

# Introducing Kernel Method in In-Context Learning

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# What is In-Context Learning (ICL)?

**Definition:** In-Context Learning is the ability of a language model to learn a new task **using only examples inside the prompt**, with **no parameter updates**.

**Example:**

Good movie	→ Positive
Bad book	→ Negative
Amazing story	→ ?

The model infers the task (sentiment classification) directly from the context.

# What Problem Are We Solving?

- Language models (like GPT) learn from examples in the prompt.
- **Problems:**
  - Output changes depending on how examples are formatted.
  - Longer prompts have better performance and but higher computations.
  - Shorter prompts have poor performance and but lower computations.
- **Problem Statement:**
  - How can we improve the model performance while reducing the computational costs?
- **Solution:**
  - Introducing Iterative Vectors or IVs and Modifying the computation by kernel Method

# Sentiment Classification Task

## Few-Shot Prompt (In-Context Learning):

### Prompt Given to the Model

Review: I love this movie!

Sentiment: Positive

Review: The book was terrible.

Sentiment: Negative

Review: It was okay, not great.

Sentiment:

**Output:** Neutral

# Prompt Variation (ICL Problem)

## Reordered Few-Shot Prompt:

### Reordered Prompt

Review: The book was terrible.

Sentiment: Negative

Review: I love this movie!

Sentiment: Positive

Review: It was okay, not great.

Sentiment:

**Model Now Predicts:** Positive

**Same examples, same task — but different output.**

# Real Evidence of ICL Instability

## Order Sensitivity:

- *Webson Pavlick (2022)* and *Min et al. (2022)* show that:
  - Reordering examples can significantly change model outputs.
  - Accuracy drops without clear reason.

## Prompt Format Sensitivity:

- Changing "Review:" to "Comment:" or "Sentiment:" to "Label:" affects predictions.

## IVs Paper (Liu et al. 2025):

- Points out that ICL results are fragile due to:
  - Prompt structure
  - Demonstration ordering
  - Token formatting

# Key Insight — Attention = Gradient Descent

## First the model processes:

- Tokenization
- Embedding layer
- Transformer layers:
  - Self-Attention (token interactions), Feedforward Network, Residual + LayerNorm

**Attention(Attn) reads all examples before the query. It extracts the pattern relating input  $\rightarrow$  label.**

$$W = W_0 + \sum_{t=1}^T e_t \otimes x_t$$

is mathematically identical to:

$$\text{Attn}(X, E, q) = \sum_t (e_t \otimes x_t) q$$

Keys ( $x_t$ ) behave like training inputs, Values ( $e_t$ ) behave like gradients  
Query ( $q$ ) receives the update

# Why This Creates ICL

Because attention behaves like gradient descent:

- Demonstrations in the prompt act like **training examples**.
- Attention layers generate **meta-gradients**.
- These meta-gradients shift the hidden representations of the query.

$$F_{\text{ICL}}(q) = (W_{\text{ZSL}} + \Delta W_{\text{ICL}}) q$$

Where:

- $q$  = representation of the query token.
- $W_{\text{ZSL}}$  = zero-shot transformation (model's original behavior without examples).
- $\Delta W_{\text{ICL}}$  = meta-gradient update induced by the in-context demonstrations.
- $F_{\text{ICL}}(q)$  = final output representation after applying the in-context update.



# Extracting Meta-Gradients ( $\Delta Act$ )

We run two forward passes:

- **Few-shot** (with demonstrations)
- **Zero-shot** (without demonstrations)
- $Act_l^{\text{few-shot}}(x_i)$  **Activation function of few shot in layer l**
- $Act_l^{\text{zero-shot}}(x_i)$  **Activation function of zero shot in layer l**
- **) Activation function can be GELU, RELU, sigmoid, softmax**

Then compute:

$$\Delta Act_l(x_i) = Act_l^{\text{few-shot}}(x_i) - Act_l^{\text{zero-shot}}(x_q)$$

- Removes the baseline behavior.
- Isolates the ICL-specific update direction.
- Represents the **meta-gradient** at layer l.

# Two types of Averaging

## Averaging into Class Vectors

For each class  $j$ :

$$v_{j,l} = \frac{1}{|C_j|} \sum_{i \in C_j} \Delta \text{Act}_l(x_i)$$

- Produces a stable class-specific meta-gradient.
- Reduces noise from individual samples.
- One vector per class per layer.

## Episode-Averaged Task Vectors

Across many episodes:

$$V_l = \frac{1}{m} \sum_{i=1}^m V_l^i$$

- Aggregates meta-gradients across episodes.
- Produces a stable task-level update direction.
- Much more reliable than a single episode.

# Iterative Vectors (IVs) = Multi-Step Refinement

During extraction of batch  $i$ :

$$Act \leftarrow Act + \alpha_1 \cdot \bar{V}_{i-1}$$

**Where:**

- $Act$  = activation at the input-output separator token during extraction.
- $\alpha_1$  = extraction strength (how strongly previous vectors are applied).
- $\bar{V}_{i-1}$  = cumulative average of IVs from all previous batches:

$$\bar{V}_{i-1} = \frac{1}{i-1} \sum_{j=1}^{i-1} V_i^j$$

- $V_i^j$  = vectors extracted from batch  $i$  at layer  $l$ .

Final refined vectors:

$$V_l = \frac{1}{m} \sum_{i=1}^m V_i^j$$

**Where:**  $V_l$  = final Iterative Vector at layer  $l$ ,  $m$  = total number of extraction batches.

# Final Takeaway

(1) Demonstrations in prompt



(2) Attention processes them → gradient-like updates



(3) These updates change hidden activations (ICL)



(4)  $\Delta\text{Act} = (\text{few-shot} - \text{zero-shot})$  captures this update



(5) Averaging  $\Delta\text{Act}$  → class-specific meta-gradients



(6) Averaging across episodes → stable task meta-gradients



(7) Iterative refinement simulates gradient descent steps → IVs

## Model

- We have used pretrained language model:

GPT-J -6B

- **No gradient updates** anywhere.
- Only **forward pass** activations are used.

## Verbalizer

- Converts class labels into natural language tokens.
- Example:

Positive  $\rightarrow$  “positive”,    Negative  $\rightarrow$  “negative”

- Used to compute probabilities via answer-token logits.

# Extraction Process

- Run model on:

Few-shot prompt,    Zero-shot prompt

- Collect relevant activations at separator tokens.
- Compute deltas:

$$\Delta\text{Act} = \text{Act}_{FS} - \text{Act}_{ZS}$$

- Compute class-wise averages:

$$V_{\text{pos}}, V_{\text{neg}}, V_q$$

# Part 1: What is an Episode?

A 2-shot (2 examples per class), 2-way (Positive/Negative) classification task.

Episode prompt  $E$ :

great movie  $\rightarrow$  Positive

terrible book  $\rightarrow$  Negative

hated it  $\rightarrow$  Negative

amazing film  $\rightarrow$  Positive

loved that movie  $\rightarrow$ ?

Contains:

Positive: great movie, amazing film

Negative: terrible book, hated it

Query: loved that movie

Zero-shot version  $E_0$  only contains the query:

loved that movie



## Part 2: Collecting Activations

We run the model on:

- 1)  $E$ (Few-shot prompt)      2)  $E_0$ (Zero-shot prompt)

For each run, we collect attention activation at the separator token.

Stored activations:  $\text{Act}_I$ (Activation function of Few-shot examples)

$\text{Act}_I^0$ (Activation function of zero shot examples)

$\text{Act}_I$ (great movie)

$\text{Act}_I$ (terrible book)

$\text{Act}_I$ (hated it)

$\text{Act}_I$ (amazing film)

$\text{Act}_I$ (loved that movie)

$\text{Act}_I^0$ (loved that movie)

These are high-dimensional vectors (e.g., 768, 1024).

## Part 3: Computing $\Delta\text{Act}$

For every example  $x$  (including query):

$$\Delta\text{Act}_I(x) = \text{Act}_I(x) - \text{Act}_I^0(q)$$

This isolates the change caused by demonstrations.

For the episode, 5 vectors:

$\Delta\text{Act}(\text{great movie})$

$\Delta\text{Act}(\text{amazing film})$

$\Delta\text{Act}(\text{terrible book})$

$\Delta\text{Act}(\text{hated it})$

$\Delta\text{Act}(\text{loved that movie})$

Each represents the demo-induced shift.

## Part 4: Averaging by Class

Positive class (2 examples):

$$v_{\text{positive}}^I = \frac{1}{2} \left( \Delta\text{Act}(\text{great movie}) + \Delta\text{Act}(\text{amazing film}) \right)$$

Negative class:

$$v_{\text{negative}}^I = \frac{1}{2} \left( \Delta\text{Act}(\text{terrible book}) + \Delta\text{Act}(\text{hated it}) \right)$$

Query vector:

$$v_q^I = \Delta\text{Act}(\text{loved that movie})$$

Episode vector:

$$V_E^I = \{v_{\text{positive}}^I, v_{\text{negative}}^I, v_q^I\}$$

# Iterative Refinement Across Batches

## Task setup:

2-way: Positive / Negative, 2-shot per class, 1 query

10 episodes total, Batch size = 2

Batches:

$$B_1 = (E_1, E_2)$$

$$B_2 = (E_3, E_4)$$

$$B_3 = (E_5, E_6)$$

$$B_4 = (E_7, E_8)$$

$$B_5 = (E_9, E_{10})$$

Each episode provides:

$$V_E^I = \{v_{\text{pos}}^I, v_{\text{neg}}^I, v_q^I\}$$

*These vectors are derived from class-wise averages of  $\Delta\text{Act}$ .*

# Batch 1 (Episodes 1 and 2)

**Step 1: Extract  $V_1'$**

$$V_1' = \frac{1}{2} (V_{E_1}' + V_{E_2}')$$

No activation editing in Batch 1.

**Step 2: Running average**

$$\bar{V}_1' = V_1'$$

This is the first rough task vector.

## Batch 2 (Episodes 3 and 4)

**Activation editing before extraction:**

$$\text{Act}_{\text{edited}} = \text{Act} + \alpha_1 \cdot \bar{V}'_1$$

**Step 1: Extract  $V'_2$**

$$V'_2 = \frac{1}{2} \left( V'_{E_3} + V'_{E_4} \right)$$

**Step 2: Running average**

$$\bar{V}'_2 = \frac{1}{2} \left( V'_1 + V'_2 \right)$$

Now Batch 1 and Batch 2 contribute to the representation.

## Batch 3 (Episodes 5 and 6)

### Activation editing:

$$\text{Act}_{\text{edited}} = \text{Act} + \alpha_1 \cdot \bar{V}'_2$$

### Step 1: Extract $V'_3$

$$V'_3 = \frac{1}{2} (V'_{E_5} + V'_{E_6})$$

### Step 2: Running average

$$\bar{V}'_3 = \frac{1}{3} (V'_1 + V'_2 + V'_3)$$

Now 3 batches contribute to meta-learning.

## Batch 4 (Episodes 7 and 8)

### Activation editing:

$$\text{Act}_{\text{edited}} = \text{Act} + \alpha_1 \cdot \bar{V}_3'$$

### Step 1: Extract $V_4'$

$$V_4' = \frac{1}{2}(V_{E7}' + V_{E8}')$$

### Step 2: Running average

$$\bar{V}_4' = \frac{1}{4} (V_1' + V_2' + V_3' + V_4')$$

The representation becomes more refined.



## Batch 5 (Episodes 9 and 10)

### Activation editing:

$$\text{Act}_{\text{edited}} = \text{Act} + \alpha_1 \cdot \bar{V}_4^I$$

### Step 1: Extract $V_5^I$

$$V_5^I = \frac{1}{2} (V_{E_9}^I + V_{E_{10}}^I)$$

### Step 2: Running average

$$\bar{V}_5^I = \frac{1}{5} (V_1^I + V_2^I + V_3^I + V_4^I + V_5^I)$$

$\bar{V}_5^I$  is the final refined representation.

**Final IV for layer  $l$ :**

$$V_{IV}^l = \bar{V}_5^l$$

**Usage during inference:**

$$\text{Attn}_l(x) \leftarrow \text{Attn}_l(x) + \alpha_2 \cdot V_{IV}^l$$

# Summary

- Each episode  $\rightarrow \Delta \text{Act} \rightarrow (v_{\text{pos}}, v_{\text{neg}}, v_q)$ .
- Batch 1  $\rightarrow V^1$ .
- Batch 2  $\rightarrow V^2$  with activation editing.
- Batch 3  $\rightarrow V^3$  with editing.
- Batch 4  $\rightarrow V^4$  with editing.
- Batch 5  $\rightarrow V^5$  with editing.
- Final IV:

$$V_{\text{IV}} = \frac{1}{5}(V^1 + V^2 + V^3 + V^4 + V^5)$$

# Final Summary

- 1 Run few-shot & zero-shot  $\rightarrow$  collect activations.
- 2 Subtract  $\rightarrow \Delta\text{Act}$  vectors.
- 3 Average by class  $\rightarrow v_{pos}, v_{neg}, v_q$ .
- 4 Each episode gives  $V_E^I$ .
- 5 Process episodes in batches  $\rightarrow V_i^I$ .
- 6 Iterative refinement using running averages.
- 7 Final IV:

$$V_{IV}^I = \frac{1}{5} \sum_{i=1}^5 V_i^I$$

Result: A small set of vectors encoding the entire task.

# Introducing Batch level Kernel-Based Averaging

**Without kernel (paper default):**

$$V_i^l = \frac{1}{|B_i|} \sum_{E \in B_i} V_E^l$$

**To improve the performance ,My code replaces this with a kernel-weighted average:**

`ivs[l] = attention_aggregate(iv_list,  $\sigma = 0.2$ )(function used in improved code)`

instead of:

$$\text{ivs}[l] = \text{mean}(\text{iv\_list})$$

Thus, the aggregation becomes:

$$V_i^l = \text{KernelWeightedAverage}(V_E^l)$$

# What is `iv_list`?

Inside the function `attention_aggregate`, the input has shape:

$$\text{iv\_list} \in \mathbb{R}^{(N_{\text{episodes}}) \times (C+1) \times d}$$

For a 2-way task (Positive/Negative):

$$\text{iv\_list}[i][c] = \begin{cases} v_{\text{pos}}^{(i)} \\ v_{\text{neg}}^{(i)} \\ v_q^{(i)} \end{cases}$$

Therefore, it aggregates:

ALL episodes within a batch

for each of the class/query vectors.

# What the Kernel Function Does

The function:

`attention_aggregate(iv_list,  $\sigma$ )`

performs:

- 1 computes a reference vector (mean)
- 2 normalizes embeddings
- 3 computes cosine similarities
- 4 turns similarity into Gaussian RBF weights
- 5 normalizes these weights
- 6 returns a weighted average

Instead of:

simple mean

it computes:

RBF-weighted mean

# Step 1: Reference Vector and Step 2: L2 Normalization

## Reference Vector

$$\text{ref} = \text{mean}(\text{iv\_list}, \text{dim} = 0)$$

This yields one “central” vector per class:

$$\text{ref}[c] = \frac{1}{N} \sum_{i=1}^N \text{iv\_list}[i][c]$$

This is the anchor for similarity measurement.

## L2 Normalization

$$\text{iv\_norm} = \frac{\text{iv\_list}}{\|\text{iv\_list}\|} \quad \text{ref\_norm} = \frac{\text{ref}}{\|\text{ref}\|}$$

Purpose:

Cosine similarity becomes dot product



## Step 3: Cosine Similarity Computation

Per episode and per class:

$$\text{sim}_{ic}(\text{Cosine similarity}) = \text{iv\_norm}_{ic} \cdot \text{ref\_norm}_c$$

Shape:

$$\text{sim} \in \mathbb{R}^{N_{\text{episodes}} \times C}$$

Interpretation:

- $\text{sim} \approx 1 \rightarrow$  very similar episode
- $\text{sim} \approx 0 \rightarrow$  unrelated
- $\text{sim} < 0 \rightarrow$  contradictory or noisy episode

## Step 4: Gaussian RBF Weight

Weights computed as:

$$w_{ic} = \exp\left(-\frac{1 - \text{sim}_{ic}}{2\sigma^2}\right)$$

**If  $\text{sim} \approx 1$  (good episode):**

$$w_{ic} \approx 1$$

**If low similarity (noisy episode):**

$$w_{ic} \rightarrow 0$$

Thus, the kernel:

- boosts good episodes
- suppresses bad/noisy episodes

## Step 5: Normalize Weights

Per class  $c$ :

$$w_{ic} \leftarrow \frac{w_{ic}}{\sum_j w_{jc}}$$

This ensures:

$$\sum_i w_{ic} = 1$$

for each class  $c$ .

## Step 6: Weighted Aggregation (Final IV)

Final aggregated vector:

$$V_{\text{agg}}^I[c] = \sum_{i=1}^N w_{ic} \cdot \text{iv\_list}[i][c]$$

Or using tensor notation:

$$V_{\text{agg}}^I = \text{einsum} ("ic, icd \rightarrow cd", w, \text{iv\_list})$$

This replaces:

$$V_i^I = \text{mean}(V_E^I)$$

with a smarter, RBF-weighted version.

# Naive Averaging vs Weighted Averaging

**Naive averaging:**

$$V_i^I = \frac{1}{N} \left( V_{E_1}^I + V_{E_2}^I + \cdots + V_{E_N}^I \right)$$

**method Introduced (weighted averaging):**

$$V_i^I = w_1 V_{E_1}^I + w_2 V_{E_2}^I + \cdots + w_N V_{E_N}^I$$

# Final Intuition and Summary

## Simple averaging (old):

$$V = \frac{1}{N} \sum_i V_i$$

treats all episodes equally.

## Kernel averaging (in code):

$$V = \sum_i w_i V_i, \quad w_i \text{ from cosine + RBF}$$

- **Good episodes** get high weight
- **Noisy/poor episodes** get low weight
- Produces **smooth, stable, robust** IVs
- Mimics **attention mechanism**

## Super Simple Summary:

KernelAggregate = WeightedAverage(weights from cosine similarity)

# Example Setup + Original IV Issue

Toy 2D vectors (Positive class):

$$v_1 = [1, 1], \quad v_2 = [2, 2], \quad v_3 = [8, -2] \text{ (noisy)}$$

Simple mean (Original IV):

$$V_{\text{mean}} = \frac{1}{3}(v_1 + v_2 + v_3) = [3.67, 0.33]$$

**Problem:** Outlier  $v_3$  strongly biases the mean.

# Kernel Step 1–2: Reference + Cosine

Reference vector (same mean):

$$R = [3.67, 0.33]$$

Cosine similarity:

$$\text{sim}_1 = \text{sim}_2 \approx 0.71, \quad \text{sim}_3 \approx 0.28$$

Interpretation:

- $v_1, v_2 \rightarrow$  aligned with reference
- $v_3 \rightarrow$  wrong direction (noise)



## Kernel Step 3–4: RBF Weights + Batch 1 IV

RBF weights:

$$w_1 = w_2 \approx 0.49, \quad w_3 \approx 0.002$$

Kernel batch vector:

$$V_{\text{kernel}}^1 \approx [1.49, 1.47]$$

**Outlier suppressed → clean class signal.**

## Batch 2 + First Refinement

$$v_4 = [3, 3], \quad v_5 = [4, 4], \quad v_6 = [10, -5] \text{ (noisy)}$$

Batch 2 kernel result:

$$V^2 \approx [4.95, 4.91]$$

Refinement after 2 batches:

$$\bar{V}^2 = \frac{1}{2}(V^1 + V^2) = [3.22, 3.19]$$

This becomes the new editing vector for Batch 3.

## Batch 3 + Second Refinement

$$E_7 : v_7 = [2, -1]$$

$$E_8 : v_8 = [3, -2]$$

$$E_9 : v_9 = [20, -10] \quad (\text{outlier})$$

$$E_{10} : v_{10} = [-20, 10] \quad (\text{opposing outlier})$$

Batch 3 kernel result:

$$V^3 \approx [6.0, 5.8]$$

Refined across first 3 batches:

$$\bar{V}^3 = [4.15, 4.06]$$

# Final Batches + Summary

$$E_9(\text{Batch} - 4) : v_9 = [20, -10] \quad (\text{outlier})$$

$$E_{10}(\text{Batch} - 4) : v_{10} = [-20, 10] \quad (\text{opposing outlier})$$

$$V^4 = [7.1, 7.0], \quad V^5 = [6.8, 6.6]$$

Final Kernel-IV vector:

$$\bar{V}^5 = \frac{1}{5}(V^1 + V^2 + V^3 + V^4 + V^5) = [5.27, 5.16]$$

Batch	Kernel IV
1	[1.49, 1.47]
2	[4.95, 4.91]
3	[6.00, 5.80]
4	[7.10, 7.00]
5	[6.80, 6.60]
<b>Final</b>	<b>[5.27, 5.16]</b>

# Why Kernel-Based Averaging is Better Than Naive Averaging

## 1. Naive Averaging Treats All Episodes Equally

$$V_{\text{mean}}^i = \frac{1}{|B_i|} \sum_{E \in B_i} V^E$$

- noisy or contradictory episodes distort the mean
- no mechanism to prefer high-quality episodes
- direction of  $V^E$  is lost when outliers dominate

## 2. Kernel Method Uses Similarity-Based Weighting

$$V_c^i = \sum_E w_{E,c} V_c^E \quad w_{E,c} = \frac{\exp\left(-\frac{1-\cos(V_c^E, R_c)}{2\sigma^2}\right)}{\sum_{E'} \exp\left(-\frac{1-\cos(V_c^{E'}, R_c)}{2\sigma^2}\right)}$$

- episodes aligned with reference  $R_c$  get higher weight
- noisy/outlier episodes get very low weight
- acts as “attention over episodes”

# Naive Averaging vs Kernel Method (Results + Explanation)

## Performance Comparison

Metric	Naive Avg	Kernel Method	Gain
Micro F1	63.79	64.26	+0.47
Macro F1	59.97	61.04	+1.07
Weighted F1	59.84	60.96	+1.12

## Metrics Explanation

- **Micro F1:** Global F1 over all samples (majority-class heavy).
- **Macro F1:** F1 averaged per class (treats all classes equally).
- **Weighted F1:** F1 per class weighted by class size.

## Why Kernel is Better

- Naive averaging treats all episodes equally  $\rightarrow$  noisy episodes distort  $V^i$ .
- Kernel method uses similarity-based weights (cosine + RBF).
- Good episodes get high weight noisy episodes get very low weight.

# Problem Statement & How Kernel Methods Help

**Goal: Improve model performance in In-Context Learning while reducing computational cost.**

**Challenges:**

- Long few-shot prompts  $\rightarrow$  expensive attention ( $O(n^2)$ ).
- Iterative Vector extraction requires multiple forward passes.
- Memory + latency increase with number of examples.

**Question:** How do we obtain the performance benefits of ICL **without** the heavy computational overhead?

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## How Kernel Methods Help

Kernel methods offer a low-cost approximation of transformer behavior.






- Approximate attention using kernel feature maps (reduces  $O(n^2) \rightarrow O(n)$ ).
- Compute ICL meta-gradients without full forward passes.
- Kernel regression matches ICL behavior in hidden space.

# Future Improvements

- **Adaptive Kernel Bandwidth** Instead of a fixed  $\sigma$ , learn or tune  $\sigma$  dynamically per batch or per class.
- **Layer-Wise Weight Learning** Assign different strengths to IVs at different layers, instead of uniform  $\alpha$ .
- **Magnitude-Aware Weighting** Combine cosine similarity with vector magnitude to suppress large-magnitude outliers.
- **Better Episode Sampling** Use curriculum-based or difficulty-aware sampling instead of random episode selection.
- **Cross-Batch Attention** Replace running average with attention over all previous batches for richer refinement.
- **Task-Adaptive Strength ( $\alpha$ )** Learn or tune  $\alpha$  per task rather than using fixed values.
- **Robustness to Noisy Demonstrations** Explore more robust kernels (e.g., Laplacian, Cauchy) for episode weighting.



# References

-  Liu, Z., Li, X., Zhou, K., Leike, J. (2025). *Iterative Vectors: In-Context Gradient Steering without Backpropagation*. arXiv:2501.XXXX.
-  Webson, A. Pavlick, E. (2022). *Do Prompt-Based Models Really Understand the Task?* arXiv:2109.01247.
-  Bishop, C. (2006). *Pattern Recognition and Machine Learning*. Springer. (Gaussian RBF Kernel Definition)
-  ArkciaTheDragon. (2024). *Iterative Vectors GitHub Repository*. <https://github.com/ArkciaTheDragon/iterative-vectors>
-  Vaswani, A. et al. (2017). *Attention is All You Need*. NeurIPS.