LINEAR ALGEBRA ASSIGNMENT

Unit 5

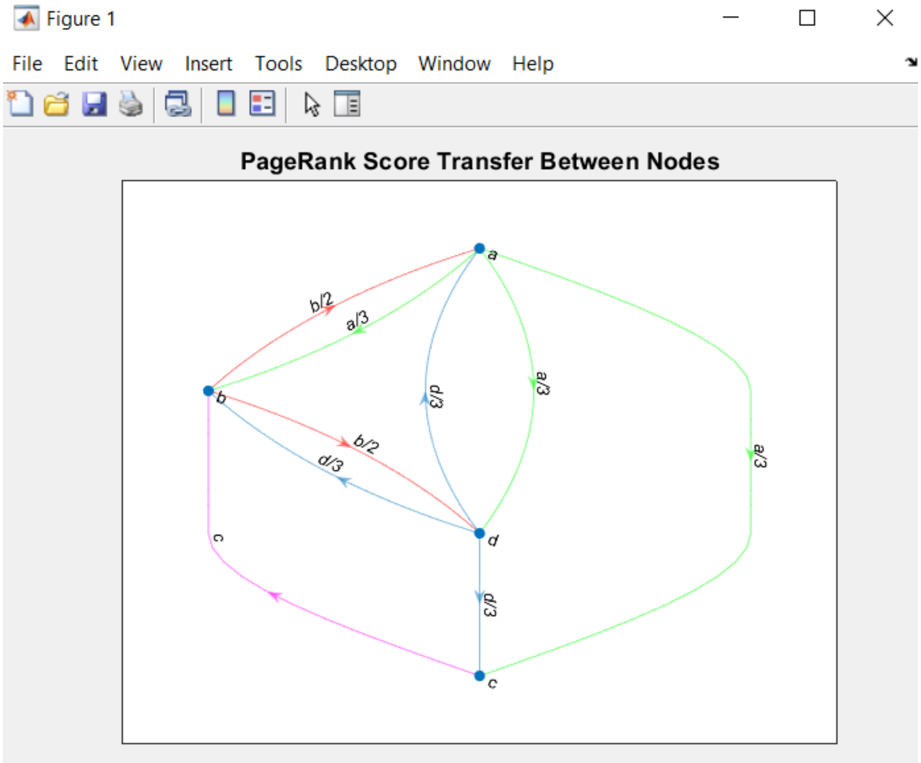
Applications of Linear algebra for Page Rank Algorithm.

a) Creating a graph that illustrates how each node confers its PageRank score to the other nodes in the graph.

Code

```
s = {'a' 'a' 'a' 'b' 'b' 'c' 'd' 'd' 'd'};
t = {'b' 'c' 'd' 'd' 'a' 'b' 'c' 'a' 'b'};
G = digraph(s,t);
labels = {'a/3' 'a/3' 'a/3' 'b/2' 'b/2' 'c' 'd/3' 'd/3'
'd/3'};
p = plot(G,'Layout','layered','EdgeLabel',labels);
highlight(p,[1 1 1],[2 3 4],'EdgeColor','g')
highlight(p,[2 2],[1 4],'EdgeColor','r')
highlight(p,3,2,'EdgeColor','m')
title('PageRank Score Transfer Between Nodes')
```

output

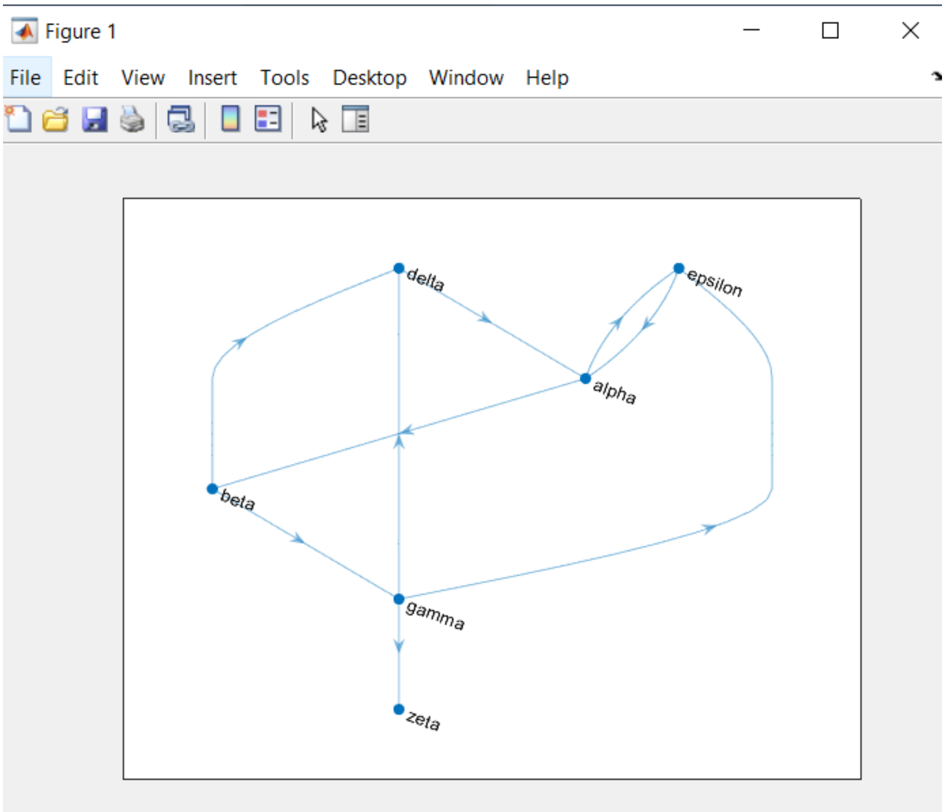


b) PageRank with 6 Nodes

Code

```
s = [1 1 2 2 3 3 3 4 5];
t = [2 5 3 4 4 5 6 1 1];
names = {'http://www.example.com/alpha',
'http://www.example.com/beta', ...
         'http://www.example.com/gamma',
'http://www.example.com/delta', ...
         'http://www.example.com/epsilon',
'http://www.example.com/zeta'};
```

	`G = digraph(s,t,[],names)`
	`plot(G,'Layout','layered', ...`
	
	`'NodeLabel',{'alpha','beta','gamma','delta','epsilon','zeta'}`
	`})`

output	`G =`
	
	`digraph with properties:`
	
	`Edges: [9×1 table]`
	`Nodes: [6×1 table]`
	

➢	Calculating the PageRank centrality score for the above graph
Code	`pr = centrality(G,'pagerank','FollowProbability',0.85)`

output	`pr =`
	
	`0.3210`
	`0.1706`
	`0.1066`
	`0.1368`
	`0.2008`
	`0.0643`

➢	View the PageRank scores and degree information for each page.

code	`G.Nodes.PageRank = pr;`
	`G.Nodes.InDegree = indegree(G);`
	`G.Nodes.OutDegree = outdegree(G);`
	`G.Nodes`

output

6×4 table

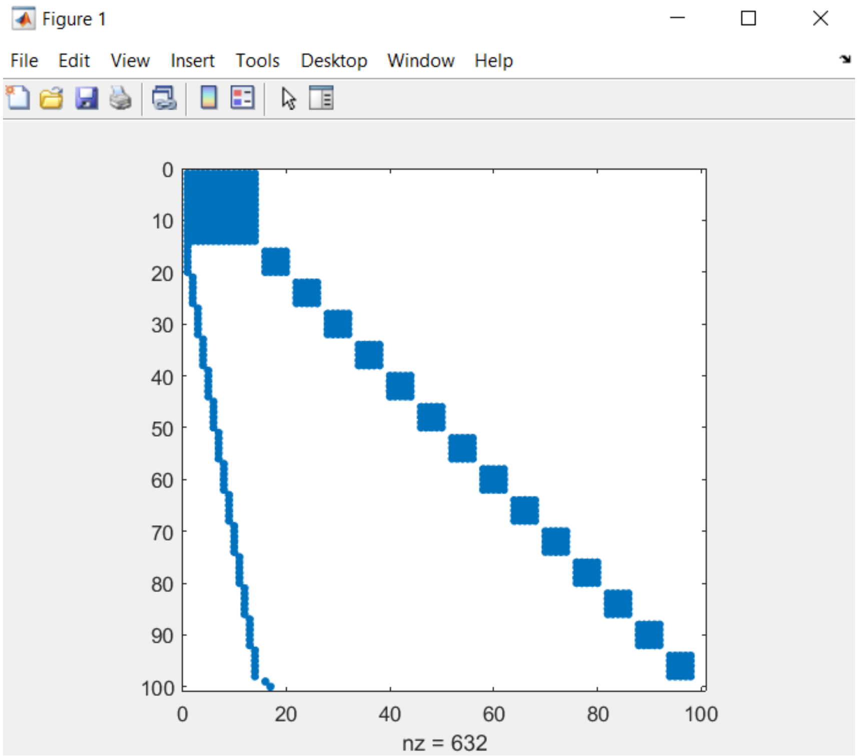
Name	PageRank	InDegree	OutDegree
{'http://www.example.com/alpha' }	0.32098	2	2
{'http://www.example.com/beta' }	0.17057	1	2
{'http://www.example.com/gamma' }	0.10657	1	3
{'http://www.example.com/delta' }	0.13678	2	1
{'http://www.example.com/epsilon'}	0.20078	2	1
{'http://www.example.com/zeta' }	0.06432	1	0

c) PageRank of Websites on mathworks.com

Code

```
load mathworks100.mat
spy(A)
```

output



d) Create a directed graph with the sparse adjacency matrix, A, using the URLs contained in U as node names.

Code

```
G = digraph(A,U)
```

output

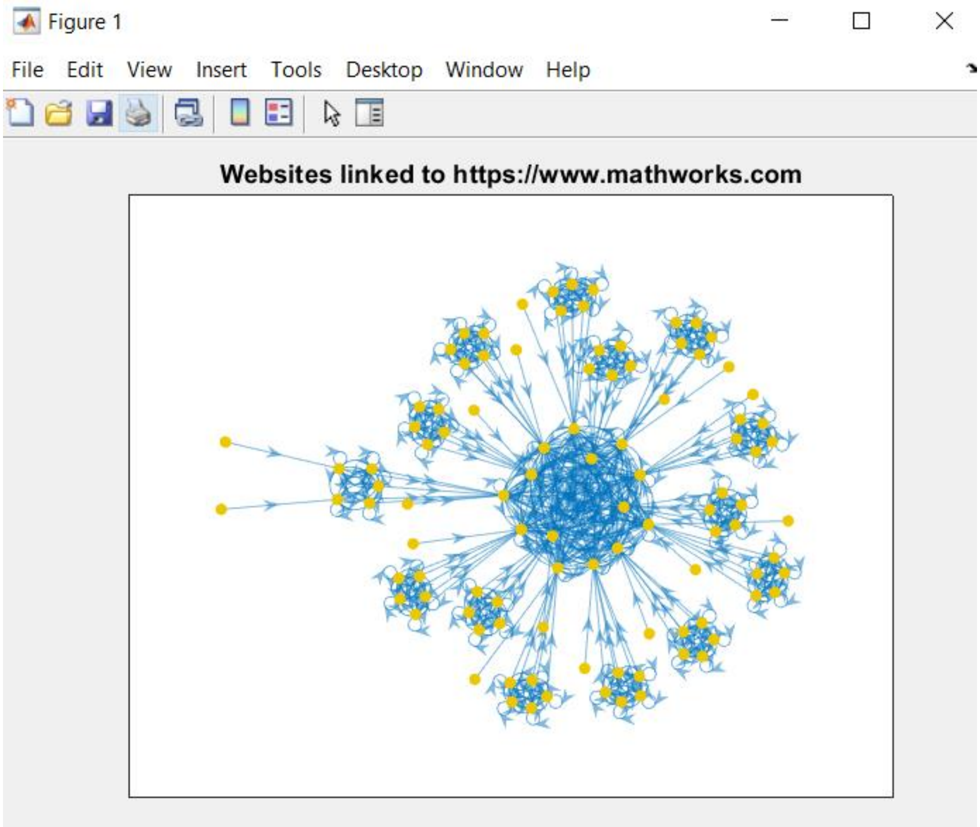
```
G =

  digraph with properties:

    Edges: [632×1 table]
    Nodes: [100×1 table]
```

➢ Plotting the graph using the force layout.

Code

```
plot(G,'NodeLabel',{},'NodeColor',[0.93 0.78 0],'Layout','force');
title('Websites linked to https://www.mathworks.com')
```

output



e) Computing the PageRank scores for the graph, G, using 200 iterations and a damping factor of 0.85. Add the scores and degree information to the nodes table of the graph.

code

```
pr =
centrality(G,'pagerank','MaxIterations',200,'FollowProbabili
ty',0.85);
G.Nodes.PageRank = pr;
G.Nodes.InDegree = indegree(G);
G.Nodes.OutDegree = outdegree(G);
G.Nodes(1:25,:) %Viewing the top 25 resulting scores.
```

output

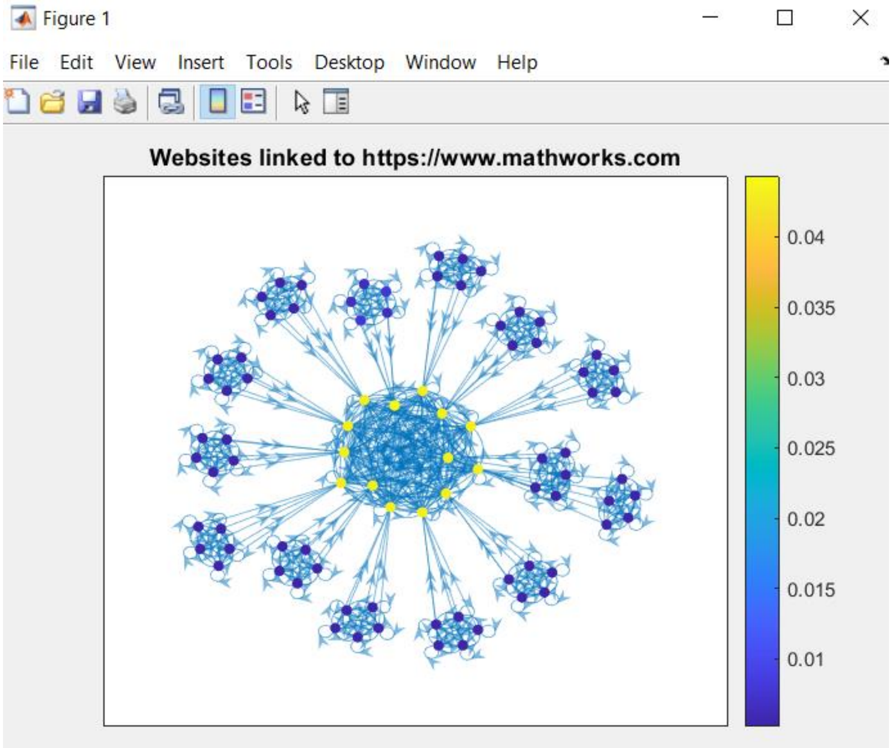
Name	PageRank	InDegree	OutDegree
{'https://www.mathworks.com'}	0.044342	20	14
{'https://ch.mathworks.com'}	0.043085	20	14
{'https://cn.mathworks.com'}	0.043085	20	14
{'https://jp.mathworks.com'}	0.043085	20	14
{'https://kr.mathworks.com'}	0.043085	20	14
{'https://uk.mathworks.com'}	0.043085	20	14
{'https://au.mathworks.com'}	0.043085	20	14
{'https://de.mathworks.com'}	0.043085	20	14
{'https://es.mathworks.com'}	0.043085	20	14
{'https://fr.mathworks.com'}	0.043085	20	14
{'https://in.mathworks.com'}	0.043085	20	14
{'https://it.mathworks.com'}	0.043085	20	14
{'https://nl.mathworks.com'}	0.043085	20	14
{'https://se.mathworks.com'}	0.043085	20	14
{'https://www.mathworks.com/index.html%3Fnocookie%3Dtrue'}	0.0015	0	1
{'https://www.mathworks.com/company/aboutus/policies_statements/patents.html'}	0.007714	6	6
{'https://www.mathworks.com/company/aboutus/policies_statements/trademarks.html'}	0.007714	6	6
{'https://www.mathworks.com/company/aboutus/policies_statements'}	0.006439	5	6
{'https://www.mathworks.com/company/aboutus/policies_statements/piracy.html'}	0.006439	5	6
{'https://www.mathworks.com/company/rss/index.html'}	0.006439	5	6
{'https://ch.mathworks.com/index.html%3Fnocookie%3Dtrue'}	0.0015	0	1
{'https://ch.mathworks.com/company/aboutus/policies_statements/patents.html'}	0.0051817	5	6
{'https://ch.mathworks.com/company/aboutus/policies_statements/trademarks.html'}	0.0051817	5	6
{'https://ch.mathworks.com/company/aboutus/policies_statements'}	0.0051817	5	6
{'https://ch.mathworks.com/company/aboutus/policies_statements/piracy.html'}	0.0051817	5	6

➢ Extracting and plot a subgraph containing all nodes whose score is greater than 0.005. Colour the graph nodes based on their PageRank score.

Code

```
H = subgraph(G,find(G.Nodes.PageRank > 0.005));
plot(H,'NodeLabel',{},'NodeCData',H.Nodes.PageRank,'Layout',
'force');
title('Websites linked to https://www.mathworks.com')
colorbar
```

output



f) **Page rank algorithm**

Code

```
% PageRank algorithm
%% Constructing adjancency matrix A
A = [...
    0 1 0 1 1;
    0 0 1 1 1;
    1 0 0 1 0;
    0 0 0 0 1;
    0 0 1 0 0];

%% Solveing the eigenvalue problem
% Diagonal matrix D contains the eigenvalues of A in the
diagonal.
% The columns of matrix V are the eigenvectors of A
[V,D] = eig(A);

% Eigenvalues of A are in this vector. The first
% one is the only real eigenvalue; that's what we
% need.
evals = diag(D);
eval1 = evals(1);

% Find proportionality constant alpha
alpha = 1/eval1;

% Find ranking vector r as the eigenvector corresponding to
```

```matlab
% the first eigenvalue
r = V(:,1);

% Normalize the ranking vector
r = r/sum(r);

% trying the same with the power method!

% First, pick some 5-vector
x0 = [3,10,pi,5,0];
x0 = x0(:) % Make vertical

% Second, iterate!
x = x0;
for iii = 1:500
    x = A*x;
end

% Third, normalize
x = x/sum(x);

% Show the result to compare
format long
disp([r.';x.'])
```

output

```
>> page_rank

x0 =

    3.0000
   10.0000
    3.1416
    5.0000
         0

   0.291473140314845   0.266433387961365   0.224884875090076   0.081678798064438   0.135529798569276
   0.291473140314845   0.266433387961365   0.224884875090076   0.081678798064438   0.135529798569276
```