

# LINEAR ALGEBRA ASSIGNMENT

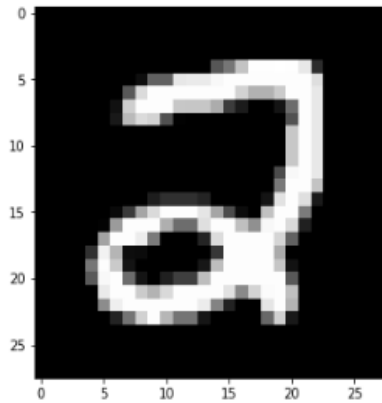
## Unit 4

1.	<p><b>Data Set:</b> MNIST (Mixed National Institute of Standards and Technology) computer vision digit Dataset or any other similar data sets.</p> <p><b>Implementation of Dimensionality Reduction Techniques:</b></p> <ol style="list-style-type: none"><li>1. Principal Component Analysis</li><li>2. Independent Component Analysis</li><li>3. Methods Based on Projections</li></ol>																																																																																																																														
Code	<p><b>Data Set:</b></p> <ul style="list-style-type: none"><li>• MNIST computer vision digit Dataset</li></ul> <p><b>Technique used:</b></p> <ul style="list-style-type: none"><li>• Implemented using principal component Analysis</li></ul>																																																																																																																														
Output screen-shots	<p><b>Implementing PCA(Principal Component Analysis) on MNIST Dataset</b></p> <p>Importing our mandatory python libraries which are required for the implementation of PCA.</p> <pre>In [145]: import numpy as np import pandas as pd import matplotlib.pyplot as plt import seaborn as sns</pre> <p>loading our MNIST dataset from our computer which is stored in .csv format.</p> <pre>In [146]: df = pd.read_csv('mnist_train.csv') print("the shape of data is :", df.shape)</pre> <p>the shape of data is : (42000, 785)</p> <pre>In [147]: #Displaying first few data's df.head()</pre> <pre>Out[147]:</pre> <table><thead><tr><th></th><th>label</th><th>pixel0</th><th>pixel1</th><th>pixel2</th><th>pixel3</th><th>pixel4</th><th>pixel5</th><th>pixel6</th><th>pixel7</th><th>pixel8</th><th>...</th><th>pixel774</th><th>pixel775</th><th>pixel776</th><th>pixel777</th><th>pixel778</th><th>pixel779</th><th>pixel780</th><th>pixel781</th><th>pixel782</th></tr></thead><tbody><tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>...</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>...</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>...</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>3</td><td>4</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>...</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>4</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>...</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></tbody></table> <p>5 rows × 785 columns</p> <div><div></div><div></div></div>		label	pixel0	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	...	pixel774	pixel775	pixel776	pixel777	pixel778	pixel779	pixel780	pixel781	pixel782	0	1	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	3	4	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0
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## Extracting label column from the dataset

## Plotting a random sample data point from The dataset using matplotlib imshow() method

```
In [148]: label = df['label']
df.drop('label', axis = 1, inplace = True)
ind = np.random.randint(0, 20000)
plt.figure(figsize = (20, 5))
grid_data = np.array(df.iloc[ind]).reshape(28,28)
plt.imshow(grid_data, interpolation = None, cmap = 'gray')
plt.show()
print('The above values is',label[ind])
```



The above values is 2

## 2D Visualization using PCA

Picking first 20K data-points out of 42k data-points to work on for time-efficiency.

```
In [149]: labels = label.head(20000)
data = df.head(20000)
print("the shape of sample data = ", data.shape)

the shape of sample data = (20000, 784)
```

## Data-preprocessing : standardizing the dataset

```
In [150]: #Column standardization of our dataset using StandardScaler class of sklearn.preprocessing module.
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler()
std_df = scaler.fit_transform(df)
std_df.shape
```

Out[150]: (42000, 784)

## Finding the Co-Variance matrix which is $AT * A$ using NumPy matmul method

```
In [151]: covar_mat = np.matmul(std_df.T, std_df)
covar_mat.shape
```

Out[151]: (784, 784)

After multiplication, the dimensions of our Co-Variance matrix is  $784 * 784$ , because  $AT(784 * 20000) * A(20000 * 784)$ .

## Finding the top two Eigen-values and corresponding eigenvectors for projecting onto a 2D surface.

```
In [152]: #converting the eigenvectors into (2,d) form for easiness of further computations
from scipy.linalg import eigh
values, vectors = eigh(covar_mat, eigvals = (782, 783))
print("Dimensions of Eigen vector:", vectors.shape)
vectors = vectors.T
print("Dimensions of Eigen vector:", vectors.shape)
```

Dimensions of Eigen vector: (784, 2)  
Dimensions of Eigen vector: (2, 784)

here the vectors[1] represent the eigenvector corresponding 1st principal eigenvector

here the vectors[0] represent the eigenvector corresponding 2nd principal eigenvector

If we multiply the two top vectors to the Co-Variance matrix, we found our two principal components PC1 and PC2.

```
In [153]: final_df = np.matmul(vectors, std_df.T)
print(" vectors:", vectors.shape, "\n", "std_df:", std_df.T.shape, "\n", "final_df:", final_df.shape)

vectors: (2, 784)
std_df: (784, 42000)
final_df: (2, 42000)
```

Now we vertically stack our final\_df and label and then Transpose them, then we found the NumPy data table so with the help of pd.DataFrame we create the data frame of our two components with class labels.

```
In [154]: final_dFT = np.vstack((final_df, label)).T
dataFrame = pd.DataFrame(final_dFT, columns = ['1st_principle component analysis', '2nd_principle component analysis', 'label'])
dataFrame
```

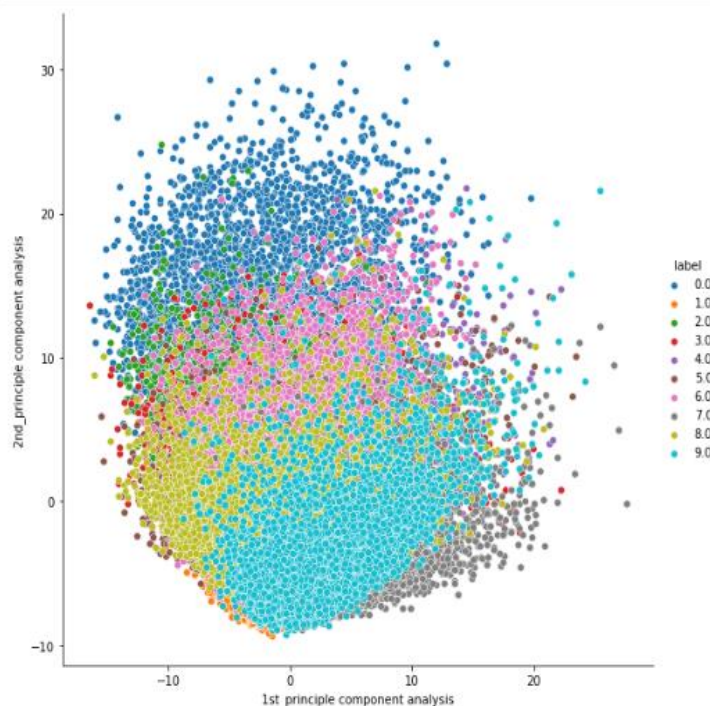
```
Out[154]:
```

	1st_principle component analysis	2nd_principle component analysis	label
0	-5.226445	-5.140478	1.0
1	6.032996	19.292332	0.0
2	-1.705813	-7.644503	1.0
3	5.836139	-0.474207	4.0
4	6.024818	26.559574	0.0
...	...	...	...
41995	-1.350386	13.678849	0.0
41996	-1.187360	-8.869582	1.0
41997	7.076277	0.495391	7.0
41998	-4.344513	2.307240	6.0
41999	1.559121	-4.807670	9.0

42000 rows × 3 columns

visualizing the final data with help of the seaborn FacetGrid method.

```
In [155]: sns.FacetGrid(dataFrame, hue = 'label', height = 8).map(sns.scatterplot, '1st_principle component analysis', '2nd_principle component analysis')
plt.show()
```



Visualized data of PCA on MNIST dataset

## PCA for dimensionality reduction

```
In [156]: # using SKlearn importing PCA
from sklearn import decomposition
pca = decomposition.PCA()
```

```
In [157]: # PCA for dimensionality reduction (non-visualization)

pca.n_components = 784
pca_data = pca.fit_transform(std_df)

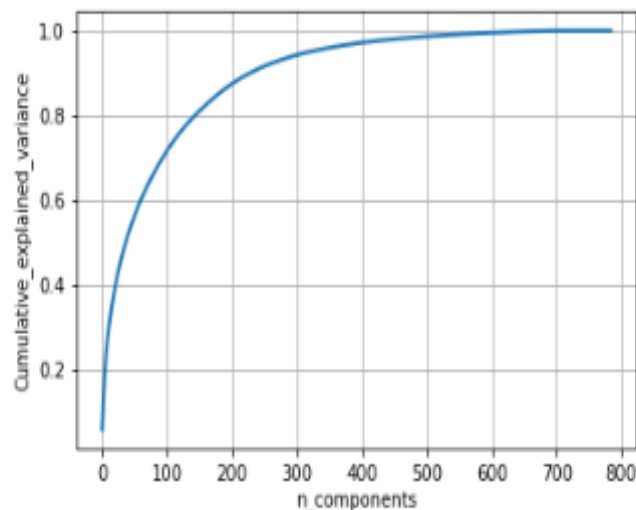
percentage_var_explained = pca.explained_variance_ / np.sum(pca.explained_variance_);

cum_var_explained = np.cumsum(percentage_var_explained)

# Plot the PCA spectrum
plt.figure(1, figsize=(6, 4))

plt.clf()
plt.plot(cum_var_explained, linewidth=2)
plt.axis('tight')
plt.grid()
plt.xlabel('n_components')
plt.ylabel('Cumulative_explained_variance')
plt.show()

# If we take 200-dimensions, approx. 90% of variance is explained.
```



```
In [158]: # directly entering parameters
pca.n_components = 2
pca_data = pca.fit_transform(std_df)

print('shape of pca_reduced data = ',pca_data.shape)

shape of pca_reduced data = (42000, 2)
```

```
In [161]: # Data massaging - adding label column to the reduced matrix
pca_data = np.vstack((pca_data.T,label)).T
```

```
In [162]: # dataframing and plotting the pca data
pca_df = pd.DataFrame(data=pca_data,columns=('1st_principal','2nd_principal','labels'))
sns.FacetGrid(pca_df,hue='labels',height=6).map(plt.scatter,'1st_principal','2nd_principal').add_legend()
plt.show()
```

