. To compare two edges, Length, θ_1 , θ_2 , $Type_1$, $Type_2$, Ridgecount parameters are used. Note that these parameters are invariant of the translation and rotation (see Table 1). Conditions for determining whether two edges match identified as the set of linear inequalities depending on the above parameters and the specified thresholds (usually derived empirically depending on image size and quality). A sample set of such conditions can be found in [15]. If the threshold is selected successfully, transformation $(\Delta\theta, \Delta x, \Delta y)$ used to align the input and template images, is obtained.

Matching using Delaunay triangulation edge is realized as follows. If one edge from an input image matches two edges from the template image, we need to consider the triangulation to which this triangle edge belongs to and compare the triangle pair. For a certain range of translation and rotation dispersion, we detect the peak in the transformation space, and record transformations that are neighbors of the peak in the transformation space. Note that those recorded transformations are close to each other, but not identical. Also note that elastic deformations should be dealt with separately as such deformations can not be ignored. Figure 2 shows the successfully matched Delaunay triangle edge pairs. Algorithm 1 shows the pseudo code for local matching.

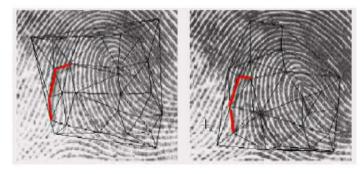


Figure 2. Delaunay triangle edges are matched, but not a single triangle is matched

Algorithm1 LocalMatching (TriangleEdgeQ,TriangleEdgeP) $1 \ k = 0;$ $2 \ for \ i = 1 : n$ // Delaunay triangle edge in Q $3 \ for \ j = 1 : m$ // Delaunay triangle edge in P $4 \ if \ TriangleEdgeQ(i)match \ TriangleEdgeP(j)$ $5 \ Calculate \ the \ transformation \ parameters \ (\theta,x,y)$ $6 \ and \ record \ this \ match \ as \ A[k];$

```
7 k++;
8 Discrete (θ,x,y) parameters into Hough space (a three dimension array Count);
9 For I = 1 : k
10 Obtain Hough space A[i]falls into, for example: (θ[m1],x[m2],y[m3]);
11 Count[m1][m2][m3]++;
12 Detect the maximum value in Hough space as (θm,xm,ym)
13 For i = 1 : k
14 if A[i]are round equal (θm,xm,ym)
15 Record A[i] as effective local matching;
```

4.2 Modeling Fingerprint Deformation

The deformation problem arises because of the inherent flexibility of the finger. Pressing or rolling a finger against a flat surface induces distortions which may vary from one impression to another. Such distortions lead to relative translations of features when comparing one fingerprint image with another. A good fingerprint identification system will always compensate for these deformations.

In [15], we proposed a framework aimed at quantifying and modeling the local, regional and global deformation of the fingerprint. The method is based on the use of Radial basic functions (RBF), that represent a practical solution to the problem of modeling of a deformable behavior. The application of RBF has been explored in medical image matching as well as in image morphing [3, 17]. For our fingerprint matching algorithm, deformation problem can be described as knowing the consistent transformations of specific *control* points from the minutiae set, and knowing how to interpolate the transformation of other minutiae which are not control points. We do not consider all the transformations obtained by the local matching. We vote for them in the transformation space and pick those consistent transformations which form large clusters.

In the following, we briefly describe the basic idea of RBF method. Given the coordinates of a set of corresponding points (control points) in two images: $\{(x_i, y_i), (u_i, v_i) : i = 1,...,n\}$, determine function f(x, y) with components $f_x(x, y)$ and $f_y(x, y)$ such that

$$u_i = f_x(x_i, y_i),$$

 $v_i = f_y(x_i, y_i), \quad i = 1, ..., n.$ (2)

Rigid transformation can be decomposed into a translation and a rotation (See Eq.(1)). In 2D, a simple affine transformation can represent rigid transformation as:

$$f_k(\vec{x}) = a_{1k} + a_{2k}x + a_{3k}y$$
 $k = 1,2$ (3)

where $\vec{x} = (x, y)$.

There are only three unknown coefficients in rigid transformation. Solution of this linear system can be found in [3, 17].

Fig.3 (c) shows the alignment of two images Fig.3 (a) and (b) when we consider the transformation of input image as a rigid transformation. We tried every transformation in rigid transformation space and found that the maximum number of

matching minutiae pairs between input image and template image is 6 (labeled by the big dashed circle in Fig. 3(c)). Circles denote minutiae of the input image after transformation, while squares denote minutiae of the template image. Knowing transformation of five minutiae (control points) in the input image, we apply the RBF to model the non rigid deformation which is visualized by the deformed grid in Fig.3 (d). The number of matching minutiae pairs is 10, which greatly increased the matching scores of these two corresponding images.

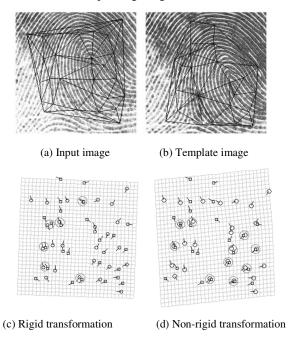


Figure 3. Comparison of rigid transformation and non-rigid transformation in fingerprint matching [15].

4.3 Global Matching

The purpose of global matching is to find the maximum number of matched features. After local matching procedure was performed and deformation model was applied to minutiae set, the numbers of paired and matched minutiae can be obtained. If two minutiae fall into the same tolerance box after identification, they are defined as paired in this paper. If paired minutiae have equal directions (within some tolerance), they become matched. So, now each minutia in both the template fingerprint and the input fingerprint is classified as paired, matched, paired but unmatched or unpaired. We propose our matching score:

$$M_{s1}(q, p) = \frac{100 \times m^r}{\sqrt{n'_q \times n'_p}}$$
 $M_{s2}(q, p) = \frac{100 \times m^r}{N_{pair}}$ (4)

where m^r , N_{pair} represent the number of matched and paired minutiae respectively, n_q' and n_p' are the number minutiae in the overlap area of input and template image respectively.

Even though the proposed formulas (4) is an improvement to the matching score in [5], how to utilize the information of matched minutiae is still not obvious and more research and experimentation should be devoted to this problem. To address the issue and get a more efficient matching algorithm, we resort to the singular points to validate our matching [13]. It is important to note that the minutiae matching and singular points validation method presented are statistically very robust. Even if some of the minutiae or singular points may not be extracted due to poor quality of image or not corrected matched, experiments show that the remaining minutiae are still sufficient in order to validate the identification. These minutiae or singular points may be matched and even validated, but they are statistically insignificant.

Optimization can be obtained is based on the number of singular points and their relationship, fingerprints are matched in singular points level, avoiding the time-consuming steps to extract minutiae if they are not of the same type. The sequential matching algorithm is presented in [15]. First singular points are extracted utilizing hierarchical block based method (see illustration of concept in Figure 4). Next, hierarchical matching algorithm utilizes two distinct sets of fingerprint information: singular points, and minutiae features.

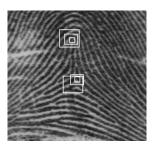


Figure 4. Singular point detection algorithm based on decreasing size blocks.

4.4 Using Ridge Geometry based on Distance Transform

We introduced a complete fingerprint matching systems in the above sections. However, additional features may bye used in conjunction with the minutiae to increase the system accuracy and robustness [see Table 3]. In this sub-section, we introduce a feature based on the spatial relationship and geometry attributes of ridge lines.

A two-dimensional binary image I of $M \times M$ pixels is a matrix of size $M \times M$ whose entries are 0 or 1. The pixel in a row i and column j is associated with the Cartesian coordinate (i, j). For a given distance function, the Euclidean distance transform of a binary image I is defined as an assignment to each background pixel (i, j) a value equal to the Euclidean distance between (i, j) and the closest feature pixel, i.e. a pixel having a value 1. The binary image is the thinning fingerprint image and it

can be transformed to a distance map to the feature points. To save the storage space and time, we only sample some points in the binary images.

Let m_{00} represent the minutiae (end or bifurcation), we can draw a vertical line perpendicular to its orientation field. Assume that the intersection with the binary image is m_{10} , m_{20} , m_{30} and m_{40} .

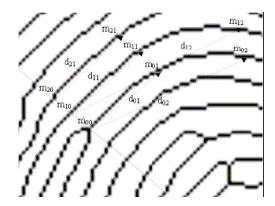


Figure 5. Minutiae and associated ridge.

Figure 5 shows the intersections with above associated ridge is m_{10} and m_{20} , but ignore the intersections m_{40} and m_{50} with below associated ridge. The local comprehensive fingerprint information for this minutiae can be a set:

$$M = \{M_0, M_1, M_2, M_3, M_4\}$$

$$M_i = \{d_{i1}, d_{i2}, d_{i3}, d_{i4}\} \ i = 0, ..., 4$$

Where M_i represent the associated *ith* ridge information. To add the curvature information of a associated *ith* ridge of minutiae m_{00} , we use m_1 , m_2 , m_3 and m_4 , the sampled points on the ridge close the m_{i0} and equally spaced from near to far, to represent the ridge. The local comprehensive fingerprint information contains the 5 associated ridges, and each ridge include the four distance from sampled points to the intersection.

Experiments show that the proposed approach increases the matching accuracy at the cost of some additional storage space for ridge. Application of new features (orientation field and ridge shape) in our matching algorithm decreases the false refused rate from 8% to 5%.

4.5 Complexity Estimates

Before showing that our algorithm performs very well in speed experimentally, theoretical analysis is performed first. We compare the computation complexity of our DT matching with the standard minutiae based matching algorithms. The overall complexity of the presented fingerprint identification system is $O(n^2)$ and is estimated as follows:

Local M	latching	Deformation tolerance		Global	Total
Delaunay	Alignment	Model	Deformation	matching	time
O(nlogn)	$O(n^2)$	$O(n^2)$	O(n)	$O(n^2)$	$O(n^2)$

Table 2. Running time estimates on the system performance

In comparison, computational complexity of standard minutiae extraction method [5] is O(n³) in Algorithm2:

Algorithm2 General minutiae matching (Minutiae Q, Minutiae P)

// minutiae in input image 1 for i = 1: nO(n)2 for j=1: m// minutiae in template image O(n)3 // register two images 4 assume minutiae i correspond minutiae j 5 get a group rigid transformation parameters 6 // compare two images compare two images under the transformation O(n)

Experimental results confirm the performance of the proposed system and are presented in the next section.

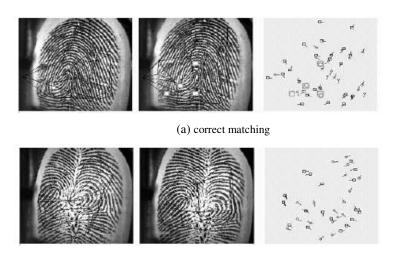
5. Experimental Analysis

The system was implemented using C++ programming environment and tested on a Pentium 4 2.8GHZ CPU, 512 RAM PC. Experiments have been performed on two fingerprint databases: Database1 is from Biometric System Laboratory at University of Bologna, which consists of 21x8 fingerprint images of 256x256 size; and on the publicly available database used in Fingerprint Verification Competition FVC2000 [4]. The captured fingerprint images vary in quality. The snap-shot of the system is presented below (Figure 6).



Figure 6. Fingerprint verification system snap-shot.

In the first series of experiments, each fingerprint image in the test set was matched with other images except itself. There are totally 168 ×167 matching. The FRR (False Refused Rate) is the complement to one of the identification rates, i.e. the number of corresponding image pairs which have not been matched with sufficient accuracy divided by the total number of corresponding image pairs. The FAR (False Acceptance Rate) is the number of non corresponding image pairs which are matched by the system as a percentage of the total number of non corresponding image pairs. Examples of correct, false positive and false negative matching are shown in the images below:



(b) incorrect not matching



(c) incorrect matching

Figure 7. The first and second columns are input and template images respectively. The third column is the results of registration of minutiae patterns and singular points

Performance of matching algorithm in comparison with traditional approach [5] is shown in Table 3. In the same database and minutiae set, our algorithm performs better both in accuracy and speed. Figure 8 shows some other results when we change some criterion threshold in the global matching (Genuine acceptance rate equals to 1-FRR).

	FRR	FAR	Time
Standard minutiae methods [5]	17.09%	0.84%	98 ms
Our DT matching	8.48%	0.18%	27ms
Our DT matching (with Ridge Geometry)	5.46%	0.19%	29ms

Table 3. FAR and FRR rates reported by DT matching algorithm

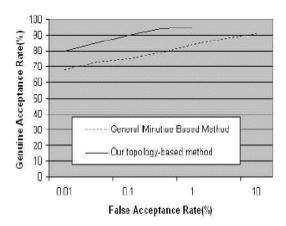


Figure 8. Acceptance Rate comparison of Delaunay triangulation based algorithm vs. traditional approach.

Note that these experimental results are significantly improved in comparison with preliminary report [15] as a result of using combination of minutia, singular points and ridge features. We can estimate the performance of our matching algorithm by comparing it with the general minutiae based matching algorithm. As it can be seen from Table 3, the performance of our method is very good in terms of time cost, and it is also very efficient in terms of a space, since we use the triangle edge as our comparing index. The application of RBF to model finger deformation and our improvement in global matching can greatly increase the matching accuracy.

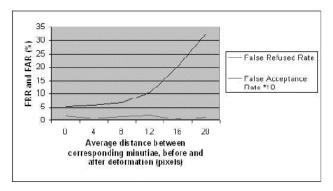


Figure 9. Performance matching algorithm in the presence of distortion.

We further conduct some experiments to test our matching algorithm's tolerance of fingerprint deformation, which will harm the accuracy of most fingerprint matching algorithms. We assume that the original images in test database are images without

non-linear distortion. In the plastic distortion model [2], there are three distinct regions in the fingerprint. A close-contact region does not allow any skin slippage. For minutiae points in other two regions, their locations will be changed. The average distance in Figure 9 represents the average distance change of minutiae points in pixels after we distort the images. When there is no distortion, the location of minutiae points does not change and the average distance is 0. Figure 9 shows that when the distortion is not very significant, i.e., average distance is less than 8 pixels, the accuracy of our matching algorithm remains almost the same. However, the FRR of the standard minutiae methods will increase in a much faster manner than ours. In other words, our algorithm is more robust against the non-linear distortion than the standard methods.

In the last series of experiments, we compared performance of the proposed method with the four most prominent research laboratories in the world that hold biannual fingerprint verification competition – FVC 2002 and 2004 [4]. Among the participants of, there are many renown researchers and organizations such as Biometric System Lab (BIOLAB), University of Bologna, Pattern Recognition and Image Processing Laboratory, Michigan State University and U.S. National Biometric Test Center, San Jose State University. We run our software on the provided database and found out very satisfactory results. For instance, the **average match time** (calculated as the average CPU time taken by a single match operation between a template and a fingerprint image) provided in the FVC 2002 summarizing Table 4 varies from 0.17 to 2.10s. Our performance was on average 0.21sec (0.18 for minutiae extraction [15], 0.03 for one matching). The EER is the value where FRR equals to FAR. Our EER on the second database of FVC 2002 is about 9.75%.

Algorithm	EER	EER*	REJ _{ENROLL}	REJ _{MATCH}	Avg Enroll	Avg Match	
					Time	Time	
Sag1	0.67%	0.67%	0.00%	0.00%	2.48 sec	0.96 sec	
Sag2	1.17%	1.17%	0.00%	0.00%	0.88 sec	0.88 sec	
Cetp	5.06%	5.06%	0.00%	0.00%	0.81 sec	0.89 sec	
Cwai	7.06%	4.27%	3.71%	3.90%	0.22 sec	0.32 sec	
Cspn	7.60%	7.60%	0.00%	0.00%	0.17 sec	0.17 sec	
Utwe	7.98%	7.98%	0.00%	0.00%	10.40 sec	2.10 sec	
Krdl	10.66%	7.35%	6.43%	6.59%	1.00 sec	1.06 sec	
Fpin	13.46%	13.46%	0.00%	0.00%	0.83 sec	0.87 sec	
Uinh	21.02%	20.65%	1.71%	5.08%	0.53 sec	0.56 sec	
Diti	23.63%	23.63%	0.00%	0.00%	0.65 sec	0.72 sec	
Nemi	49.11%	49.15%	0.00%	0.12%	1.13 sec	1.34 sec	
					·		

Table 4. Algorithm performance of FVC 2002 [4]

6. Conclusions

We have developed a novel minutiae matching algorithm based on Delaunay Triangulation approach and demonstrated that it outperforms traditional minutiae based methods. Delaunay triangle edge matching employed is fast and overcomes the non-linear deformation problems, because it is independent on the fingerprint rigid transformation. To overcome the deformations that are present in the fingerprint image pairs, we utilize RBF model that is capable of dealing with elastic distortions. We also introduced a new method for ridge detection based on distance transform. Compared with most methods that rely on rigid transformation assumptions, these modifications make presented system more robust and computationally efficient. Our experiments show that fingerprint images can be well matched using the minutiae based matching method, and that it is a fast, reliable and efficient technique.

Our future objectives is to consider new features based on fingerprint topology for augmenting matching technique and to concentrate on the fingerprint image enhancement algorithms in order to extract those features more accurately.

Acknowledgements

Authors would like to acknowledge generous support from CFI, NSERC and GEOIDE Canadian granting agencies.

References

- [1] Bebis, G., Deaconu, T and Georiopoulous, M. Fingerprint identification using Delaunay triangulation, ICIIS99, Maryland, Nov, pp. 452-459, 1999.
- [2] Cappelli R., Maio D. and Maltoni D., "Modelling Plastic Distortion in Fingerprint Images", ICAPR, LNCS 2013, pp. 369-376, 2001.
- [3] Fornefett, M., Rohr, K. and Stiehl, H. Radial basic functions with compact support for elastic registration of medical images, Image and Vision Computing, 19: pp. 87-96, 2001.
- [4] FVC2002: the Third International Fingerprint Verification Competition http://bias.csr.unibo.it/fvc2002/
- [5] Jain, A., Hong, L. and Bolle, R. On-line fingerprint verification, IEEE TPAMI, vol. 4, pp. 302–313, 1997.
- [6] Jiang, X., Yau, W.-Y, 2000. Fingerprint minutiae matching based on the local and global structures. In: Proc. 15th Internet. Conf. Pattern Recognition (ICPR, 2000) 2. pp.1042– 1045, 2000.
- [7] Kovacs-Vajna, Zs. Miklos A Fingerprint Verification System Based on Triangular Matching and Dynamic Time Warping, IEEE Trans. on PAMI, Vol.22, No.11, pp.1266-1276, 2000.
- [8] Liang, X. and Asano, T. A Linear Time Algorithm for Binary Fingerprint Image Denoising Using Distance Transform, IEICE TRANSACTIONS on Information and Systems, vol. E89-D, no. 4, pp. 1534-1542, 2006.
- [9] M. Gavrilova, Computational Geometry and Biometrics: on the path to convergence, in Proceedings of the International Workshop on Biometric Technologies 2004, Calgary, AB, Canada, pp. 131-138, 2004.
- [10] Ratha, N.K., Karu, K., Chen, S. and Jain, A. A Real-Time Matching System for Large Fingerprint Databases, PAMI Vol.18, No.8, pp. 799-813, 1996.
- [11]Ramo P., Tico M., Onnia, V., Saarinen, J. "Optimized singular point detection algorithm for fingerprint images", Proceedings. International Conference on Image Processing, vol.3, pp. 242 –245, 2001.
- [12] Senior, A.W., Bolle, R. Improved Fingerprint Matching by Distortion Removal, In IEICE Transactions on Information and Systems, special issue on biometrics. Volume E84-D No. 7, pp. 825-832, 2001.

- [13] Wang, C. and Gavrilova, M. A Multi-Resolution Approach to Singular Point Detection in Fingerprint Images, International Conference of Artificial Intelligence, I, pp. 506-511, 2004
- [14] Wang, C. and Gavrilova, M. An efficient algorithm for fingerprint matching, (submitted), ICPR 2006.
- [15] Wang, C. and Gavrilova, M., A Novel Topology-based Matching Algorithm for Fingerprint Recognition in the Presence of Elastic Distortions. International Conference on Computational Science and its Applications, LNCS, Springer-Verlag, vol.1, pp.748-757, 2005
- [16] Wayman, J, Jain. A, Maltoni, D. and Maio, D. "Biometric Systems: Technology, Design and Performance Evaluation," Book, Springer-Verlag, 2006.
- [17] Wirth, M.A., Choi, C. and Jennings, A. A nonrigid-body approach to matching mammograms, in IEEE 7th International Conference on Image Processing and its Applications, Manchester UK, pp.484-487, 1999.
- [18] Xiao, Y. and Yan, H. Facial Feature Location with Delaunay Triangulation/Voronoi Diagram Calculation, Conferences in Research and Practice in Information Technology, 11. Feng, D. D., Jin, J., Eades, P. and Yan, H., Eds., ACS. 103-108, 2002.
- [19] Yanushkevich, S., Gavrilova, M. and Shmerco, V. Editors, Proceedings of the International Workshop on Biometric Technologies 2004, Special Forum on Modeling and Simulation in Biometric Technologies, University of Calgary, Calgary, AB, Canada, 2004.