Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
A		2010736

Fish migration model based on seawater temperature field and cellular automaton

Summary

Nowadays, the rise of global ocean temperature caused the migration of Scottish herring and mackerel. We conducted a study on the relation between temperature change and migration of Scottish herring and mackerel, and put forward proposals for fishery companies to adjust their fishing strategies.

To solve the 1st problem, we use a seawater stratification model to reduce the high-dimensional temperature field equation by one dimension in space, set up the field equation and fit. Based on the characteristic that the fish always migrate to the place with the most suitable temperature for them, we set up cellular automata to simulate the process that organisms actively choose the migration route, and finally give the most likely location in the next 50 years.

In view of the 2nd problem, we established the profit function of small fishing companies based on the distribution of the fish groups. Then we take the maximum range of fishing boats as the feasible region and the profit function as the objective function, respectively calculate the maximum and minimum profit of each year, and give the best and worst cases And the time between them.

For problem 3, we consider two strategies: moving fishery companies and replacing vessels with small fishing boats with refrigeration and fresh-keeping functions. According to these two strategies, we modify the profit function, use the method in question 2 to find the maximum profit, and give the specific relocation location and ship replacement plan.

In question 4, we add the factor of territorial sea, divide the sea area, reduce the feasible region, and find the optimal solution of profit function and the fishing position with the maximum profit in our territorial sea under the condition of adopting the optimization strategy in question 3.

For task 5, based on our models, we wrote an article about the adverse effects of fish migration caused by temperature changing on fisheries for the magazine to explain the severity of the problem and put forward corresponding improvement strategies.

In addition, in the model optimization, we gave a more accurate and stable temperature field model. We use partial differential equation to establish a four-dimensional model about temperature, time and space.

Keywords: Cellular Automata, Sea Stratification Model, Temperature Field, Fish Migration Route, Nonlinear Programming, Partial Differential Equation

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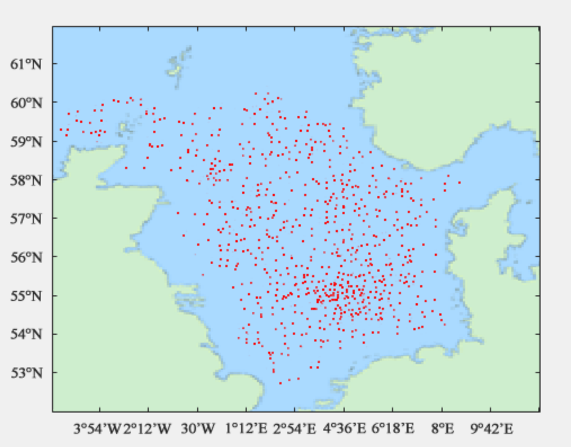
Fishermen's measures

Confronted with fish moving north in North Sea

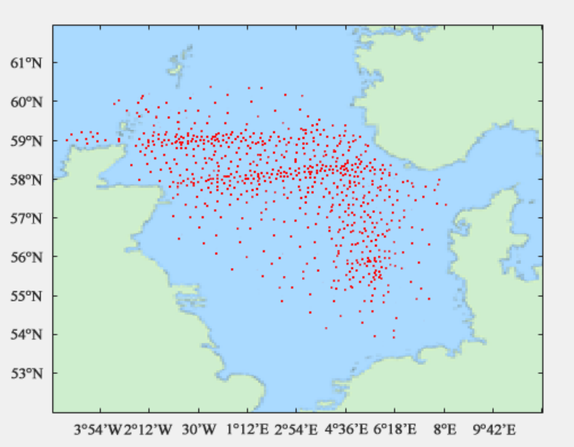
The earth is becoming warming at an unprecedented rate, especially the species living in the north and south poles are more aware of the habitat destruction caused by the temperature rise. In early 2020, researchers recorded for the first time in Antarctica 20.75 ℃ (69.35 ℉). Many people around the world are panicking about global warming trends, and the Arctic is warming much faster than Antarctica. Climate change poses potential threats to biodiversity and ecosystems.

The North Sea fishing ground, located in the northeast of the Atlantic Ocean, is one of the world's four largest fishing grounds. Scientists from the University of East Anglia in the UK, Allison L. Perry, and others investigated the impact of climate change on the distribution of fish in the North Sea, an important ecosystem. It was found that in the past 25 years, many fish species in the North Sea have a clear tendency to move north. Many important fishes of commercial value, such as cod turtle, tooth cod, angler, etc., have migrated 500-800 km northward. These fish responded significantly to the rising sea temperature caused by climate warming, which had a great impact on the development of fisheries around the North Sea.

We chose to use the fisheries in Scotland for forecasting. From NOAA / ESRL / PSD Climate Data Repository, we obtained the global average monthly ocean surface temperature of the earth from 1980 to 2009, and based on this to predict the ocean temperature from 2020 to 2070. It is initially known that the temperature in the North Sea region has growth trend. After obtaining the distribution of herring and mackerel populations in the North Sea, we used cellular automata to launch fish swarms for computer simulation. We can see that the fish swarms migrated northward and away from the coastline, and gradually moved to the ocean temperature of 9 ℃ ~ 12 ℃ in the sea area.



Herring Cluster distribution prediction in 2035



Herring Cluster distribution prediction in 2070

This change brings economic uncertainty to small fisheries companies densely distributed on the coastline. This threat is manifested in the following aspects:

1. Many herring and mackerel are distributed in shallow seas of 20m ~ 200m. The separation from the coastline increases the difficulty of fishing for fishermen. The small fishing boats have limited fuel and cannot support long-distance navigation. This will inevitably increase the difficulty of fishing for herring and mackerel and reduce its Catches.

2. The transportation cost of fishing vessels is increasing, and most small fishing vessels have a limited carrying capacity. The freshness and high quality of the catches cannot be guaranteed without refrigeration equipment, which causes the phenomenon of rising costs but decreasing profits.

3. The North Sea region has a temperate marine climate, and it is also in the extreme north-south position. Cyclones are frequent, annual rainfall is large, and storms often occur in winter. If fishing boats go to deep sea areas for fishing, weather risks may cause hidden safety hazards.

4. If global warming is not controlled, herring and mackerel populations may migrate to countries such as Norway and Iceland. When fish clusters are distributed in the territorial waters of certain countries, it will be detrimental to the balanced development of fisheries in the countries around the North Sea. The fisheries of the countries located in the south of the sea area will even be severely hit in the future.

In this regard, it is necessary for the countries in the southern part of the North Sea to take measures to ensure the healthy development of local fisheries. They need to follow a precautionary idea and mainly transform small-scale fisheries companies that depend on offshore fisheries, including the hardware capacity of fishing vessels, and the preservation of goods, or any other hardware facilities.

Fisheries companies with a better economic foundation can adopt the method of common development of size and size, purchasing large vessels with a length of more than 50m and a main engine power of 400 kilowatts or more to take charge of fishing in deep waters, while small fishing vessels can be equipped with refrigeration facilities or drying facilities. Dry facilities to ensure the needs of voyages in the middle and long distances, thereby reducing the damage to the fishery economy caused by fish migration due to global warming.

1 Introductions

1.1 Problem Background

Nowadays, global warming is a tough problem. Recently, the temperature in Antarctica has even exceeded 20 ℃. According to relevant research, the temperature of seawater is rising year by year. The rise of global ocean temperature affects the habitat quality of some marine organisms. When the temperature changes too much for them to continue to reproduce, these species will migrate to other habitats more suitable for their current and future life and reproduction success. This migration of marine life seriously disrupts the livelihoods of companies that depend on the stability of marine life species. For example, herring and mackerel in Scotland contribute a lot to the economy of Scottish fisheries. Their migration will greatly affect Scottish fisheries and even the survival of some small fishing companies. As a consultant to the North Atlantic Fisheries Management consortium in Scotland, we conducted a study on the relationship between temperature change and migration of herring and mackerel in Scotland. We set up a mathematical model to predict the position of the fish group, and analyze the prediction results, and give the economic proposal of the fishing company to adjust the fishing strategy.

1.2 Problem Restatement

- Obtain data on changes in sea surface temperature over a long enough period of times, build a mathematical model to predict the distribution of herring and mackerel in the waters around Scotland in the next 50 years through analysis.
- Based on the rate of change in seawater temperature, it is predicted that stock migration will cause the best and worst conditions and time intervals for the development of small fisheries companies, until the fish migration distance is too long to catch.
- Determining whether small fishing companies should change their operations and prove the judgment, and help to develop practical strategies if changes are needed.
- Consider the sovereignty of neighboring countries and explain the impact of fish migration into the territorial waters of other countries on the previous proposal.
- Write an article for Hook Line and Sinker magazine to help fishermen understand the seriousness of the problem and propose improvement plan to them.

2 Assumptions

- The migration of herring and mackerel in Scotland is mainly affected by seawater temperature.
- Scottish herring and mackerel only move in the sea area with a depth of 20-200 meters.
- The most suitable temperatures for herring and mackerel were as follows: 12℃ and 10℃.
- Each sea area can only hold a certain number of fish.
- When fish migrate, they always move towards the place with proper temperature.

3 Notations

We use the nomenclature in Table 1.

Table 2.Notations

Symbol	Definition
G_i	Gradient of layer i
T	Temperature
Z	Depth
$T(z)$	Sea temperature at a certain depth
n	Total layers
i	Layers
r_i	Relative gradient
G_{max}	Maximum gradient value
SST	Sea surface temperature
MLD	Depth of the mixed layer
UL	Upper boundary of the thermocline
LL	Lower boundary of the thermocline
G_{mld}	Temperature gradient of the mixed layer

4 The Model

4.1 Requirement 1

4.1.1 Problem 1-Analysis

In question one, we build a temperature field model based on the seawater stratification model and nonlinear fitting of multivariate functions. According to this, we use cellular automata to simulate the migration route of fish groups. We mainly considered the impact of sea water temperature on the migration of Scottish herring and mackerel, so we need to establish a temperature field in a specific studied area.

We obtained the temperature data of the sea water surface, and then use a temperature stratification model based on the relative gradient method to get the temperature of the sea water at different depths. Because herring and mackerel live in the depth of 20-200m, this depth cannot reach the thermocline, and the temperature is basically the same within 20-200m. Therefore, the temperature field in three-dimensional space can be approximately reduced to two-dimensional. When we add the time factor into the equation, the temperature field evolves into a ternary function of latitude, longitude and time. According to the image of the temperature field, we make a hypothesis about the equation of the temperature field, and use the data of different years to fit the temperature field in a multi-element and non-linear way, and finally successfully establish the temperature field of the sea water. Using the temperature field equation, we have made the prediction of the sea water temperature change in the next 50 years. After obtaining the temperature field, we considered the influence of temperature on the migration of Scottish herring and mackerel. We got that the most suitable temperature for herring is 12℃ and for mackerel is 10℃. We knew that when fish migrate, they always tend to move towards the most appropriate temperature point, that is, they will choose their own migration route. We simulated this choice. We refined the bit grid of the studied area, get the corresponding temperature of each dot, and subtracted the temperature from the optimum temperature of herring and mackerel respectively, and took the absolute value, then gave the value to the dot respectively. We used points to represent fish. We put these points in the sea area near Scotland in the picture. In the process of migration, fish swarm along the network. When fish go to the next node, they will choose the one with minimum value. In the process of migration, the fish will not stop until they reach the optimal position. Each sea area can only hold a certain number of fish. When the number of fish at a certain point reaches this the number cannot be increased any more, and the rest of the fish can only move towards the secondary best node. Through continuous iteration, the position of a certain year can be determined. As the temperature field changes with time, the temperature difference of each point will change constantly. We simulate the migration of fish in many years and get the position of fish in different years.

4.1.2 Model establishment and solution

First of all, we need to establish the temperature field model of seawater in the study area. We get the temperature data of seawater surface in different periods. In order to build the model, we need to use the surface temperature to calculate the sea water temperature at different depths.

According to the characteristics of the vertical structure of sea water temperature, scholars at home and abroad have carried out the research of using sea surface information to retrieve the vertical structure of sea water temperature in the 1990s. At present, the commonly used inversion method is: segment fitting method, that is, according to the characteristics of each layer, the ocean is divided into 2-3 from top to bottom, and different equations are used for fitting. There are three main methods of seawater stratification: temperature difference method, gradient method and curvature method. The delamination standard we choose is based on the relative gradient method or gradient ratio method. The reason is that the relative gradient method can not only overcome the shortcoming that curvature method is easy to be affected by data error to some extent, but also be more objective than temperature difference method and gradient method. [1]

The gradient calculation adopts unequal distance difference method:

$$G_i = (\frac{dT}{dz})_i = a_i\left(\frac{T_i - T_{i-1}}{z_i - z_{i-1}}\right) + (1 - a_i)\left(\frac{T_{i+1} - T_i}{z_{i+1} - z_i}\right), (i = 2,3,4, \dots, n-1)$$

In the above formula: G_i is the gradient of layer i; $\alpha_i = \frac{z_{i+1} - z_i}{z_{i+1} - z_{i-1}}$ is the weight; T is the temperature; z is the depth; n is the total number of layers; i is the number of layers. The gradient of the surface layer and the lower layer respectively adopts the formula of front difference and back difference:

$$(\frac{dT}{dz})_n = \frac{T_n - T_{n-1}}{z_n - z_{n-1}}$$

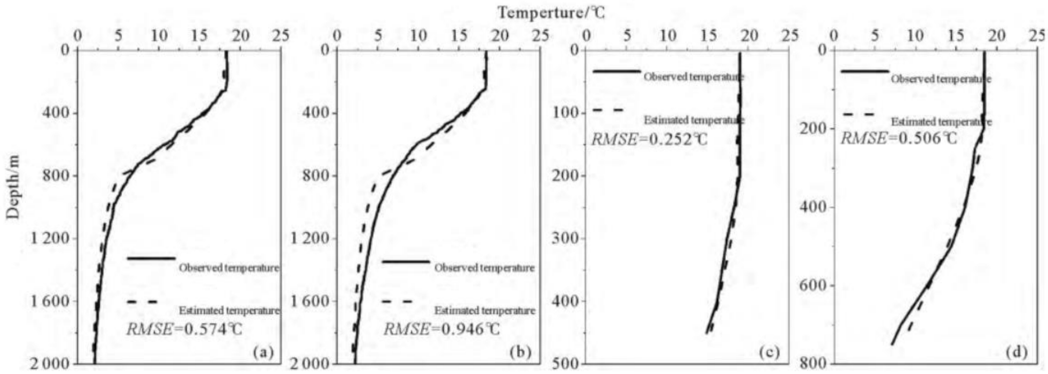
The formula for calculating the relative gradient r of each depth of the profile is as follows:

$$r_i = \frac{G_i}{G_{max}}$$

In the above formula: r_i is the relative gradient; G_{max} is the maximum gradient. The relative gradient of 0.25 and 0.5 can be used to judge the depth of mixing layer and thermocline layer objectively. We divide the ocean into mixed layer region, thermocline region and deep ocean region. The vertical temperature profile equation of seawater can be expressed as follows:

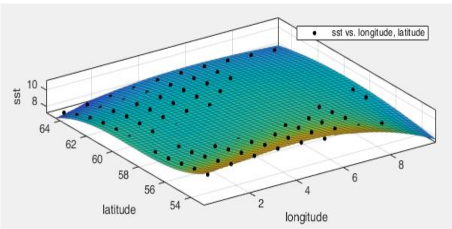
$$\begin{cases} T(z) = SST - G_{mld}z; 0 < z < MLD \\ T(z) = SST\left(ae^{-\frac{z-ULD}{LL-UL}} + be^{-\frac{z-ULD}{LL-UL}} + c\right); z_{UL} < z < z_{LL} \\ T(z) = SST\left(Ae^{B\frac{z}{MaxDepth}} + Ce^{D\frac{z}{MaxDepth}}\right); z > z_{LL} \end{cases}$$

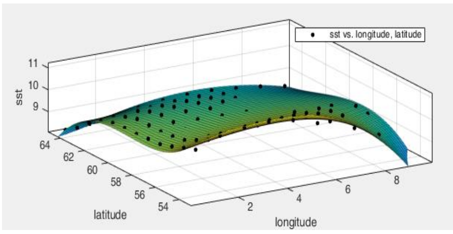
In the above formula: T(z)represents the sea water temperature of a certain depth; SST represents the sea surface temperature; MLD represents the depth of the mixed layer; UL represents the upper boundary of the thermocline; LL represents the lower boundary of the thermocline; G_{mld} represents the temperature gradient of the mixed layer; A, B, C, D represents the fitting coefficient of the deep ocean; MaxDepth represents the maximum depth of the profile (2000m here); a, b, c represents the fitting coefficient. Linear interpolation method is used between layers. According to the formula, we can get the relation between sea surface temperature and temperature at different depths.

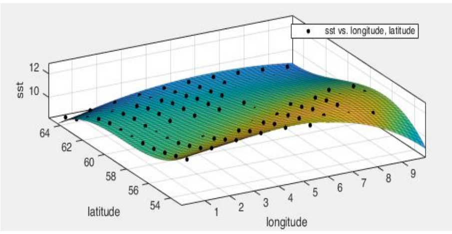


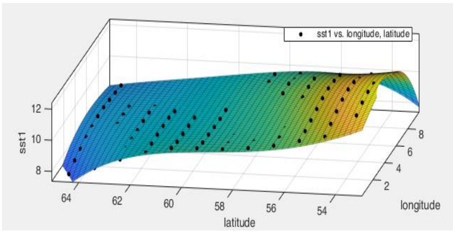
We found that the depth of herring and mackerel live in (20~200m) did not reach the thermocline, and the water temperature remained basically the same between 20 and 200m.In other words, we can reduce the temperature field equation to two-dimensional in space.

We fitted the temperature field equations of seawater in different years (1970, 1990, 2015, 2018) and got the following results:









Because the temperature of seawater changes with time, we must take the time factor into account, and turn the temperature field equation into a ternary function of latitude, longitude and time: $\mathrm{u} = \mathrm{f}(\mathrm{x}, \mathrm{y}, \mathrm{t})$

We assumed that the equation of temperature field as a ternary quartic equation:

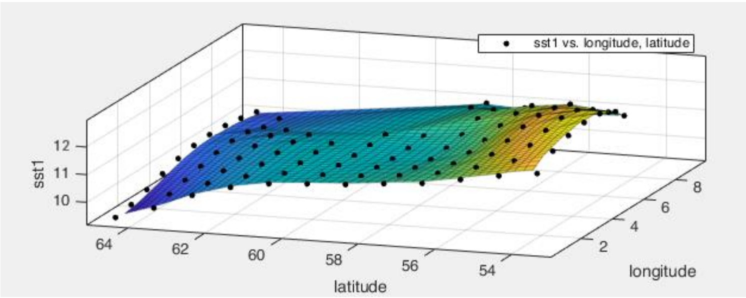
$$
\begin{aligned}
\mathrm{u} = {} & p_{000} + p_{100}x + p_{010}y + p_{001}t + p_{200}x^2 + p_{020}y^2 + p_{002}t^2 + p_{110}xy + p_{011}yt \\
& + p_{101}xt + p_{210}x^2y + p_{201}x^2t + p_{120}xy^2 + p_{102}xt^2 + p_{012}yt^2 \\
& + p_{021}y^2t + p_{111}xyt + p_{310}x^3y + p_{301}x^3t + p_{130}xy^3 + p_{103}xt^3 \\
& + p_{013}yt^3 + p_{031}y^3t + p_{112}xyt^2 + p_{121}xy^2\mathrm{t} + p_{211}x^2yt \\
& + p_{220}x^2y^2 + p_{202}x^2t^2 + p_{022}y^2t^2 + p_{400}x^4 + p_{040}y^4 + p_{004}t^4
\end{aligned}
$$

Then the data of different years are used for multivariate nonlinear fitting, and the specific values of each coefficient are obtained.

Parameter estimates

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
p000	-384.773	p011	-12.415	p111	.007	p121	5.943E-9
p100	-14.561	p101	29.816	p310	.000	p211	-2.808E-6
p010	98.347	p210	.011	p301	1.035E-6	p220	-.252
p001	46.751	p201	.000	p103	2.570E-6	p202	-.019
p200	-.500	p120	-.220	p130	.001	p022	.004
p020	9.706	p102	-.015	p013	-1.096E-6	p400	.001
p002	-.181	p012	.007	p031	5.086E-5	p040	.000
p110	6.246	p021	-.009	p112	-1.710E-6	p004	7.684E-9

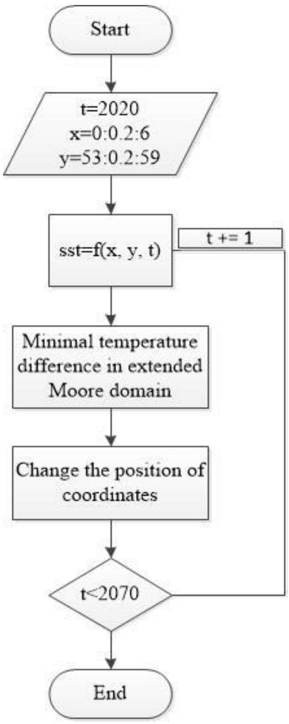
According to this result, we predicted the seawater temperature field in each year for the next 50 years.



According to this result, we use cellular automata to simulate fish migration.

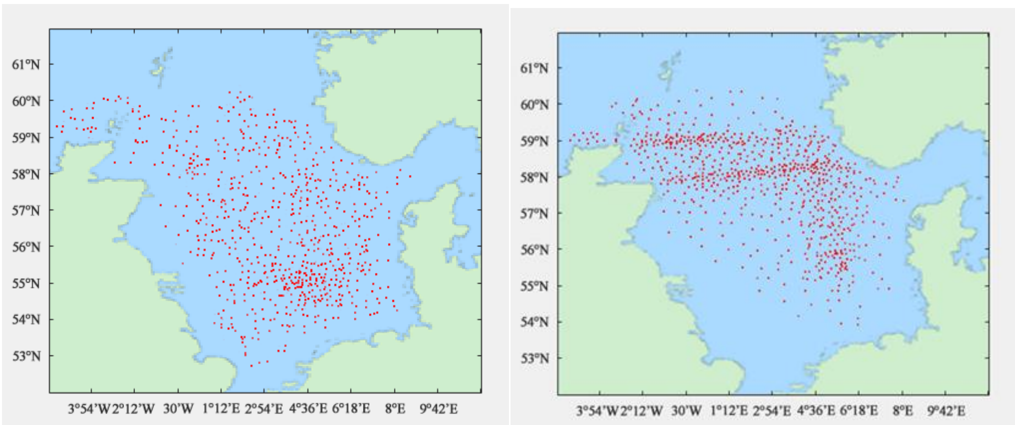
Cellular automata is a kind of grid dynamic model with discrete time, space and state. The spatial interaction and time causality are local. It has the ability to simulate the evolution process of complex system in time and space.

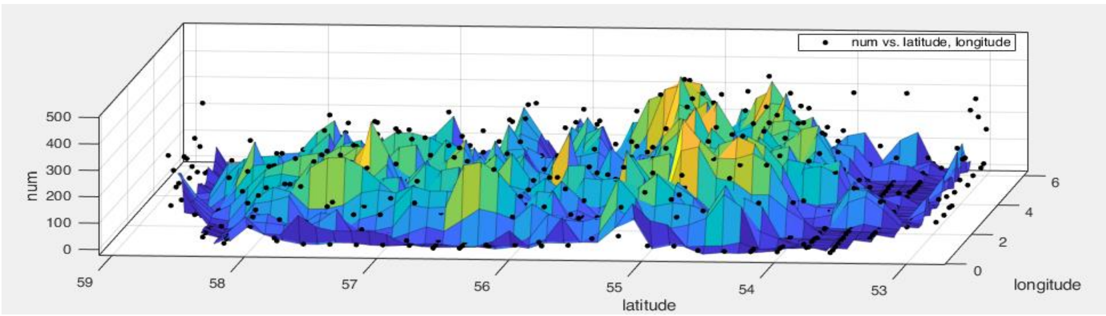
The core of cellular automata model for fish migration is the optimal location selection and automatic location update.

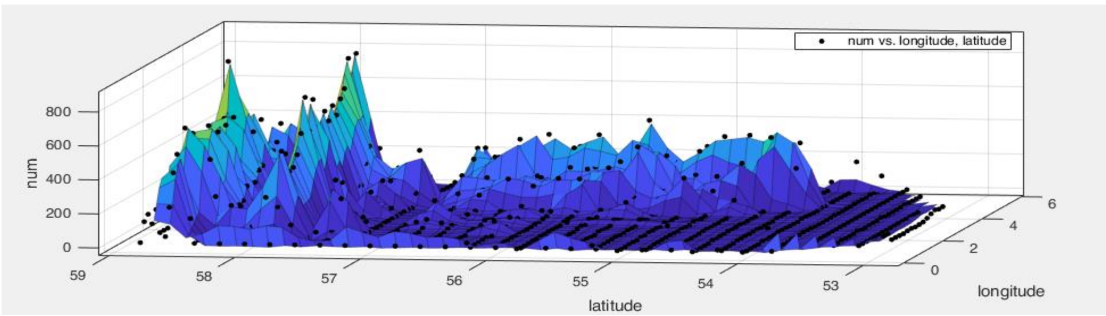


By looking up the data, we get that the most suitable temperature for herring is 12℃ and for mackerel is 10℃. We knew that when fish migrate, they always tend to move towards the most appropriate temperature point, that is, they will choose their own migration route. We simulated this choice. We refined the bit grid of the studied area, get the corresponding temperature of each dot, and subtracted the temperature from the optimum temperature of herring and mackerel respectively, and took the absolute value, then gave the value to the dot respectively. We used points to represent fish. We put these points in the sea area near Scotland in the picture. In the process of migration, fish swarm along the network. When fish go to the next node, they will choose the one with minimum value. In the process of migration, the fish will not stop until they reach the optimal position. Each sea area can only hold a certain number of fish. When the number of fish at a certain point reaches this the number cannot be increased any more, and the rest of the fish can only move towards the secondary best node. Through continuous iteration, the position of a certain year can be determined. As the temperature field changes with time, the temperature difference of each point will change constantly.

We simulated the migration of fish in each year for the next 50 years and get the position of fish in different years.







In the above four pictures, the first and second pictures are the distribution of fish groups on the map, in which the red dot represents fish groups, and the denser the red dot represents the denser and more fish groups, and the rarer the red dot represents the less fish groups; the third and fourth pictures are pictures with longitude and latitude as independent variables and fish groups as dependent variables, and the height of the picture represents the number of fish, and the higher the figure is, the higher the value is Large, more fish.

Through the analysis, we can see that the fish are densely distributed around the coastline from the beginning, and scattered in the middle of the sea area. With the passage of time, the temperature field changes, and the fish begin to move away from the coastline towards the high latitude sea area. According to the distribution image of fish groups in each year in the next 50 years, we predict that the most likely position of fish groups is between 57 °N and 59 °N.

4.2 Requirement 2

4.2.1 Problem 2 Assumption

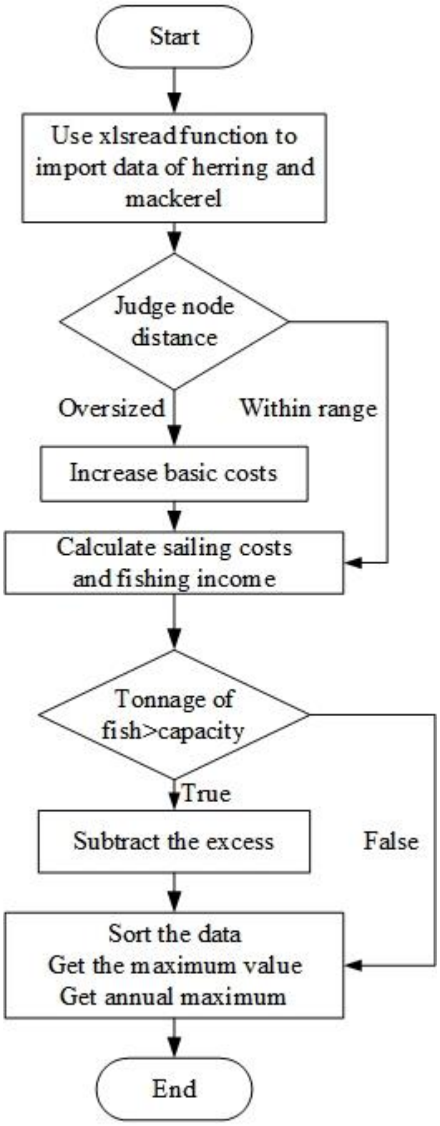
- Fishing vessels with limited fuel and navigation range can only be fished in the studied sea area.
- Fishing vessels have limited carrying capacity and can only catch once at a time. Small fishing companies choose to put out to sea when the sea is in good condition, and small fishing vessels can travel along the straight line to the designated location.
- Small fishing company's fishing boat needs to make basic preparation (mainly labor, maintenance, etc.) every time it goes out to sea, which costs M pounds, and it costs D pounds to sail every kilometer (mainly fuel, etc.).
- Small fishing companies have 4 vessels of the same type, all of which can be fished normally at sea. Their fishing range is within the radius of R meters. Four ships can no longer fish in the same places.
- The number of herring and mackerel caught by small fishing vessels is directly proportional to the number of fish in the fishing range.
- The income of small fishing companies is directly proportional to the number of herring and mackerel caught by fishing boats.

4.2.2 Problem 2-Analysis

In Question One, we have obtained a map of the fish population distribution in the studied area. Based on our assumptions, we can establish the cost function $C(x,y)$, the fishing quantity function $n(x,y)$, and the income function $I(x,y)$, and then we can calculate the profit function $P(x,y)$. The profit function $P(x,y)$ is taken as the objective function and the studied area as the feasible region. Find the best solution of the profit function $P(x,y)$ for each year. In order to accomplish this mission, we refine the graticules and disperse the continuous data. Then calculate the profit of each point by programming. The next step is rank the data and find the best and worst case, and calculate the time interval between them.

4.2.3 Model establishment and solution

We assumed that the small fishing company is located at Peterhead port. Peterhead port is the busiest port in Scotland. Setting the studied fishing company located at Scotland's busiest port will provide sufficient universality, authenticity and credibility to predict changes in earnings for most small fishing companies. The location coordinate of this port is (x_0, y_0).



We assumed that the longitude and latitude of the fishing vessel's position at sea is (x, y) and starting from Peterhead port.

Since we establish the coordinate system in longitude and latitude, we cannot use Euclidean geometry distance formula to calculate the distance. We use the distance function in MATLAB to calculate the distance (Unit: km):

$$d(x, y) = \frac{6371\pi}{180} distance(x, y)$$

Because small fishing companies need basic preparation (mainly labor, maintenance, etc.) for each voyage, which costs M=550 pounds and D=1.65 pounds for each voyage (mainly fuel, etc.), then the cost of this voyage is:

$$C(x,y) = 2 \times D \times d(x,y) + M = 2 \times D \times \frac{6371\pi}{180} distance(x,y) + M$$

As the number of herring and mackerel caught by fishing vessels is proportional to the number of fish in the fishing range. The number of herring in the range is $H(x,y)$. The number of mackerel is $m(x,y)$. a is the fishing coefficient. Take a=0.2. The number of herring and mackerel caught is:

$$n_h(x,y) = a \times h(x,y)$$
$$n_m(x,y) = a \times m(x,y)$$

Since the income of small fishing companies is directly proportional to the number of herring and mackerel caught by fishing boats, the income of this fishing is:

$$I(x,y) = k_1(x,y) \times n_h(x,y) + k_2(x,y) \times n_m(x,y)$$
$$= k_1(x,y) \times a \times h(x,y) + k_2(x,y) \times b \times m(x,y)$$

Because the freshness of the fish will decrease with the increase of the distance when the voyage distance exceeds L $(L = 360\text{km})$, and the price of the fish will also decrease accordingly, then:

If the distance from the fishing site to the departure site is less than L, K is a constant:

$$\begin{cases} k_1(x,y) = k_{10} \\ k_2(x,y) = k_{20} \end{cases}$$

If the distance exceeds L, the price of the fish will also decrease:

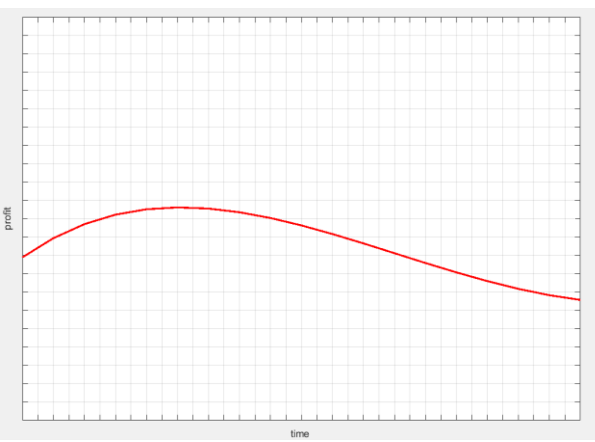
$$\begin{cases} k_1(x,y) = k_{10} - 0.1[d(x,y) - L] \\ k_2(x,y) = k_{20} - 0.15[d(x,y) - L] \end{cases}$$

In summary, the total profits of the four ships on the voyage are:

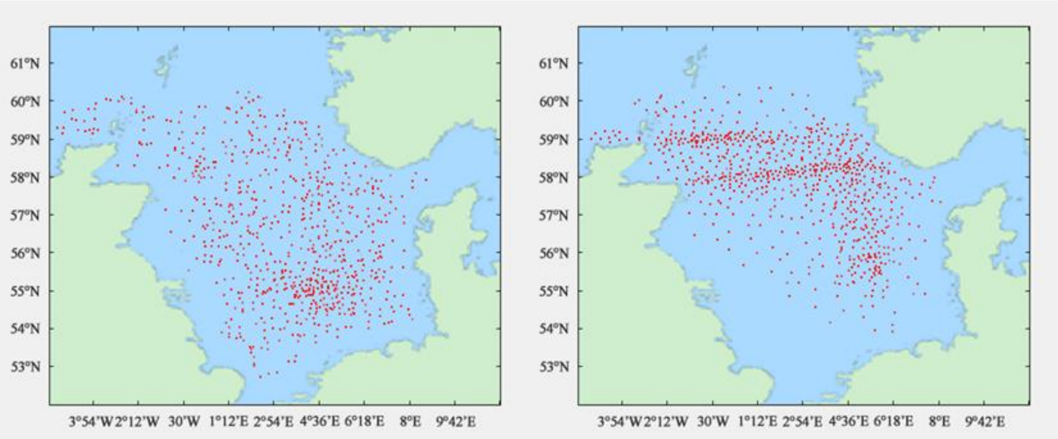
$$P = \sum_{n=1}^{4} [I(x_n,y_n) - C(x_n,y_n)]$$

$$= \sum_{n=1}^{4} [ak_1(x_n,y_n)g(x_n,y_n) + ak_2(x_n,y_n)m(x_n,y_n) - 2 \times D \times \frac{6371\pi}{180} distance(x,y) + M]$$

The profit function $P(x,y)$ is taken as the objective function and the studied area as the feasible region. Find the best solution of the profit function $P(x,y)$ for each year.



The picture above shows the optimal solution of profit function changing with the year. We get the best situation (the maximum optimal solution of profit function) in 2035 and the worst situation (the minimum optimal solution of profit function) in 2070. The following is the distribution of fish in these two years:



According to the analysis of fish distribution map, because the fish migration is controlled by the sea water temperature, the temperature increases with the time migration, the fish have to move away from the coastline, towards the high latitude sea area to find the suitable temperature habitat. Because of the migration of fish, the distance from fishing place to port first decreases and then increases, the cost first decreases and then increases, so the profit first increases and then decreases.

The best is in 2035 and the worst is in 2070, 35 years apart.

4.3 Requirement 3

4.3.1 Problem 3 Assumption

- In order to obtain more profits, small fishing companies may migrate based on the predicted results of the migration of fish stocks to enable them to fish in the areas with the most dense stocks.
- Data show that the cost of replacing a vessel with a small fishing boat with fresh-keeping function is S pound and the price is relatively expensive.
- The maximum number of fish that can be transported by the small fishing boat being replaced with is m. The basic cost of each trip is M' and D' for each kilometer.

4.3.2 Problem 3-Analysis

In order to compensate for the loss caused by the migration, we consider the following two optimization strategies to increase the profits of small fishing company: (1) replacing some original fishing vessels with small fishing vessels with preservation function to reduce the loss caused by the decrease of freshness; (2) relocating small fishing company to the sites where fish groups gather to reduce Cost of shipping. For the first strategy, we reconstruct the profit function, take the number of new fishing vessels x as the variable, and use the method in model 2 to find the optimal solution of the profit function in the feasible region, and find the corresponding x when the maximum profit occurs. For the second proposal, we take the coordinates of the relocation location as variables to find the optimal solution of the profit function and find the coordinates of the relocation location when we get the maximum profit.

4.3.3 Model establishment and solution

First of all, we considered the situation of small fishing company replacing some vessels with small fishing boat.

Assume that the com replace x fishing boats with small ones.

So the profits function becomes:

$$P' = \sum_{n=1}^{x}[I_s(x_n, y_n) - C_s(x_n, y_n)] + \sum_{n=1}^{4-x}[I(x_n, y_n) - C(x_n, y_n)] - xS$$

In the above formula:

$$\begin{aligned} & I_s(x_n, y_n) - C_s(x_n, y_n) \\ &= ak_{10}g(x_n, y_n) + ak_{20}m(x_n, y_n) - D \times distance(x_n, y_n) - M - xS \end{aligned}$$

Using the same solution method in model 2, the optimal solution of total profit is obtained.

The results show that the cost of replacing the ship is too large, and the benefits brought by the preservation function of small fishing boats are not enough to make up for the cost of replacing the ship no matter where they are fishing. It will take years of operation to recover the cost, which small fishery companies cannot afford. Therefore, we do not recommend the replacement of fishing boats to deal with the migration of herring and mackerel in Scotland.

It is assumed that the coordinate of small-scale fishery company after relocation are (x_0, y_0). The coordinate of the company (x_0, y_0) satisfies the boundary on land.

Distance function:

$$d(x_0, y_0) = \frac{6371\pi}{180} distance(x_0, y_0)$$

Income function:

$$\begin{aligned} I(x_0, y_0) &= k_1(x_0, y_0) \times n_h + k_2(x_0, y_0) \times n_m \\ &= k_1(x_0, y_0) \times a \times h + k_2(x_0, y_0) \times b \times m \end{aligned}$$

If the distance from the fishing site to the departure site is less than L, K is a constant:

$$\begin{cases} k_1 = k_{10} \\ k_2 = k_{20} \end{cases}$$

If the distance exceeds L, the price of the fish will also decrease:

$$\begin{cases} k_1(x_0, y_0) = k_{10} - 0.2[d(x_0, y_0) - L] \\ k_2(x_0, y_0) = k_{20} - 0.1[d(x_0, y_0) - L] \end{cases}$$

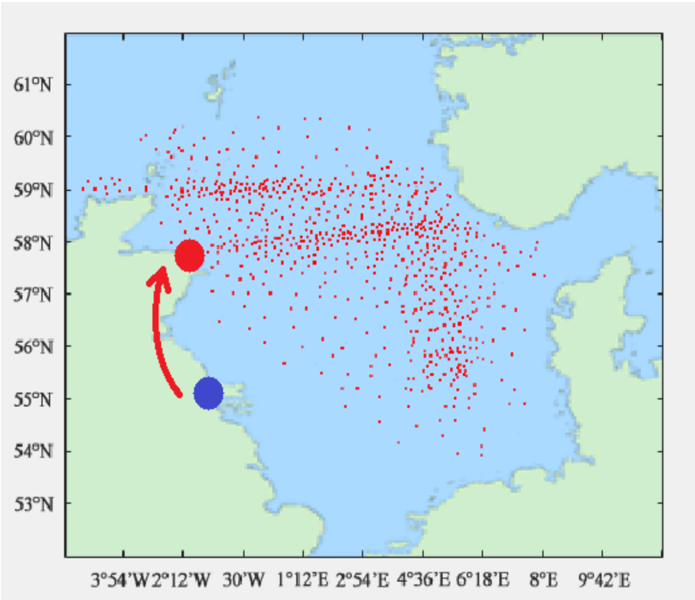
Cost function:

$$C(x_0, y_0) = D \times d(x_0, y_0) + M = D \times distance(x_0, y_0) + M$$

Profit function:

$$P(x_0, y_0) = I(x_0, y_0) - C(x_0, y_0)$$
$$= k_1(x_0, y_0) \times a \times g + k_2(x_0, y_0) \times b \times m \ - D \times distance(x_0, y_0) \ - M$$

We take the coordinates (x_0, y_0) of the relocated company as variables to find the optimal solution for the profit function. We derive the profit values of each point on the map and rank them. At last, we get the best place to move: Moving fishing companies to a coastline between 57 and 58 degrees north will maximize profits.



4.4 Requirement 4

4.4.1 Problem 4 Assumption

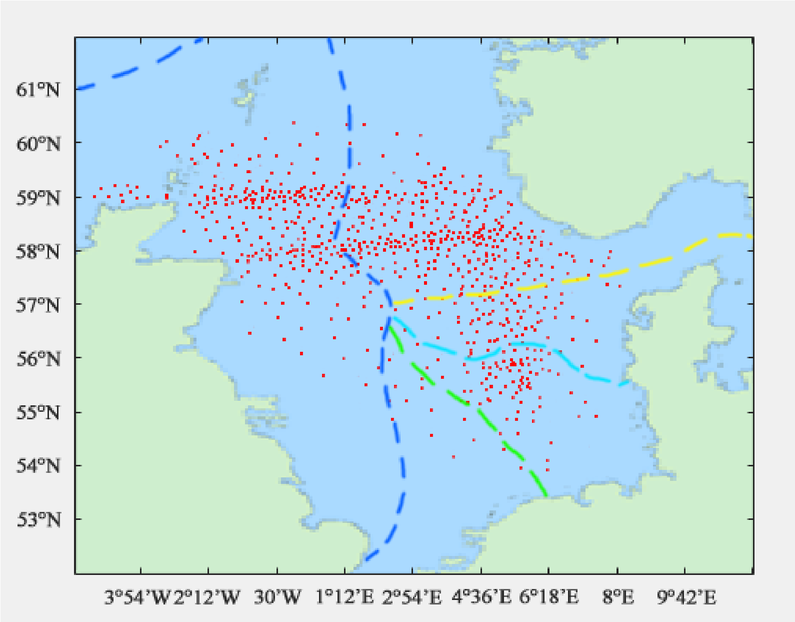
- The territorial sea is regarded as the sovereign territory of a state. Although foreign vessels (military and civil) are allowed to pass through the territorial sea innocuously, fishing vessels cannot legally fish
- Because the UK has left the EU, it cannot continue to fish in the territorial waters of EU countries.
- The migration trend of fish is based on the model we have established.

4.4.2 Problem 4-Analysis

We divided the sea area, changed the feasible region of the nonlinear programming, and found the optimal solution of the profit function and the fishing position to obtain the maximum profit in the domestic territorial sea with the optimization strategy in question 3, and compared it with the model in the previous question.

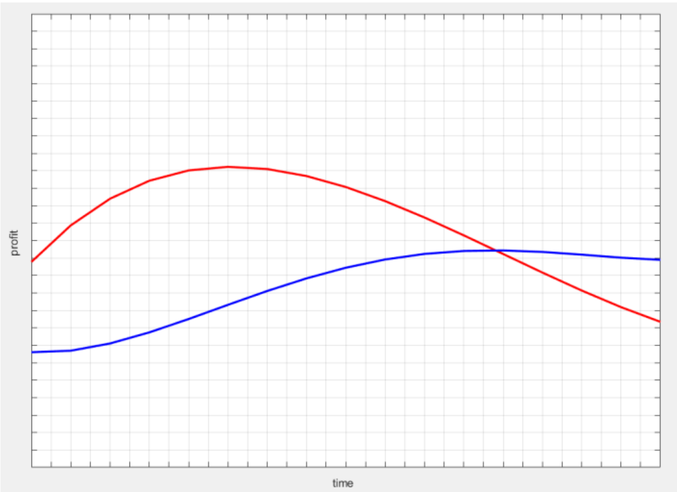
4.4.3 Model establishment and solution

We have divided the territorial waters and fishing vessels are allowed to fish in the British territorial waters. That means we need to establish a new feasible region, the British territorial waters area. In the picture, the left side of the blue dashed line shows the territorial waters of the United Kingdom, while the upper side of the yellow and blue dashed lines shows the territorial waters of Norway and the lower right corner shows the territorial waters of the Netherlands, Germany and Denmark.



The migration trend of fish follows the model we established.

Let's look at the impact within 50 years. Based on our model and the rate of migration of herring and mackerel from Scotland, by 2070 about two-thirds of herring and mackerel were outside the British territorial waters and entered the territorial waters of other countries. To make up for the loss caused by fish migration as much as possible, we used the strategy proposed in Model 3 to migrate fishing companies to ports at 57.6 °N. Then, in the new feasible region, the profit function $P(x, y)$ is used as the objective function for nonlinear programming, and the optimal solution for each year in the next 50 years is obtained. The optimal solution for each year is compared with the optimal solution for Model 2, which does not take the territorial sea factor into account, and an image is drawn as follows:



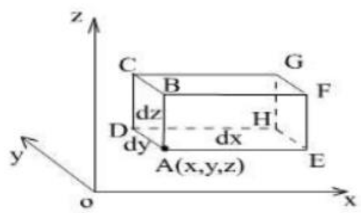
In the figure above, the red curve represents the relationship between profit in model 2 and time, showing a trend of increasing first and then decreasing; the blue curve is the relationship between profit in model 4 and time, showing an increasing trend. That is, in the next 50 years, with the optimization strategy in Model 3, the profit of Fishery Company will increase. Therefore, the adjustment strategy given in Model 3 is effective.

More than 50 years later, the future will be unstable and hard to predict. If the greenhouse effect continues to increase, herring and mackerel will continue to migrate to higher latitudes, fully entering the waters of Norway and Iceland, at that time, Scottish fishing boats will be barely able to catch herring and mackerel. Unfortunately, these two fisheries are two very large components of the Scottish fisheries economy. Scottish herring and mackerel leaving the British territorial sea and entering the territorial waters of other countries will have a huge impact on Scottish fisheries. Most fisheries companies, especially small fisheries companies which are unable to fish in a long distance, will even face a huge risk of bankruptcy. So the economic losses to the UK are enormous.

5 Model Improvement

We can use the partial differential equation to describe the temperature change of the ocean, so the prediction will be more accurate and stable. We use the heat transfer equation to describe the temperature field of seawater and mainly consider the influence of ocean current, solar thermal radiation and other factors on the study area. The heat conduction equation involves four latitudes, three dimensions of x, y, z and one dimension of time t, which greatly increases the difficulty of numerical solution. At the same time, the Neumann condition required for solving the heat conduction equation must know the heat flow entering or leaving the boundary, that is to say, we need the data of heat flow input, such as the North Atlantic warm current, solar radiation, etc., but at present There is a little lack of research and papers in this area, so it is difficult for us to obtain relevant data. Because of the time, we cannot give accurate results completely. Next, we will give the idea of establishing and solving the heat conduction equation model to describe the temperature of sea water. At the same time, this model is also the focus of future research.

As shown in the figure, consider a small cuboid with side length dx, dy and dz, its six planes are parallel to three coordinate planes.



In dt time, the heat flowing into the cuboid through the plane element $ABCD$ in x direction is as follows:

$$q^x|_x dydzdt$$

The total heat flow into the cuboid in x direction in dt time is as follows:

$$q^x|_x dydzdt - q^x|_{x+dx} dydzdt$$
$$= -(q^x|_{x+dx} - q^x|_x) dydzdt$$

$$= -\frac{\partial q^x}{\partial x}(dxdydzdt)$$

$$= \frac{\partial(ku_x)}{\partial x}dxdydzdt$$

Similarly, the total heat flow into the cuboid in y and z direction in dt time is as follows:

$$\frac{\partial(ku_y)}{\partial y}dxdydzdt, \frac{\partial(ku_z)}{\partial z}dxdydzdt$$

The heat flowing into the cuboid and the heat generated by the heat source inside the cuboid increase the temperature of the cuboid.

If the mass density of the object is ρ and the specific heat is c, then the heat needed for the temperature of the cuboid to rise du is:

$$dQ = c \times dm \times du = c \times \rho \times dxdydz \times du$$

If there is a heat source in the object, the heat emitted in unit time and unit volume is $f(x, y, z, t)$, then the heat generated in the cuboid considered in dt time is:

$$F(x, y, z, t)dx \times dy \times dz \times dt$$

From conservation of energy:

$$dQ = c \times \rho \times dxdydz$$

$$= \left[\frac{\partial(ku_x)}{\partial x} + \frac{\partial(ku_y)}{\partial y} + \frac{\partial(ku_z)}{\partial z}\right]dxdydzdt + F(x, y, z, t)dxdydzdt$$

Divide both sides by $dx \times dy \times dz \times dt$:

$$c\rho du/dt = \left[\frac{\partial(ku_x)}{\partial x} + \frac{\partial(ku_y)}{\partial y} + \frac{\partial(ku_z)}{\partial z}\right] + F(x, y, z, t)$$

Or:

$$c\rho u_t = \nabla(k\nabla u) + F(x, y, z, t)$$

If the object is uniform, then k is a constant, so the equation can be written as:

$$\frac{\partial u}{\partial t} = a^2\nabla^2 u + f(x, y, z, t)$$

In the above equation： $a^2 = \frac{k}{c\rho}$, $f(x, y, z, t) = \frac{F(x,y,z,t)}{c\rho}$

This is the derivation of the heat transfer equation.

In order to solve the heat transfer equation, it is necessary to give the initial conditions and boundary conditions.

Because the heat transfer equation only contains the first derivative of the unknown function u to time, only one initial condition is needed to solve the heat transfer equation:

$$u(x, y, z, t)|_{t=0} = \varphi(x, y, z)$$

$\varphi(x, y, z)$ is a known function, which gives the initial state of the whole system (i.e. the initial temperature distribution of the interior of the object at $t = 0$), not only the initial state of individual places in the system.

To solve the heat transfer equation, boundary conditions are also needed. There are three kinds of boundary conditions:

- Given the value of the unknown function u on the boundary.
- Given the normal first partial derivative value of unknown function u on the

boundary.

- Given the value of a linear combination of the unknown function u and its normal first partial derivative on the boundary.

We select the ocean near Scotland as latitude range $(x_1,\ x_2)$, longitude range $(y_1,\ y_2)$, and seawater depth range $(z_1,\ z_2)$ as our research objects. Other sea areas connected by this ocean and air are used as external heat sources.

Since there is no heat source in the seawater in the selected study area:

$$f\ (x,y,z,t)\ =\ 0$$

Therefore, the temperature of the sea area we studied can be described by the following partial differential equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = a^2\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right), x_1 \le x \le x_2, y_1 \le y \le y_2, z_1 \le z \le z_2, 0 \le t \le 50 \\ u|_{t=0} = \varphi(x,y,z), x_1 \le x \le x_2, y_1 \le y \le y_2, z_1 \le z \le z_2 \\ u_x|_{x=x_1} = q_1(t), u_x|_{x=x_2} = q_2(t), y_1 \le y \le y_2, z_1 \le z \le z_2, 0 \le t \le 50 \\ u_y|_{y=y_1} = q_3(t), u_y|_{y=y_1} = q_4(t), x_1 \le x \le x_2, z_1 \le z \le z_2, 0 \le t \le 50 \\ u_z|_{z=z_1} = q_5(t), u_z|_{z=z_1} = q_6(t), x_1 \le x \le x_2, y_1 \le y \le y_2, 0 \le t \le 50 \end{cases}$$

This partial differential equation means that:

We used the heat transfer equation as the universal equation to describe the temperature field of the seawater. The initial temperature field is determined by the function $\varphi(x,y,z)$. It describes the temperature distribution in the sea area at the initial time, which is fitted by the initial data. The boundary conditions are determined by the Neumann condition, which gives the heat flow $q_n(t)$ into the sea area on each boundary, such as the North Atlantic warm current, the radiation of the sun to the sea area and so on.

Next, we use the finite difference method to solve the above partial differential equation.

The continuous problem must be discretized to solve the partial differential equation with the finite difference method. Therefore, the first step is to mesh the solution area. We deal with the equation by difference, and discretize the definite solution conditions. Using MATLAB for iterative calculation and get the final result.

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