Control Systems:Question 35

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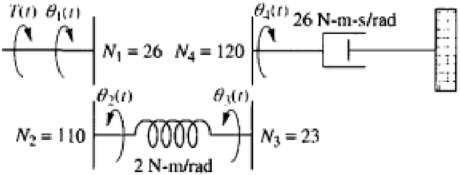
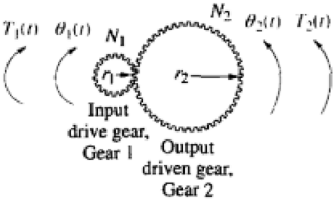
Problem Statement

- Find the transfer function, $G(s)=\theta_4(s)/T(s)$, for the rotational system shown in the following figure.



Formulae involved in a gear system



- In the above figure, N_1 and N_2 are the number of teeth on Gear 1 and Gear 2 respectively.
- As the gears turn,the distance travelled along each gear's circumference is same.Therefore $r_1\theta_1 = r_2\theta_2$ which in turn gives $\frac{\theta_1}{\theta_2}=\frac{N_2}{N_1}$.
- Assuming the gears are lossless,the energy into Gear 1 equals the energy out of Gear 2.Therefore, $T_1\theta_1 = T_2\theta_2$.

Therefore,

$$\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1} \tag{3.1}$$

Also,rotational impedances can be reflected through gear trains by multiplying the mechanical impedances by the following ratio:

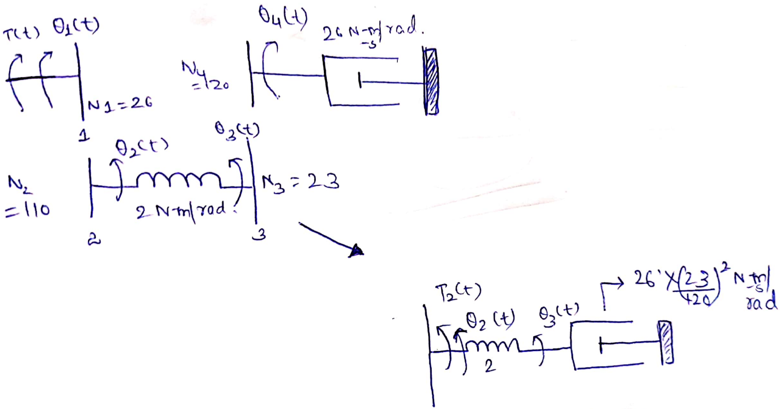
$$(\frac{N_1}{N_2})^2$$

where N_1 is the number of teeth of gear on destination shaft and N_2 is that of the source shaft.

- We reflect the impedance connected to shaft 4,to shaft 3,θ_3.It is reflected as 26*$(\frac{23}{120})^2$=0.955 N-m-s/rad.(as per the ratio given earlier in slide 5).
- Let the torque acting at shaft 2 be T_2(t).According to equation 3.1,

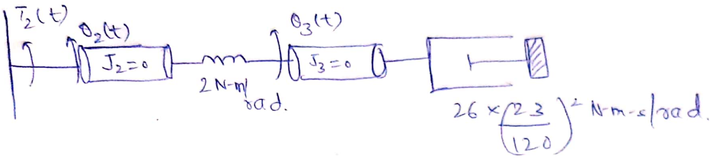
$$T_2(t) = T(t) * (\frac{110}{26}) \tag{4.1}$$

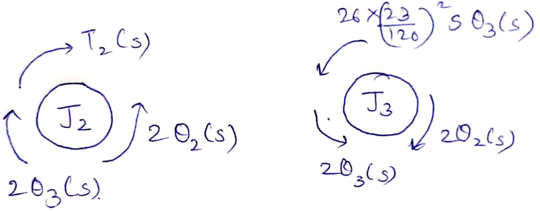
The obtained equivalent system is shown in the following figure.



$$T_2(t) = \frac{T(t) \times 110}{26}$$

- Let us assume an inertia of value J_2=0 connected in between the shaft 2 and the spring,which undergoes a displacement of θ_2(t).
- Similarly consider an inertia of J_3=0 between the spring and the viscous damper.The system after adding J_2 and J_3 and the various torques acting on them are shown in the following figure.


$T_2(t)$
$\theta_2(t)$
$J_2=0$
2 N-m/rad.
$\theta_3(t)$
$J_3=0$
$26\times\left(\frac{23}{120}\right)^2$ N-m-s/rad.


$T_2(s)$
J_2
$2\theta_2(s)$
$2\theta_3(s)$
$26\times\left(\frac{23}{120}\right)^2 s\,\theta_3(s)$
J_3
$2\theta_2(s)$
$2\theta_3(s)$

From the fig. we get the following equations of motion:

$$T_2(s) = 2(\theta_2(s) - \theta_3(s)) \tag{4.2}$$

$$2(\theta_2(s) - \theta_3(s)) = 0.955s\theta_3(s) \tag{4.3}$$

Solving 4.2 and 4.3 we get

$$T_2(s) = 0.955s\theta_3(s) \tag{4.4}$$

From equation 4.1 we express $T_2(s)$ in terms of T(s).Also,from equation 3.1

$$\theta_3(s)=\theta_4*(\tfrac{120}{23})$$

Upon substituting for T_2(s) and θ_3(s),we obtain

$$T(s) * \frac{110}{26} = 0.955 * s\theta_4(s) \tag{4.5}$$

which gives the required ratio as

$$\frac{\theta_4(s)}{T(s)} = \frac{0.848}{s} \tag{4.6}$$