Aish Goyal
Section F
23

gn and Analysis of Algorithm.

Tutorial - 1

Ques: 1. What do you understand by Asympto-tic notations. Define different Asymptotic notation with examples.

Ans: 1. Asymptotic notations are those notations that describing the limiting behaviour of a function

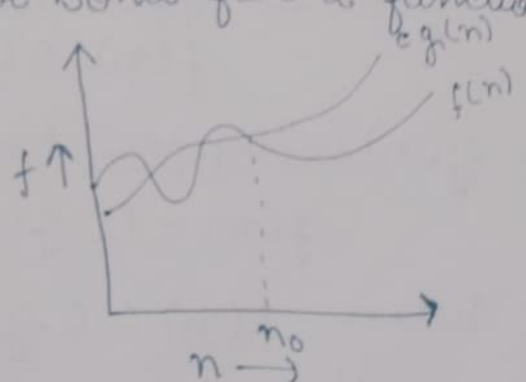
There are three different types of notations :-

- Big Oh (O)
- Big (Ω).
- Big (Θ).

→ Big Oh (O).

*Big - Oh (O) notation gives an upper bond for a function f(n) to within a constant factor.

$f(n) = O(g(n))$



$g(n)$ is "tight" upper bound

$f(n) = O(g(n))$.

iff $f(n) \leq c \cdot g(n)$

$\forall \; n \geq n_0$.

ex:- for (i=1; i≤n; i++).
{
sum += i
}.

$\Rightarrow O(1+n+n+n)$
$= O(n)$.

* Big Omega Notation

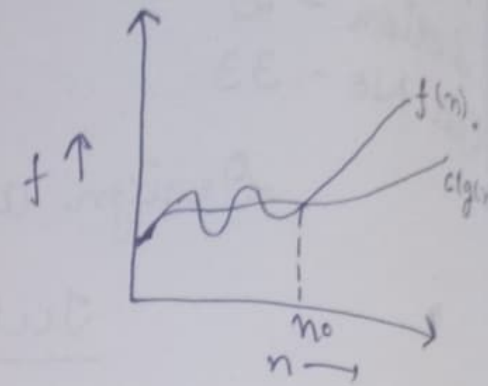
$f(n) = \Omega(g(n))$.

$g(n)$ is "tight" lower bound.

$f(n) = \Omega(g(n))$

iff $f(n) \geq c \cdot g(n)$.

$\forall \; n \geq n_0$.



* Θ (Theta

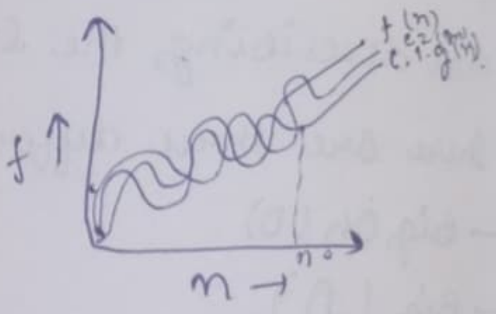
$f(n) = \Theta(g(n))$.

$g(n)$ is both "tight" upper and lower bond of $f n$.

$f(n) = \Theta(g(n))$.

iff $c_1 g(n) \leq f(n) \leq c_2 \cdot g(n)$.

$\forall \; n \geq \max(n_1, n_2)$.



* Small Oh (o).

$f(n) = o(g(n))$.

$g(n)$ is upper bond of the function $f(n)$.

$f(n) = o(g(n))$.

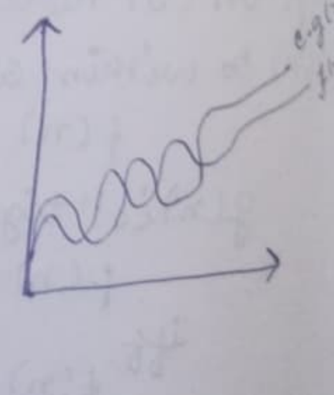
$g(n)$ is "upper" bound of $f n$

$f(n) = o(g(n))$.

when $f(n) < c \cdot g(n)$.

$\forall \; n \geq n_0$

$\forall \; c > 0$.

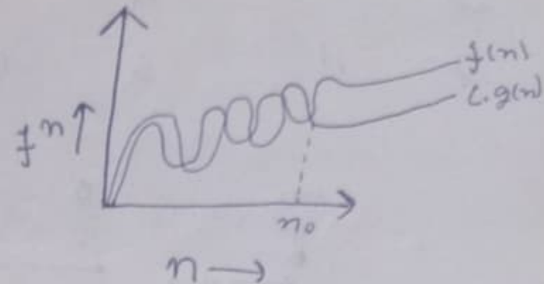


* Small Omega (ω)

$f(n) = \omega(g(n))$.

$g(n)$ is "lower" bound of fn

$f(n) = \omega(g(n))$.



when $f(n) > c \cdot g(n)$.

$\forall\ n > n_0$

if $\forall\ c > 0$

Ques:-2 What should be time complexity

for ($i = 1$ to n) $\{i = i * 2\}$.

for ($i = 1$ to n) // $i = 1, 2, 4, 8 --- n$.

$\{i = i * 2\}$ // $O(1)$.

$\Rightarrow \sum_{i=1} 1 + 2 + 4 + 8 + --- + n$.

K^{th} term of GP $\Rightarrow T_K = a r^{K-1}$.

$n = 1 * 2^{K-1}$.

$n = 2^{K-1}$

$n = \frac{2^K}{2}$

$2^n = 2^K$

$\log_2(2n) = K(\log_2 2)$

$K = \log_2(2n)$

$K = \log_2 2 + \log_2 n$

$K = 1 + \log_2 n$

$K \neq 1 + \log_2 n$

$O(\log_2 n)$

$\Rightarrow O(n)$.

Ques:-3

$T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$$
\begin{aligned}
T(n) &= 3T(n-1). \\
&= 3(3T(n-2)). \\
&= 3^2 T(n-2) \\
&= 3^3 T(n-3) \\
&\vdots \\
&= 3^n T(n-n). \\
&= 3^n T(0). \\
&= 3^n
\end{aligned}
$$

$\Rightarrow O(3^n).$

Ques:-4

$T(n) = \{2T(n-1) - 1$ if $n>0$, otherwise $1\}$.

$$
\begin{aligned}
T(n) &= 2T(n-1) - 1 \\
&= 2(2T(n-2)-1) - 1 \\
&= 2^2(T(n-2)) - 2 - 1 \\
&= 2^2(2T(n-3)-1) - 2 - 1 \\
&= 2^3 T(n-3) - 2^2 - 2^1 - 2^0 \\
&\vdots \\
&= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3} \\
&\quad \cdots\; 2^2 - 2^1 - 2^0. \\
&= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} \\
&\quad \cdots\; 2^2 - 2^1 - 2^0 \\
&= 2^n - (2^n - 1).
\end{aligned}
$$

$T(n) = 1.$

$\Rightarrow O(1).$

Ques:-5 What should be the time Complexity

```
int i = 1, s=1;
while (s<=n) {
  i++;
  s= s+i;
printf("#");
```

$i = 1, 2, 3, 4, 5, 6, \ldots k$

$s = 2 + 2 + 3 + 4 + 5 \ldots k.$

When $s >= n$, then loop will stop at k^{th} iterations

$\Rightarrow s >= n$

$s = n.$

$\rightarrow 2 + 2 + 3 + 4 + \ldots + k = n$

$= 1 + (k * (k+1))/2 = n$

$= k^2 = n$

$k = \sqrt{n}$

$= O(\sqrt{n}).$

Ques:-6 Time complexity of

```
void function(int n) {
    int i, count = 0;
  for(i=1; i*i<=n; i++)
      count ++;        //O(1).
```

as $i^2 <= n.$

$i <= \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}.$

$\sum_{i=1}^{n} 1 + 2 + 3 + \ldots + \sqrt{n}.$

$$T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$$

$$T(n) = \frac{n\sqrt{n}}{2}$$

$$T(n) = O(n).$$

Ques:-7

```
void function(int n){
 int i, j, K, count=0;
 for(i=n/2; i<=n; i++).
   for(j=1; j<=n; j-j*2)
     for(K=1; K<=n; K=K*2).
        count++.
```

for $K = K*2$

$K = 1, 2, 4, 8, ---n.$

$G.P \Rightarrow a=1, \ r=2$

$\frac{a(r^n-1)}{r-1}$

$\Rightarrow \frac{1(2^k-1)}{1}$

$n \Rightarrow 2^k$

$\log n = k.$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$.

$\Rightarrow O(n * \log n * \log n)$.

$\Rightarrow O(n \log^2 n)$.

Q:- 8.

```
funtion (int n)
{
   int (n==1)
    return;
for (i=1 to n)
{
  for (j=1 to n).
}
   function (n-3);
}.
```

$T(n) = T(n/3) + n^2$

$a=1,\ b=3,\ f(n)=n^2$

$c = \log_3 1 = 0$

$n^0 = 1 > (f(n) = n^2)$

$T(n) = \Theta(n^2)$.

Ques:- 9

```
void function (int n)
{
  for (i = 1 to n)
  {
    for (j = 1; j <= n; j = j+1).
        printf(" * ").
  }
}.
```

for $i = 1 \Rightarrow j = 1, 2, 3, 4, --- n.$
for $i = 2 \Rightarrow j = 1, 3, 5, 7, --- n$
for $i = 3 \Rightarrow j = 1, 4, 7, ---- n.$
for $i = n \Rightarrow j = 1 ---$

$\Rightarrow \sum_{i=n}^{1} n + 1/2 + n/3 + n/4 + --- + 1.$

$\Rightarrow \sum_{j=n}^{1} n[1 + 1/2 + 1/3 + 1/4 + ---- 1/n].$

$\Rightarrow \sum_{j=n} n[\log n]$

$\Rightarrow T(n) = (n \log n)$

$T(n) = O[n \log n]$

Ques'.- 10

as given n^k & c^n

relation b/w n^k & c^n is

$n^k = O(c^n).$

as $n^k \leq c^n$

$\forall n \geq n_0$ and some constant $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a 2^1$

$\Rightarrow n_0 = 1$ & $c = 2.$