



BQ: A Lock-Free Queue with Batching

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Concurrent data structures provide fundamental building blocks for concurrent programming. Standard concurrent data structures may be extended by allowing a sequence of operations to be submitted as a batch for later execution. A sequence of such operations can then be executed more efficiently than the standard execution of one operation at a time. In this article, we develop a novel algorithmic extension to the prevalent FIFO queue data structure that exploits such batching scenarios. An implementation in C++ on a multicore demonstrates significant performance improvement of more than an order of magnitude (depending on the batch lengths and the number of threads) compared to previous queue implementations.

CCS Concepts: • **Computing methodologies** → **Shared memory algorithms; Concurrent algorithms; Theory of computation** → **Data structures design and analysis;**

Additional Key Words and Phrases: Concurrent algorithms, concurrent data structures, lock-freedom, linearizability, FIFO queue

ACM Reference format:

Gal Milman-Sela, Alex Kogan, Yossi Lev, Victor Luchangco, and Erez Petrank. 2022. BQ: A Lock-Free Queue with Batching. *ACM Trans. Parallel Comput.* 9, 1, Article 5 (March 2022), 49 pages.
<https://doi.org/10.1145/3512757>

1 INTRODUCTION

The era of multi-core architectures has been having a huge impact on software development: exploiting concurrency has become the main challenge of today's programming. Concurrent data structures provide the basic blocks for concurrent programming; hence, it is crucial that they are efficient and scalable. In this article, we consider a setting in which threads sometimes execute a *sequence* of operations on a shared concurrent data structure (rather than a single operation each time). This scenario occurs either because the threads are willing to delay execution of operations to improve performance or they deliberately want operations to be executed later.

As an example, consider a server thread that serves requests of remote clients. Such a thread may accumulate several relevant operations required by some client, generate a sequence of these

This work was supported by the Israel Science Foundation (grant no. 1102/21).

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2329-4949/2022/03-ART5 \$15.00

<https://doi.org/10.1145/3512757>

operations, submit them for execution on shared data, finish handling them, and then proceed to handle other clients whose operations can be accumulated similarly.

Kogan and Herlihy [17] formulated batching of operations in a concurrent setting using the *future* programming construct. *Batching* means grouping a sequence of standard operations to a single *batch operation*, which applies them together to the shared object. They formalized correctness (linearization) guarantees and demonstrated the advantages of batching even when using naive batching strategies.

In this article, we present a novel extension to the concurrent lock-free queue by Michael and Scott [22] (henceforth MSQ) that can handle a sequence of operations in a batch. Our queue extension, denoted BQ (which stands for Batching Queue), provides faster execution for operation sequences. Kogan and Herlihy suggested to apply each sequence of operations of the same type to the shared queue at once. Specifically, they execute each subsequence of enqueues-only together by appending adequate nodes at the end of the queue, and each subsequence of dequeues-only by unlinking several nodes from the head of the queue. The advantage of this method degrades when operations in the batch switch frequently between enqueues and dequeues, which is the case with general sequences. We present an algorithm that handles any batch of enqueues and dequeues locally and applies it at once to the shared queue to reduce contention. Using novel observations on the effect of a mixed sequence on the shared queue, we achieve a fast application of the sequence on the shared queue with low synchronization.

Concurrent queues are typically not scalable because they have two points of contention: the head and the tail. However, batching of operations provides an excellent opportunity to combine operations locally and improve scalability. Such local computation reduces the number of accesses to the shared structure, which yields an overall reduced contention. As shown in the measurements, BQ improves the performance and scalability over MSQ and over the simpler batching method of Kogan and Herlihy.

We also extend Kogan and Herlihy's formal treatment of systems that only execute batch operations, to allow simultaneous execution of standard (single) operations, while still satisfying an extended form of linearizability that we present.

BQ is lock-free. It uses only compare-and-swap (CAS) atomic operations (which can easily be replaced with LL/SC instructions) and can thus be ported to other existing platforms. The original MSQ we build upon is widely known as a well-performing queue for general hardware and is included as part of the Java Concurrency Package [20]. Measurements for BQ demonstrate a significant performance improvement of more than an order of magnitude compared to MSQ when threads employ batch operations to update the queue.

Batching provides a performance improvement for operations that the user agrees to delay. Additionally, BQ guarantees that deferred operations of a certain thread will not take effect until that thread performs a non-deferred operation or explicitly requests an evaluation of previous future operations. This is useful when the user wishes to call several operations and knowingly delay their execution to a chosen time.

The rest of the article is organized as follows. Section 2 introduces the model we work with and surveys the work we build on. In Section 3, we define linearizability and its extensions to objects with batch operations. Having set the terminology, we discuss related work in Section 4. Section 5 presents an overview of the BQ algorithm, whose implementation details are described in Section 6. The memory management mechanism we used is covered in Section 7. The algorithm's correctness is laid out in Section 8. Section 9 describes measurement results. Section 10 lays out a possible portability adjustment of the algorithm. We conclude the article in Section 11.

2 PRELIMINARIES

Model. We consider a standard shared memory setting, with a set of threads accessing the shared memory using the atomic primitives Read, Write, and CAS. A CAS primitive is defined by a triplet consisting of a target memory address, an expected value, and a new value. A CAS operation compares the value stored in the target address to the expected value. If they are equal, the value in the target address is replaced with the new value, and the Boolean value true is returned. In such a case, we say that the CAS is successful. Otherwise, the shared memory remains unchanged, and false is returned. A single-word CAS is supported on nearly all platforms (possibly using the equivalent LL/SC instructions). Some platforms also support a double-width CAS, which applies to data residing on two adjacent words.

Future. A *future* is an object returned by an operation whose execution might be delayed. The user may call an *Evaluate* method to ensure the operation's execution and get its result. We assume for simplicity, following Kogan and Herlihy [17], that a future may be evaluated by its creator thread only.

Lock-freedom. A concurrent object implementation is *lock-free* [12] if each time a thread executes an operation on the object, some thread (not necessarily the same one) completes an operation on the object within a finite number of steps. Thus, lock-freedom guarantees system-wide progress. Our implementation is lock-free.

MS-queue. BQ extends MSQ to support future operations. MSQ is a lock-free algorithm for a FIFO queue, which supports *Enqueue* and *Dequeue* operations. It implements the queue as a singly linked list with *head* and *tail* pointers. As such, *head* points to the first node of the list, which functions as a dummy node. The following nodes, starting with the node pointed to by the dummy node's *next* pointer and ending with the node whose *next* pointer's value is NULL, contain the queue's items. The queue is initialized as a list containing a single (dummy) node, to which both *head* and *tail* point. This setup represents an empty queue.

Dequeuing is implemented as follows. If *head*->*next* is NULL, the queue is empty, and hence the dequeue operation returns without extracting an item from the queue. Otherwise, an attempt is made to update *head* to point to its successive node in the list, using CAS. On the occasion that the CAS fails, the dequeue operation starts over.

Enqueuing requires two CAS operations. Initially, a node with the item to enqueue is created. Then, an attempt to set *tail*->*next* to the address of the new node is made using a first CAS. The CAS fails if the current value of *tail*->*next* is not NULL. In such a case, *tail* is advanced to the current value of *tail*->*next* using an assisting CAS, to help an obstructing enqueue operation complete. Then, a new attempt to perform the first CAS starts. After the first CAS succeeds, a second CAS is applied to update *tail* to point to the new node. There is no need to retry this CAS, since it fails only if another thread has already performed the required update, trying to help our operation complete to next apply its own operation.

3 LINEARIZABILITY AND FUTURES

We describe the original linearizability [14], defined for a setting of no future operations, and generalize it for a setting with future operations. Some basic terms are required first. A *method call* is described by two events: its *invocation*, which refers to the call to the method, and its *response*, which refers to the return from the method. Each object has a *sequential specification*, which describes its behavior in sequential executions, where method calls do not overlap.

3.1 Linearizability

An execution is considered *linearizable* [14] if each method call appears to take effect at once, between its invocation and its response events, in a way that satisfies the sequential specification of the objects.

3.2 Medium Futures Linearizability (MF-Linearizability)

Medium futures linearizability is defined by Kogan and Herlihy [17] as an extension of linearizability to futures, which we adopt and extend. For each future method m , whose future evaluation returns a result object T , we assume there is a corresponding single method that returns a T object, and that the meaning of m is given by the object's sequential specification for that corresponding method. For each future operation, we look in the history at two associated method calls: the future one, which creates a future and returns it, and *Evaluate*, which is called with the future returned by the first method call and ensures the operation's execution. MF-linearizability requires the following:

- (1) Each operation takes effect (according to the object's sequential specification for its corresponding single method) at some instant between the invocation of its first related method (which produces the future) and the response of its second related method (which evaluates the future).
- (2) Two operations issued by the same thread to the same object take effect in the order of their first method calls (i.e., their future method calls).

3.3 Extended Medium Futures Linearizability (EMF-Linearizability)

The original paper did not refer to histories containing single operations. We extend the MF-linearizability definition to cover such histories. As mentioned in Section 3.2, the original paper assumes in its formal model that for each future method there is a corresponding single method. We extend the MF-linearizability definition to cover data structures that supply the user with future-returning operations and also corresponding single operations. We allow only single operations that correspond to the interface future operations. For example, the new definition covers a queue that supplies single dequeue and enqueue operations in addition to future dequeue, future enqueue, and *Evaluate* operations. We define the extension by reduction to MF-linearizability: informally, we transform an execution that possibly contains single calls into one that contains only future-returning operations, by replacing each single operation call with an adequate future call followed by an *Evaluate* call.

Next, we define *extended medium futures linearizability* (EMF-linearizability) formally.

Definition 3.1. Let ob be an object that supplies future-returning operations and also corresponding single operations, and let H be a history of ob , consisting of operation invocation and response events. We construct a new history H_f , denoted the *future history*, as follows. The invocations and responses of all future-returning operations as well as *Evaluate* calls are copied to H_f unchanged. Each single call op in H is rewritten in H_f : its invocation is replaced with an invocation and an immediate response of the corresponding future operation, and its response is replaced with an invocation and an immediate response of an *Evaluate* call that evaluates this future.

Definition 3.2. A history H is EMF-linearizable if its future history H_f is MF-linearizable.

3.4 Atomic Execution

We define *atomic execution*, a property of an EMF-linearizable object with future methods regarding its linearization, and then describe its benefit.

Definition 3.3. Let ob be an EMF-linearizable object. ob satisfies *atomic execution* if for each history H of ob there exists a linearization L of the corresponding future history H_f that satisfies MF-linearizability, such that for each thread t in the history the following holds:

- (1) Future operations of t on ob in H are linearized in L only during a later $ob.Evaluate$ call by t , or during a later single operation call on ob by t . (Namely, future operations may not be immediately linearized but rather only during following non-future calls.)
- (2) Let op be a call on ob in H by t (of either *Evaluate* or a single operation) during which some future operations of t on ob , denoted op_1, \dots, op_n , are linearized in L . op_1, \dots, op_n are linearized successively without any other operation on ob linearized between them. Moreover, if op is a single operation, then op is also linearized together with op_1, \dots, op_n successively (in their original invocation order), without any other operation on ob linearized between them. (Namely, t 's operations linearized during the same method call are linearized successively, without any operation on ob by another thread linearized between them.)

For an EMF-linearizable object ob , when a single method op is called by a thread t , all future operations of t that have not yet been applied to ob must take effect prior to op to satisfy EMF-linearizability. Atomic execution dictates that they are executed at once together with op .

Similarly, when t calls the *Evaluate* method for some future operation op , all t 's future operations preceding op must take effect prior to op to satisfy EMF-linearizability. Additional future operations by t (ones that succeed op) may be applied as well during this *Evaluate* call. Atomic execution dictates that all operations applied during this *Evaluate* call are executed at once.

An example of a potential benefit of atomic execution is that it can achieve locality for a producers-consumers system, where consumer servers handle requests of remote producer clients. In such scenario, the clients enqueue their requests, possibly several at a time, to a shared queue. Each server consumes requests regularly by performing a batch operation consisting of a certain number of dequeues. Both clients and servers perform batch operations by calling several future operations, followed by an *Evaluate* operation of the last future operation.

Serving requests of the same client consecutively may allow for more efficient processing due to locality of the client's data. A queue that supports batching and satisfies atomic execution would enable the servers to exploit locality and successively serve several requests by the same client, which he applied in the same batch-of-enqueues operation. This is thanks to the atomic execution's guarantee that both a batch-of-enqueues operation by a client and a batch-of-dequeues operation by a server take effect instantaneously.

4 RELATED WORK

Various works introduce lock-free linearizable FIFO queues, which use different strategies to outperform MSQ.

Tsigas and Zhang [27] present a queue that allows the head and tail to lag at most m nodes behind the actual head and tail of the queue, so the amortized number of CAS executions per operation is $1 + 1/m$. Their algorithm is limited to bounded queues due to their static allocation. Additional cyclic array queues are described in other works [3, 6, 26]. Moir et al. [24] present a queue that uses elimination as a back-off strategy to increase scalability: pairs of concurrent enqueue and dequeue method calls may exchange values without accessing the shared queue. However, to keep the FIFO queue semantics, the enqueue method can be eliminated only after all items of preceding enqueue operations have been dequeued, which makes the algorithm practical only for nearly empty or highly overloaded queues. Hoffman et al. [15] present the baskets queue, which increases scalability by allowing concurrent enqueue operations to insert nodes at adjacent positions at the end of the linked list, regarded as baskets. Such insertion, however, is done only after a failed initial

attempt to append the node to the tail. Thus, the contention on the tail is only partially diminished, and there is also contention on the baskets.

Ladan-Mozes and Shavit [19] present an optimistic queue, which replaces one of the two CAS operations performed during an enqueue operation with simple stores. Like the original msq, this algorithm does not scale, due to synchronization on the head and tail variables that allows only one enqueue operation and one dequeue operation to be applied concurrently. Gidenstam et al. [7] present a cache-aware queue that stores the items in fixed-size blocks, connected in a linked list. This allows for a lazy update of the head and tail, only once per block. Nevertheless, at least one CAS per operation is still required, making the queue non-scalable under high contention. Morrison and Afek [25] present a queue based on a linked list of ring queue nodes. To reduce contention, it relies on the fetch-and-add primitive to spread threads among items in the queue and let them operate in parallel. Yang and Mellor-Crummey [28] utilize fetch-and-add as well, to form a wait-free queue. Queues that improve scalability by relaxing the sequential specification of the queue appear in other works [1, 10, 16]. For example, Basin et al. [1] suggest to trade fairness for lower contention by relaxing the FIFO semantics of the queue. The extension of linearizability BQ adheres to could be viewed as a relaxation, but a stricter one, as it forces FIFO semantics and preserves process order.

Previous works [e.g., 4, 5, 8, 9, 11, 13, 18] present concurrent constructs that combine multiple operations into a single operation on the shared object. We chose to combine operations and apply them as batches, to increase scalability. The work of Kogan and Herlihy [17] is the closest to this work. They propose alternative definitions for linearizability of executions with batches, including MF-linearizability, which we use. They describe very simple implementations of stacks, queues, and linked lists that demonstrate the benefits of using futures. In this work, we propose a novel implementation of the queue that obtains better scalability and performance. Moreover, BQ satisfies atomic execution, whereas Kogan and Herlihy's simple queue does not.

5 ALGORITHM OVERVIEW

We present BQ, an extension to msq, which supports deferred operations and satisfies EMF-linearizability. Unlike standard operations, deferred operations need not be applied to the shared queue before their responses occur. When a future method is called, its details are recorded locally together with previous deferred operations that were called by the same thread. A *Future* object is returned to the caller, who may evaluate it later.

Deferred operations allow to apply pending operations (namely, future operations that have not yet been applied to the shared queue) in batches: BQ delays their execution until the user explicitly evaluates a future of one of them or calls a standard method. When that happens, all pending operations of the same thread are gathered to a single batch operation. This operation is then applied to the shared queue. Afterward, the batch execution is finalized by locally filling the futures' return values. This mechanism reduces synchronization overhead by allowing fewer accesses to the shared queue, as well as less processing in the shared queue—thanks to the preparations performed locally by the initiating thread during the run of each future operation, and the local pairing of applied futures with results following the batch execution.

5.1 Batch Execution

Whenever a deferred enqueue operation is called, the executing thread appends its item to a local list. This way, when the thread has to perform a batch operation, the list of nodes to be linked to the shared queue's list is already prepared.

The key to applying all operations of a batch at once to the shared queue is to set up a moment in which the state of the queue is “frozen.” Namely, we establish a moment in which we hold both ends of the queue so that we know its head and tail, and its size right before the batch takes effect.

This way we can unambiguously determine the queue's shape after applying the batch, including its new head and tail. We achieve a hold of the queue's ends by executing a batch operation in stages, according to the following scheme.

The thread first creates an announcement describing the required batch operation. An announcement is an auxiliary object used to announce an ongoing batch operation so that other threads will not interfere with it but rather help it complete. Then, the thread modifies the shared queue's head to point to the created announcement. This marks the head so that further attempted dequeues will help the batch execution be completed before executing their own operations. Now we hold one end of the queue.

Next, the initiating thread or an assisting thread links the list of items, which the initiating thread has prepared in advance, after the shared queue's tail. This determines the tail location after which the batch's items are enqueued. Thus, now we hold both ends of the queue, as required. We then update the shared queue's tail to point to the last linked node.

As a last step that would uninstall the announcement and finish the batch execution, we update the shared queue's head. It is possible that during the execution of the required enqueues and dequeues the queue becomes empty and that some of the dequeues operate on an empty queue and return NULL. We make a combinatorial observation that helps quickly determine the number of non-successful dequeues. This number is used to determine the node to which the queue's head points following the batch execution. By applying this fast calculation, we execute the batch with minimal interference with the shared queue, thus reducing contention. This computation is described in Section 5.2.

The entire algorithm, including the process of setting futures' results, is discussed in detail in Section 6.2.

5.2 A Key Combinatorial Property of Batches on Queues

Let us state combinatorial observations that help us execute the local batch quickly on the queue. The enqueued items in the batch are kept as a linked list so that they can be attached at the end of the list in a single CAS. This list is added to the tail of the queue and then *#successfulDequeues* dequeues are executed by pushing the head *#successfulDequeues* nodes further in the shared linked-list representing the queue, then dequeued items are privately matched with the batch dequeue operations. In the simplest scenario, *#successfulDequeues* equals the number of future dequeues in the batch. The problem is that some dequeues may operate on an empty queue and thus must return a NULL value. The following discussion explains how the adequate *#successfulDequeues* can be computed.

Definition 5.1. We call a future dequeue a *failing dequeue* with respect to a given state of the shared queue, if the application of the batch that contains it (as well as the other local pending operations) on this shared queue makes this dequeue operate on an empty shared queue. A future dequeue that is not failing is called a *successful dequeue*.

Note that a failing dequeue does not modify the queue, and its future's result is NULL.

We start by analyzing the execution of a batch on an empty queue (which can be analyzed independently of the current shape of the shared queue) and then we show that this analysis can be extended to a general shared queue, simply by plugging the shared queue size.

Definition 5.2. An *excess dequeue* is a future dequeue operation that is a failing dequeue with respect to an empty queue.

For example, if the sequence of pending operations (i.e., not-yet-applied future operations) in some thread is EDDEEDDEDDEE, where E and D represent enqueue and dequeue operations respectively, then the thread has three excess dequeues (the second, fifth, and seventh).

An excess dequeue is a special case of a failing dequeue. We start by computing how many excess dequeues there are in a batch.

LEMMA 5.3. *Let B be a batch of queue operations. The number of excess dequeues in this batch equals the maximum over all prefixes of this batch, of the number of dequeues in the prefix minus the number of enqueues in this prefix.*

PROOF. First, we note that if, for some prefix p of the batch operations, the number of dequeues minus the number of enqueues is k , then the overall number of excess dequeues must be at least k . This is simply because when executing the prefix p on the empty queue, the number of items that enter the queue is $\#enqueues$, the number of enqueues in the prefix, whereas $\#dequeues$, the number of dequeues that are executed in this prefix, is larger by k . So at least k dequeues must operate on an empty queue (returning NULL).

However, we show by induction on the number of excess dequeues that in the prefix that ends in the ℓ^{th} excess dequeue, $\#dequeues - \#enqueues \geq \ell$. We inspect the execution of the prefix on an empty queue. The base of the induction follows from the fact that the first excess dequeue must happen when the number of dequeues so far exceeds the number of enqueues. (Otherwise, there is an item to dequeue.) For the induction step, we look at the prefix of the batch that ends in the $\ell - 1^{st}$ excess dequeue. By the induction hypothesis, for that prefix $\#dequeues - \#enqueues \geq \ell - 1$. In addition, the queue must be empty after (any excess dequeue and in particular after) the $\ell - 1^{st}$ excess dequeue. So the subsequence of operations between the $\ell - 1^{st}$ excess dequeue and the ℓ^{th} excess dequeue operates on an empty queue and has an excess dequeue at the end, which means that for this subsequence $\#dequeues - \#enqueues \geq 1$ (like in the base case of the induction), and we are done. \square

Now we proceed to discuss a batch applied to a queue of any size.

CLAIM 5.4. *Let n be the size of the queue right before a given batch is operated on it. The number of failing dequeues in the batch with respect to a queue of size n equals to the maximum value of $(\#dequeues - \#enqueues - n)$ over all prefixes of the batch's operation sequence, or 0 if this maximum is negative.*

The claim can be proven by adjusting the proof of Lemma 5.3 to failing dequeues instead of excess dequeues, and to a queue of general size n rather than 0. Note that the first n excess dequeues are not failing dequeues because they can dequeue the n items in the original queue. Any additional excess dequeues will become failing dequeues.

Claim 5.4 and Lemma 5.3 yield the following corollary.

COROLLARY 5.5. *Let n be the size of the queue right before a given batch is operated on it. The number of failing dequeues in the batch equals to $\max\{\#excessDequeues - n, 0\}$.*

It immediately follows that the number of successful dequeues in a batch with respect to a queue of size n equals:

$$\#successfulDequeues = \#dequeues - \max\{\#excessDequeues - n, 0\}.$$

5.2.1 Using the Combinatorial Property in BQ. To optimize the calculation of the new head after a batch is applied, each thread maintains three local counters: the quantities of *FutureEnqueue* and *FutureDequeue* operations that have been called but not yet executed on the shared queue, and the number of excess dequeues. The thread updates these counters on each of its future operation calls. When a thread executes a batch operation, it includes its local counters in the batch's announcement, to allow any helping thread to complete the batch execution.

In addition, we let the shared queue's head and tail contain not only a pointer but also a successful dequeue and enqueue counters, respectively. When applying a batch, they are updated using the announcement's counts. The difference between the queue's enqueue and dequeue counts prior to a batch execution yields the queue's size n in its "frozen" state right before linking the batch's items.

These counters in the batch's announcement and in the head and tail are used to quickly calculate the number of successful dequeues according to Corollary 5.5. This number helps discovering the new head, by iterating over *#successfulDequeues* nodes, and avoids a heavier simulation of the batch enqueues and dequeues one by one to discover the shape of the resulting shared queue.

Indeed, to determine the result of each future dequeue in the batch, the thread that initiated the batch operation will need to simulate these future operations according to their order. Nevertheless, it will conduct this simulation after the announcement is removed from the shared queue, without delaying other threads that access the shared queue.

6 ALGORITHM DETAILS

We now turn to the details of the algorithm. In Section 6.1, we elaborate on the principal underlying data structures, and in Section 6.2, we describe the algorithm. Memory management is covered in Section 7.

6.1 Underlying Data Structures

Table 1 describes the fields of the data structures used in BQ's implementation, out of which the *Future* and *Queue* structures are the only ones exposed to the user of the *Queue* object, whereas all others are internal to the queue's implementation.

`std::atomic<T>` is a type created by the `std::atomic` template of the C++ atomic standard library. For any type T , the `std::atomic` library implements every access (read, write, or CAS) to a variable of type `std::atomic<T>` atomically.

6.1.1 Queue. Similarly to *MSQ*, the shared queue is represented as a linked list of nodes in which the first node is a dummy node, and all nodes thereafter contain the values in the queue in the order they were enqueued. We maintain pointers to the first and last nodes of this list in the queue's head and tail, respectively.

Batch operations require the size of the queue for a fast calculation of the new head after applying the batch's operations. To this end, *Queue* maintains counters of the number of enqueues and the number of successful dequeues applied so far. These are kept side-by-side with the tail and head pointers respectively, and they are updated atomically with the respective pointers using a double-width CAS. The tail, denoted *SQTail* (which stand for *Shared Queue's Tail*), is implemented using an atomic Pointer and Count object (`std::atomic<PtrCnt>`, which is atomically modified).

Batch operations that enqueue at least one item install an announcement in the head. The queue's head, denoted *SQHead*, can either hold the aforesaid *PtrCnt* object with a pointer to the head of the queue, or a pointer to an *Ann* object, described next. Therefore, *SQHead* is an atomic Pointer and Count or Announcement object (`std::atomic<PtrCntOrAnn>`). *PtrCntOrAnn* is a 16-byte union that may consist of either *PtrCnt* or an 8-byte tag and an 8-byte *Ann* pointer. Whenever it contains an *Ann*, the tag is set to 1. Otherwise, it contains a *PtrCnt* (the tag overlaps *PtrCnt.node*, whose least significant bit is 0 since it stores either NULL or an aligned address).

For brevity, we will mostly avoid specific mention of the counter; however, when we refer to an update of the head's pointer, it means that the head's counter is updated as well, and likewise for the tail.

Table 1. Underlying Data Structures

struct Node	{item: Item*, next: std::atomic<Node*>}
struct BatchRequest	{firstEnq: Node*, lastEnq: Node*, enqsNum: unsigned int , deqsNum: unsigned int , excessDeqsNum: unsigned int }
struct PtrCnt	{node: Node*, cnt: unsigned int }
struct Ann	{batchReq: BatchRequest, oldHead: std::atomic<PtrCnt>, oldTail: std::atomic<PtrCnt>}
union PtrCntOrAnn	{ptrCnt: PtrCnt, struct {tag: unsigned int , ann: Ann*}}
struct Queue	{SQHead: std::atomic<PtrCntOrAnn>, SQTail: std::atomic<PtrCnt>}
struct Future	{result: Item*, isDone: Boolean }
struct FutureOp	{type: {ENQ, DEQ}, future: Future*}
struct ThreadData	{opsQueue: Queue of FutureOp, enqsHead: Node*, enqsTail: Node*, enqsNum: unsigned int , deqsNum: unsigned int , excessDeqsNum: unsigned int }

It is possible to avoid the double-width CAS in platforms that do not support such an operation. This can be accomplished by replacing the *PtrCnt* object with a pointer to a node, replacing the *PtrCntOrAnn* object with a pointer to either a node or an announcement (with a least significant bit mark indicating the type of the pointed object), and have the *Node* object contain a counter. We describe this variation of BQ in Section 10. Measurements demonstrate that it does not incur a significant performance degradation.

Thread-local data. A *threadData* array holds local data for each thread. First, the pending operations details are kept, in the order they were called, in an operation queue *opsQueue*, implemented as a simple local non-thread-safe queue. It contains *FutureOp* items. Second, the items of the pending enqueue operations are kept in a linked list in the order they were enqueued by *FutureEnqueue* calls. This list is referenced by *enqsHead* and *enqsTail* (with no dummy nodes here). Last, each thread keeps record of the number of *FutureEnqueue* and *FutureDequeue* operations that have been called but not yet applied, and the number of excess dequeues.

Each thread can access its local data in *threadData* using its thread ID as an index. In the pseudo-code, *threadData[threadId]* is abbreviated to *threadData* for brevity.

6.1.2 Future. A *Future* contains a *result*, which holds the return value of the deferred operation that generated the future (for dequeues only, as enqueue operations have no return value) and an *isDone* Boolean value, which is true only if the deferred computation has been completed. When *isDone* is false, the contents of *result* may be arbitrary.

6.1.3 BatchRequest. A *BatchRequest* is prepared by a thread that initiates a batch and consists of the details of the batch's pending operations: *firstEnq* and *lastEnq* are pointers to the first and last nodes of a linked list containing the pending items to be enqueued; *enqsNum*, *deqsNum*, and *excessDeqsNum* are, respectively, the numbers of enqueues, dequeues, and excess dequeues in the batch.

6.1.4 Announcement. An *Ann* object represents an announcement. It contains a *BatchRequest* instance, with all the details required to execute the batch operation it stands for. Thus, any operation that encounters an announcement may help the related batch operation complete before proceeding with its own operation.

In addition to information regarding the batch of operations to execute, *Ann* includes *oldHead*, the value of the head pointer (and dequeue counter) before the announcement was installed, and *oldTail*, an entry for the tail pointer (and enqueue counter) of the queue right before the batch is applied (i.e., a pointer to the node to which the batch's list of items is linked).

6.2 Algorithm Implementation

We detail the algorithm implementation, accompanied by pseudo-code. First, we describe the core operations, performed on the shared queue. Then, we outline the enclosing methods, which call the core methods and carry out complementary local computations. Finally, we refer to the special case of a dequeues-only batch operation.

6.2.1 Internal Methods Operating on the Shared Queue. The following methods are used internally to apply operations to the shared queue: *EnqueueToShared*, *DequeueFromShared*, and *ExecuteBatch*. To help a concurrent batch execution and obtain the new head, they call the *HelpAnn-AndGetHead* auxiliary method. To carry out a batch, the *ExecuteAnn* auxiliary method is called. Its caller may be either the batch's initiating thread or a helping thread that encountered an announcement when trying to execute its own operation.

Let us elaborate on each of these methods.

EnqueueToShared. *EnqueueToShared* appends an item after the tail of the shared queue, using two CAS operations, in a similar manner to *msq*'s *Enqueue*: it first updates *SQTail.node->next* to point to a node consisting of the new item and then updates *SQTail* to point to this node. An obstructing operation might enqueue its items concurrently, causing the first CAS (in line 5) to fail. In this case, *EnqueueToShared* would try to help complete the obstructing operation, before starting a new attempt to enqueue its own item. This assistance is performed in lines 9 through 13. Herein lies the distinction between *EnqueueToShared* in *BQ* and *Enqueue* in *msq*. In *msq*, the first CAS might fail only due to an obstructing enqueue operation, and thus only the equivalent to line 13 of *BQ* is executed. In *BQ*, however, the obstructing operation may be either a standard enqueue operation or a batch operation.

Listing 1. *EnqueueToShared*

```

1 EnqueueToShared(item)
2   newNode = new Node(item, NULL)
3   while (true)
4     tailAndCnt = SQTail
5     if (CAS(&tailAndCnt.node->next, NULL, newNode))
6       // newNode is linked to the tail
7       CAS(&SQTail, tailAndCnt, (newNode, tailAndCnt.cnt + 1))
8     break
9   head = SQTail

```

```

10     if head consists of Ann: // (head.tag & 1 != 0)
11         ExecuteAnn(head.ann)
12     else
13         CAS(&SQTail, tailAndCnt, ⟨tailAndCnt.node->next, tailAndCnt.cnt + 1⟩)

```

DequeueFromShared. If the queue is not empty when the dequeue operation takes effect, *DequeueFromShared* extracts an item from the head of the shared queue and returns it; otherwise, it returns NULL. The only addition to the MSQ's *Dequeue* is helping pending batch operations complete first by calling the *HelpAnnAndGetHead* method.

Listing 2. DequeueFromShared

```

14 Item* DequeueFromShared()
15     while (true)
16         headAndCnt = HelpAnnAndGetHead()
17         headNextNode = headAndCnt.node->next
18         if (headNextNode == NULL)
19             return NULL
20         if (CAS(&SQHead, headAndCnt, ⟨headNextNode, headAndCnt.cnt + 1⟩))
21             return headNextNode->item

```

HelpAnnAndGetHead. This auxiliary method assists announcements in execution, as long as there is an announcement installed in *SQHead*.

Listing 3. HelpAnnAndGetHead

```

22 PtrCnt HelpAnnAndGetHead()
23     while (true)
24         head = SQHead
25         if head consists of PtrCnt: // (head.tag & 1 == 0)
26             return head.ptrCnt
27         ExecuteAnn(head.ann)

```

ExecuteBatch. *ExecuteBatch* is responsible for executing the batch. Before it starts doing so, it checks whether there is a colliding ongoing batch operation whose announcement is installed in *SQHead*. If so, *ExecuteBatch* helps it complete (line 31). Afterward, it stores the current head in *ann* (line 32), installs *ann* in *SQHead* (line 33), and calls *ExecuteAnn* to carry out the batch. The batch execution's steps are illustrated in Figure 1.

Listing 4. ExecuteBatch

```

28 Node* ExecuteBatch(batchRequest)
29     ann = new Ann(batchRequest)
30     while (true)
31         oldHeadAndCnt = HelpAnnAndGetHead()
32         ann->oldHead = oldHeadAndCnt // Step 1 in Figure 1
33         if (CAS(&SQHead, oldHeadAndCnt, ann)) // Step 2 in Figure 1
34             break
35         ExecuteAnn(ann)
36     return oldHeadAndCnt.node

```

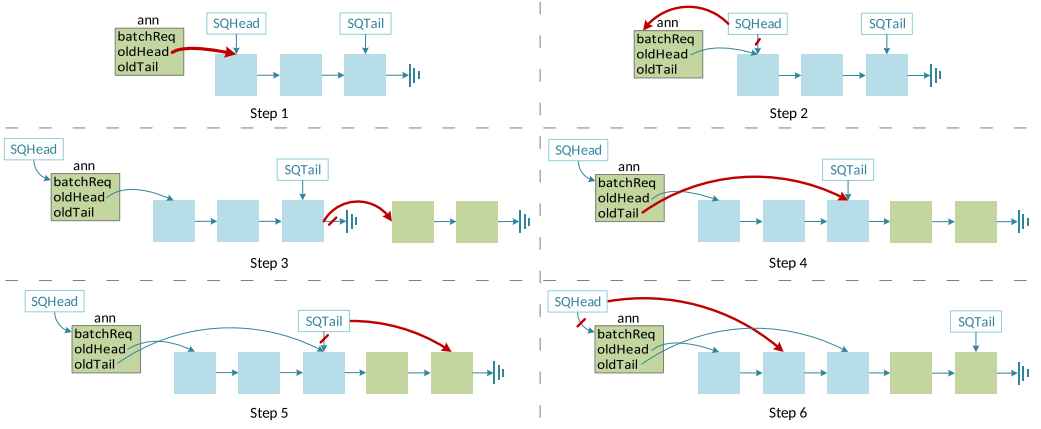


Fig. 1. Steps of the batch execution: (1) Setting `ann`→`oldHead` to the head of the queue right before the batch. (2) Installing `ann` in `SQHead`. (3) Linking the batch's items to `SQTail`.node→`next`. (4) Setting `oldTail` field in the installed announcement `ann`. (5) Advancing `SQTail` to point to the last node enqueued by the batch operation (and increasing its enqueue count by the number of enqueues). (6) Setting `SQHead` to point to the last node dequeued by the batch operation in place of `ann` (and increasing its dequeue count by the number of successful dequeues).

ExecuteAnn. *ExecuteAnn* is called with `ann` after `ann` has been installed in `SQHead`, and `ann`'s `oldHead` field consists of the value of `SQHead` right before `ann`'s installation. *ExecuteAnn* carries out `ann`'s batch. If any of the execution steps has already been executed by another thread, *ExecuteAnn* moves on to the next step. Specifically, if `ann` will have been removed from `SQHead` by the time *ExecuteAnn* is executed, `ann`'s execution will have been completed, and all the steps of this run of *ExecuteAnn* would fail and have no effect.

ExecuteAnn first makes sure that `ann`'s enqueued items are linked to the queue, in the while loop in line 39. If they have already been linked to the queue, and the old tail after which they were linked has also been recorded in `ann`, it follows that another thread has completed the linking, and thus we break out of the loop in line 43. Otherwise, we try to link the items by performing a CAS operation on the next pointer of the node pointed to by the tail in line 44. In line 45, we check whether the items were linked after `tail`, regardless of which thread linked them. If so, we record `tail`, to which the items were linked, in `ann`. Otherwise, we try to help the obstructing enqueue operation complete in line 50 and start over with a new attempt to link the batch's items.

The next step is `SQTail`'s update in line 52. There is no need to retry it, since it fails only if another thread has written the same value on behalf of the same batch operation. Last, we call *UpdateHead* to update `SQHead` to point to the last node dequeued by the batch. This update uninstalls the announcement and completes its handling.

The *UpdateHead* method calculates *successfulDeqsNum* as described in Corollary 5.5. It then determines the new head according to the following optimization. If the number of the batch's successful dequeues is at least the size of the queue before applying the batch, which implies that the new dummy node is one of the batch's enqueued nodes, the new head is determined by passing over *successfulDeqsNum* − *oldQueueSize* nodes, starting with the node pointed to by the old tail. Otherwise, it is determined by passing over *successfulDeqsNum* nodes, starting with the old dummy node. Finally, *UpdateHead* updates `SQHead` (and as in `SQTail`'s update, there is no need to retry the CAS).

Listing 5. ExecuteAnn

```

37 ExecuteAnn(ann)
38   // Link items to tail and update ann
39   while (true)
40     tailAndCnt = SQTail
41     annOldTailAndCnt = ann->oldTail
42     if (annOldTailAndCnt.node != NULL)
43       break
44     CAS(&tailAndCnt.node->next, NULL, ann->batchReq.firstEnq) // Step 3 in Figure 1
45     if (tailAndCnt.node->next == ann->batchReq.firstEnq)
46       // Step 4 in Figure 1:
47       ann->oldTail = annOldTailAndCnt = tailAndCnt
48       break
49     else
50       CAS(&SQTail, tailAndCnt, <tailAndCnt.node->next, tailAndCnt.cnt + 1>)
51     newTailAndCnt = <ann->batchReq.lastEnq, annOldTailAndCnt.cnt + ann->batchReq.
      enqsNum>
52     CAS(&SQTail, annOldTailAndCnt, newTailAndCnt) // Step 5 in Figure 1
53     UpdateHead(ann)
54
55 UpdateHead(ann)
56   oldQueueSize = ann->oldTail.cnt - ann->oldHead.cnt
57   successfulDeqsNum = ann->batchReq.deqsNum
58   if (ann->batchReq.excessDeqsNum > oldQueueSize)
59     successfulDeqsNum -= ann->batchReq.excessDeqsNum - oldQueueSize
60   if (successfulDeqsNum == 0)
61     CAS(&SQHead, ann, ann->oldHead) // Step 6 in Figure 1
62     return
63   if (oldQueueSize > successfulDeqsNum)
64     newHeadNode = GetNthNode(ann->oldHead.node, successfulDeqsNum)
65   else
66     newHeadNode = GetNthNode(ann->oldTail.node, successfulDeqsNum - oldQueueSize)
67   CAS(&SQHead, ann, <newHeadNode, ann->oldHead.cnt + successfulDeqsNum>) // Step 6 in
      Figure 1
68
69 Node* GetNthNode(node, n)
70   repeat n times:
71     node = node->next
72   return node

```

6.2.2 Interface Methods. The queue's interface methods exposed to the user are *Enqueue*, *Dequeue*, *FutureEnqueue*, *FutureDequeue*, and *Evaluate*. These methods wrap the methods that access the shared queue, which are detailed in Section 6.2.1. After describing them, we will elaborate on the *PairFuturesWithResults* auxiliary method, which is called by *Evaluate*, and locally sets the futures' results to complete the batch operation.

Enqueue. *Enqueue* checks whether the thread-local operation queue *opsQueue* is empty. If it is, it directly calls *EnqueueToShared*. Otherwise, to satisfy EMF-linearizability, the pending operations in *opsQueue* must be applied before the current *Enqueue* is applied. Hence, *Enqueue* calls *FutureEnqueue* with the required item, which in turn returns a future. It then calls *Evaluate* with that future. This results in applying all preceding pending operations, as well as applying the current operation.

Listing 6. Enqueue

```

73 Enqueue(item)
74     if (threadData.opsQueue.Empty())
75         EnqueueToShared(item)
76     else
77         Evaluate(FutureEnqueue(item))

```

Dequeue. The implementation of *Dequeue* is similar to the one of *Enqueue*. If *Dequeue* succeeds, it returns the dequeued item; otherwise (the queue is empty when the operation takes effect), it returns NULL.

Listing 7. Dequeue

```

78 Item* Dequeue()
79     if (threadData.opsQueue.Empty())
80         return DequeueFromShared()
81     else
82         return Evaluate(FutureDequeue())

```

FutureEnqueue. *FutureEnqueue* adds the item to be enqueued to thread's list of items pending to be enqueued. This list will be appended directly to the end of the shared queue's list of nodes when a batch operation is executed by this thread. This is the reason these items are stored in a linked list of nodes rather than directly in *opsQueue*. *FutureEnqueue* also updates the local numbers of pending enqueue operations. In addition, *FutureEnqueue* enqueues a *FutureOp* object representing an enqueue operation to the thread's *opsQueue*. A pointer to the *Future* object encapsulated in the created *FutureOp* will be returned by the method, so the caller could later pass it to the *Evaluate* method.

Listing 8. FutureEnqueue

```

83 Future* FutureEnqueue(item)
84     AddToEnqsList(item)
85     ++threadData.enqsNum
86     return RecordOpAndGetFuture(ENQ)
87
88 AddToEnqsList(item)
89     node = new Node(item, NULL)
90     if (threadData.enqsHead == NULL)
91         threadData.enqsHead = node
92     else
93         threadData.enqsTail->next = node
94     threadData.enqsTail = node
95
96 Future* RecordOpAndGetFuture(futureOpType)
97     future = new Future()
98     threadData.opsQueue.Enqueue((futureOpType, future))
99     return future

```

FutureDequeue. *FutureDequeue* updates the local numbers of pending dequeue operations and excess dequeues. The latter is calculated based on Lemma 5.3. *FutureDequeue* then enqueues a

FutureOp object representing a dequeue operation to the thread's *opsQueue*. Like *FutureEnqueue*, *FutureDequeue* returns a pointer to a *Future* object.

Listing 9. FutureDequeue

```

100 Future* FutureDequeue()
101     ++threadData.deqsNum
102     threadData.excessDeqsNum = max(threadData.excessDeqsNum, threadData.deqsNum -
        threadData.enqsNum)
103     return RecordOpAndGetFuture(DEQ)

```

Evaluate. *Evaluate* receives a future and ensures it is applied when the method returns. We assume for simplicity, following Kogan and Herlihy [17], that a future may be evaluated by its creator thread only.

If the future has already been applied from the outset, its result is immediately returned. Otherwise, all locally pending operations found in *threadData.opsQueue* are applied to the shared queue at once. After the batch operation's execution completes, while new operations may be applied to the shared queue by other threads, the batch operation results are paired to the appropriate futures of operations in *opsQueue*.

If *opsQueue* consists of at least one enqueue operation, the batch operation's execution and the results-to-futures pairing are accomplished by calling *ExecuteBatch* and *PairFuturesWithResults* (described next), respectively. If all pending operations are dequeues, we pursue a different course of action, on which we elaborate in Section 6.2.3.

Listing 10. Evaluate

```

104 Item* Evaluate(future)
105     if (!future->isDone)
106         ExecuteAllPending()
107     return future->result
108
109 ExecuteAllPending()
110     if (threadData.enqsNum == 0)
111         // No enqueues. Execute a dequeues-only batch
112         <successDeqsNum, oldHeadNode> = ExecuteDeqsBatch()
113         PairDeqFuturesWithResults(oldHeadNode, successDeqNum)
114     else
115         // Execute a batch operation with at least one enq
116         oldHeadNode = ExecuteBatch(
117             <threadData.enqsHead,
118             threadData.enqsTail,
119             threadData.enqsNum,
120             threadData.deqsNum,
121             threadData.excessDeqsNum>)
122         PairFuturesWithResults(oldHeadNode)
123         threadData.enqsHead = NULL
124         threadData.enqsTail = NULL
125         threadData.enqsNum = 0
126         threadData.deqsNum = 0
127         threadData.excessDeqsNum = 0

```

PairFuturesWithResults. *PairFuturesWithResults* receives the old head. It simulates the pending operations one by one according to their original order, which is recorded in the thread's *opsQueue*. Namely, it simulates updates of the head and tail of the shared queue. This is done by advancing *nextEnqNode* (which represents the value of *tail->next* in the current moment of the simulation) on each enqueue, and by advancing *currentHead* on dequeues that occur when the queue in its current state is not empty. The simulation is run to set results for future objects related to the pending operations and mark them as done.

Listing 11. PairFuturesWithResults

```

128 PairFuturesWithResults(oldHeadNode)
129   nextEnqNode = threadData.enqsHead
130   currentHead = oldHeadNode
131   noMoreSuccessfulDeqs = false
132   while threadData.opsQueue is not empty:
133     op = threadData.opsQueue.Dequeue()
134     if (op.type == ENQ)
135       nextEnqNode = nextEnqNode->next
136     else // op.type == DEQ
137       if (noMoreSuccessfulDeqs ||
138         currentHead->next == nextEnqNode)
139         // The queue is currently empty
140         op.future->result = NULL
141       else
142         currentHead = currentHead->next
143         if (currentHead == threadData.enqsTail)
144           noMoreSuccessfulDeqs = true
145         op.future->result = currentHead->item
146   op.future->isDone = true

```

6.2.3 Dequeues-Only Batch. The batch execution scheme outlined in Section 5.1 and detailed in Section 6.2.1 does not work if the batch operation consists solely of dequeue operations, since such a batch does not link items after the tail and does not update the tail. Thus, steps 3 through 5 in Figure 1 that handle the enqueues and the tail become irrelevant. This also poses a problem for the new head's calculation in the end of the batch execution (in Method *UpdateHead* in Listing 5), since it uses the counter of the tail to which the batch's items were linked to compute the number of successful dequeues.

As most of the mechanism we presented to handle a batch is redundant when it contains no enqueues, we propose a simpler mechanism for applying a dequeues-only batch. For such a batch, the *Evaluate* method calls *ExecuteDeqsBatch* to apply the batch operation. The *ExecuteDeqsBatch* method first assists a colliding ongoing batch operation if there is any (in line 149). It then calculates the new head and the number of successful dequeues by traversing over the items to be dequeued in the loop in line 152. If there is at least one successful dequeue, the dequeues take effect at once using a single CAS operation in line 160. The CAS pushes the shared queue's head *successfulDeqsNum* nodes forward.

Then *Evaluate* calls *PairDeqFuturesWithResults* to pair the successfully dequeued items to futures of the appropriate operations in *opsQueue*. The remaining future dequeues are unsuccessful, and thus their results are set to NULL.

Listing 12. ExecuteDeqsBatch

```

147 <unsigned int, Node*> ExecuteDeqsBatch()
148     while (true)
149         oldHeadAndCnt = HelpAnnAndGetHead()
150         newHeadNode = oldHeadAndCnt.node
151         successfulDeqsNum = 0
152         repeat threadData.deqsNum times:
153             headNextNode = newHeadNode->next
154             if (headNextNode == NULL)
155                 break
156             ++successfulDeqsNum
157             newHeadNode = headNextNode
158         if (successfulDeqsNum == 0)
159             break
160         if (CAS(&SQHead, oldHeadAndCnt, <newHeadNode, oldHeadAndCnt.cnt +
161             successfulDeqsNum>))
162             break
163     return <successfulDeqsNum, oldHeadAndCnt.node>

```

Listing 13. PairDeqFuturesWithResults

```

163 PairDeqFuturesWithResults(oldHeadNode, successfulDeqsNum)
164     currentHead = oldHeadNode
165     repeat successfulDeqsNum times:
166         currentHead = currentHead->next
167         op = threadData.opsQueue.Dequeue()
168         op.future->result = currentHead->item
169         op.future->isDone = true
170     repeat threadData.deqsNum - successfulDeqsNum times:
171         op = threadData.opsQueue.Dequeue()
172         op.future->result = NULL
173         op.future->isDone = true

```

7 MEMORY MANAGEMENT

We utilized the optimistic access scheme [2], which extends the hazard pointers scheme [21], as a lock-free manual memory management mechanism for BQ. All measurements include use of memory reclamation.

We describe memory management of lock-free data structures in general in Section 7.1 and explain the optimistic access mechanism. Then, we describe how this mechanism is utilized in BQ in Section 7.2.

7.1 Lock-Free Manual Memory Management

A lock-free data structure requires a delicate memory reclamation mechanism. Such mechanism should prevent two risks posed by reclamation: an access to shared memory that has been freed by another thread and the ABA problem (comparing a pointer to an expected value that has been recycled). No efficient lock-free automatic garbage collector exists in the literature. Therefore, to manage memory of lock-free data structures in a lock-free fashion, one should employ a manual memory management scheme. In such schemes, an object that is part of the shared data structure is reclaimed in coordination between the data structure's algorithm and the reclamation procedure.

First, the algorithm unlinks the object from the data structure. Next, to declare that the object is no longer needed, the algorithm announces it as retired. This implies that the object should be reclaimed when with certainty no one might access it or compare its address anymore. A reclamation procedure runs periodically or when there is not enough free memory space, and reclaims the nodes that were announced as retired so far *and* are guaranteed not be accessed or compared later. Each manual memory management scheme dictates a different approach to determine which retired nodes are safe to reclaim.

The lock-free memory management scheme we utilize in our measured implementations of BQ, MSQ, and Kogan and Herlihy's queue is optimistic access [2]. It employs hazard pointers [21] for write operations. In the hazard pointers scheme, each thread owns single-writer multi-reader shared pointers called *hazard pointers*. A thread assigns hazard pointers indicating memory locations it might later access or compare, to protect them from reclamation. This scheme considers a retired node as safe to reclaim if no hazard pointer points to it.

In more detail, in the hazard pointers scheme, each time a node is about to read or write to a memory location, it first points a hazard pointer at that location, then applies an expensive memory fence to ensure that the hazard pointer is visible to all threads, and only afterward validates that the location is still safe to access—has not been retired before the hazard pointer was set—before accessing it. The memory fence is crucial, as without it our set hazard pointer might become visible to other threads only after we run our validation test. In such a case, the validation might pass, although afterward another thread might retire and reclaim the location we are about to access, before our hazard pointer becomes visible to the reclaiming thread. This way the validation does not ensure that the location has not been reclaimed before the hazard pointer was published to other threads, and we might end up accessing a reclaimed location.

Read operations are performed in the optimistic access method without installing hazard pointers and setting memory fences. A read operation first reads the data without a prior check and only then verifies that the read memory location has not been reclaimed. The verification will mostly succeed, but when it does not, the operation should be restarted because it might have read reclaimed memory. To enable reading a possibly deallocated address without triggering a segmentation fault, the algorithm utilizes a user-level allocator. This allocator maintains a pool of objects and does not return pages back to the operating system.

A verification is carried out in both read and write operations: in read operations, after reading a value we verify its memory has not been reclaimed; in write operations, after setting a hazard pointer to an address we verify this address is still safe to use. The verification in optimistic access is performed by confirming that a thread-local flag is not set: to signal that a reclamation phase has started and every object retired so far might be recycled, optimistic access maintains a local warning flag per thread. This flag is set during the reclamation process.

7.2 Applying the Optimistic Access Scheme to BQ

We adapted the optimistic access scheme presented in the work of Cohen and Petrank [2] to our needs. To begin with, we extended it to support both requisite objects (*Node* and *Ann*). For additional details about the adjustment of the original optimistic access mechanism to BQ, refer to Section 7.2.1.

When applying memory management to BQ, we had to make sure that a dequeued item is read before it is retired so that it could be returned to its dequeuer. This requires some delicate manipulations, described in Section 7.2.2 under Retirements.

We also made some optimizations upon the conservative optimistic access usage scheme. In writes to shared locations, we assign hazard pointers only to relevant addresses that might be retired and not to all related addressed. Moreover, we do not use a CAS where it is not necessary in

contrast to the specification of the original scheme. We further explain about write optimizations in Section 7.2.2 under Writes.

We apply another optimization when finding out, during a batch operation's execution, that a warning flag is set. In such cases, we refrain from starting the execution from scratch and instead perform an additional check to determine if we may proceed. See details in Section 7.2.2 in the part that discusses avoiding batch execution restart.

Next, we elaborate further on the adjustment of the optimistic access mechanism to BQ and its usage throughout BQ's algorithm.

7.2.1 Adjusting the Optimistic Access Mechanism. We utilize the optimistic access mechanism described in Section 4 in the work of Cohen and Petrank [2], and we extend it to handle two object sizes: *Node*'s size and *Ann*'s size. We call *MM.Retire* to trigger the mechanism's *Reclaim* function, and *MM.AllocateNode* and *MM.AllocateAnn* to trigger its *Allocate* function for the appropriate object size.

BQ's *ThreadData* entry held by each thread is extended to include the memory management related data: a *warning* Boolean flag as well as hazard pointers (*nodeHp* and *annHp*).

7.2.2 Adjusting BQ's Code. We cover the necessary modifications that must be applied to allocations, retirements, writes, and reads from the shared memory in BQ. The code modifications are presented thereafter.

Allocations. Objects are allocated by a user-level allocator. It allocates a node for a standard enqueue operation in line 175, and for a future enqueue operation in line 377. An announcement object is allocated in line 237.

Retirements. An announcement is retired in line 253 by the same method that created it. Nodes could be retired in several occasions, depending on the way they are dequeued. A node dequeued by a single dequeue operation is retired in line 219 right after advancing the queue's head to the next node. A node dequeued by a batch operation is retired when traversing it during the pairing process of the batch's applied future operations with results (in lines 434 and 454 in case of a batch that includes both enqueues and dequeues, or in lines 470 and 474 for nodes dequeued by a dequeues-only batch operation).

Retiring nodes should be done carefully to ensure retrieving a dequeued item prior to the recycling of its enclosing node. Next, we describe the difficulties in achieving this goal and then detail how we accomplish it.

Recall that the queue's head points to a dummy node. Consequently, each node's matching item lies in its successor node. Let A be an address of a node pointed to by the head, and let $newHead$ equal $A \rightarrow next$, which is a pointer to A 's successor. A is dequeued by setting the head to $newHead$, retiring A , and returning $newHead \rightarrow item$ as the dequeued item. This dequeue operation, which returns $newHead \rightarrow item$, is not the same one that retires $newHead$. We should make sure to read $newHead \rightarrow item$ when $newHead$ is still certainly not retired. Otherwise (i.e., if another dequeue or batch operation retire $newHead$ beforehand), the node pointed to by $newHead$ may be recycled, in which case the dequeuing thread might not be able to get a hold of its dequeued item.

In a single dequeue operation, we make sure to read the item prior to its recycling by reading $newHead \rightarrow item$ before applying a CAS to the queue's head. If we read $newHead \rightarrow item$ and then successfully CAS the head, we know for certain that when we read $newHead \rightarrow item$, $newHead$ has not yet been retired: a node is retired only during the execution of its dequeuing by a dequeue or a future dequeue operation, and the node pointed to by $newHead$ could not have been dequeued before the CAS of head from A to $newHead$ succeeded.

Accomplishing the goal of retrieving an item dequeued by a future operation prior to its recycling is more tricky. The reason is that during a batch execution, we do not read all dequeued items before applying a CAS to the queues' head, because we aim to minimize the synchronization time. Thus, only after CASing the head to complete the batch's effect on the shared queue does the thread that initiated the batch pair its applied futures with results locally. During the pairing process, the initiating thread traverses its successfully dequeued nodes, reads their corresponding items, and retires them. Reading all dequeued items but the last one can be easily performed before retiring the nodes that contain them, as the initiating thread is the one responsible for their retirement. We read the items' values in lines 440 and 460 of *PairFuturesWithResults* and line 472 of *PairDeqFuturesWithResults*, before retiring the node that holds them in the next loop iteration in lines 434 and 454 of *PairFuturesWithResults* and lines 470 and 474 of *PairDeqFuturesWithResults*.

The last dequeued item is problematic: it lies in the node that is pointed by the head after the batch execution. This node is retired by the thread that performs the subsequent dequeue or batch operation. This might not be our batch's initiator but rather another thread.

Therefore, like in a single dequeue operation, the thread that executes the batch operation should read the last dequeued item before performing a CAS of the head, since after this CAS occurs, the node that holds the last dequeued item might be recycled. The remaining question is how this read value is later paired with the appropriate future. The answer depends on the kind of batch that the future dequeue is a part of.

Let us examine a dequeues-only batch first. It requires a single modification to the shared queue's state: a CAS of the head. Therefore, no helping is involved in its execution. The batch's initiator is the only one to apply it to the shared queue and then match the futures with the results. It reads the last dequeued item, which lies in the node that is about to be pointed to by the new head, in line 279. Just like in a single dequeue operation, the item is read before applying a CAS to the queue's head, which enables a recycling of the node that holds this item. Then, as the initiating thread is also the one to pair the batch's futures with results, it simply sets the future's result of the last successful dequeue to this read value in line 476.

The case of a batch operation that contains both dequeues and enqueues is more complicated, because the thread that initiates such batch operation is the one responsible for pairing its futures with results, but the batch execution itself may be carried out by a helping thread. The thread that executes the batch operation should read the last dequeued item before performing the CAS that uninstalls the announcement from the head, and inform the initiating thread of this item. This is mandatory since after the CAS of the head, by the time the future of the last successful dequeue is paired with the appropriate item, the node that holds this item might have already been recycled. To inform the initiating thread about the read item, the thread that executes the batch should keep it in some shared location. We chose to place it in the node that was pointed to by the queue's head prior to the current announcement's installation. This node is under the responsibility of the batch's initiator who should retire it, and thus no one else relies on its content (due to the same arguments). The last dequeued item is read in line 344 and placed in the node pointed to by the old head in line 348 (after verifying that the item was read before its node was possibly changed by a subsequent batch operation or retired). Later, the future of the last successful dequeue is paired with this item in line 463 of Listing 25.

The batch's applied futures are paired with results by the thread that initiated the batch (in Method *PairFuturesWithResults* in Listing 25). However, this thread has not necessarily carried out the batch on its own, so it might not know which item is the last to be successfully dequeued from the queue. Therefore, when it traverses over the batch's operations and pairs them with results, it does not know if a current successful dequeue is the last successful one and its item was stored in the node pointed to by the old head, or it is not the last and its item lies in the next node.

To circumvent this problem, on each successful dequeue the initiator encounters, it sets the result of the previous successful dequeue, which is now revealed not to be the last. When the traversal is over, the last successful future dequeue is revealed, and its result is set to the item that was stored in the node pointed to by the old head node in advance.

Writes. Before each write to a shared location, we set a hazard pointer to the hazardous reference, apply a memory fence to ensure that the hazard pointer is visible to all threads, and check the warning flag to verify we may proceed.

The original optimistic access paper described a conservative approach to protect writes to shared locations: writing only using a CAS and setting hazard pointers to all associated pointers (the location that is about to be written, its expected value, and the new value). However, in most writes, optimizations may apply, and so we avoid any of these hazard pointers where they are not required, as well as refrain from using an avoidable CAS.

One kind of write we guard is to long-lasting shared locations: the shared queue's current head and tail pointers. This kind of write carries the risk of the ABA problem, in case the CAS's expected value is recycled. Hence, we install a hazard pointer to the expected pointer value of head before performing a CAS of the shared queue's head (in lines 207, 241, and 258, where head's previous value is a pointer to a node, and in lines 192 and 228, where its previous value is a pointer to an announcement) and similarly for tail (in lines 179, 293, and 301). The location to which we write is not subjected to an access hazard, since it is not freed during the whole run.

However, an access to a short-lasting shared object (a node or an announcement) is subjected to an access hazard. For this type of write, we install a hazard pointer to the target object's address prior to the access, to prevent recycling of the target object. We guard a tail pointer before the next field of the node it points to is updated when linking new nodes (in lines 179 and 301). A node to which we wish to write a batch's last dequeued item is guarded first (in line 330). Likewise, an announcement object is protected before its batch execution, which involves setting its *oldTail* field (in lines 192 and 228).

The two kinds of writes we described sometimes overlap. In such cases, an installation of a certain hazard pointer serves as a safeguard against both potential problems.

Reads. Reads are treated differently. After reading a node's next pointer, we set a compiler fence to prevent compiler reordering. We then check the warning flag to verify the node has not been recycled, and thus the address we read is valid. This occurs when traversing over the nodes to be dequeued for finding the new head after a batch operation (in lines 269 and 356).

Avoid batch execution restart. We apply the following optimization throughout the execution of a batch in Method *ExecuteAnn* (Listing 19). If we test the warning flag and find it set, then according to the optimistic access scheme guidelines we should unset it and restart the operation. Restarting in the current stage of execution would mean to reread the queue's head value and, if it contains an announcement, start its execution from the beginning.

Instead of restarting, when discovering that our warning flag is set, we check whether the announcement we hold is still installed in the queue's head. If not, the batch execution has anyhow been completed by another thread, so we may immediately return. Otherwise, we may proceed with the batch execution. When we find the warning set in line 356 after reading *node->next*, the still-installed announcement implies that when we read the next pointer it has not been retired, because it is located in a later node in the queue that cannot be retired as long as the announcement is installed. Similarly, we might find the warning set after assigning a hazard pointer to the node that was pointed to by the head or tail prior to the batch execution. In such case, the still-installed

announcement implies that when we set the hazard pointer, the node pointed to by the old head or tail has not been retired.

The modified code. The additions to the algorithm presented in Section 6 are colored in red in the following code snippets.

We begin with the implementation of the internal methods operating on the shared queue.

Listing 14. EnqueueToShared

```

174 EnqueueToShared(item)
175     node = MM.AllocateNode()
176     node.item = item; node.next = NULL
177     while (true)
178         tailAndCnt = SQTail
179         threadData.nodeHp = tailAndCnt.node
180         _memoryFence
181         if (threadData.warning)
182             threadData.warning = false
183             threadData.nodeHp = NULL
184         continue
185     if (CAS(&tailAndCnt.node->next, NULL, node))
186         // Linked node to tail
187         CAS(&SQTail, tailAndCnt, (node, tailAndCnt.cnt+1))
188         break
189     head = SQHead
190     if head consists of Ann: // (head.tag & 1 != 0)
191         threadData.nodeHp = NULL
192         threadData.annHp = head.ann
193         _memoryFence
194         if (threadData.warning)
195             threadData.warning = false
196             threadData.annHp = NULL
197         continue
198         ExecuteAnn(head.ann)
199         threadData.annHp = NULL
200     else
201         CAS(&SQTail, tailAndCnt, (tailAndCnt.node->next, tailAndCnt.cnt+1))
202         threadData.nodeHp = NULL
203     threadData.nodeHp = NULL

```

Listing 15. DequeueFromShared

```

204 Item* DequeueFromShared()
205     while (true)
206         headAndCnt = HelpAnnAndGetHead()
207         threadData.nodeHp = headAndCnt.node
208         _memoryFence
209         if (threadData.warning)
210             threadData.warning = false
211             goto cleanup
212         headNextNode = headAndCnt.node->next
213         if (headNextNode == NULL)

```

```

214         threadData.nodeHp = NULL
215     return NULL
216     dequeuedItem = headNextNode->item
217     if (CAS(&SQHead, headAndCnt, ⟨headNextNode, headAndCnt.cnt+1⟩))
218         threadData.nodeHp = NULL
219         MM.Retire(headAndCnt.node)
220     return dequeuedItem
221 cleanup:
222     threadData.nodeHp = NULL

```

Listing 16. HelpAnnAndGetHead

```

223 PtrCnt HelpAnnAndGetHead()
224 while (true)
225     head = SQHead
226     if head consists of PtrCnt: // (head.tag & 1 == 0)
227         return head.ptrCnt
228     threadData.annHp = head.ann
229     _memoryFence
230     if (threadData.warning)
231         threadData.warning = false
232         threadData.annHp = NULL
233         continue
234     ExecuteAnn(head.ann)
235     threadData.annHp = NULL

```

Listing 17. ExecuteBatch

```

236 Node* ExecuteBatch(batchRequest)
237     ann = MM.AllocateAnn()
238     ann->batchReq = batchRequest
239     while (true)
240         oldHeadAndCnt = HelpAnnAndGetHead()
241         threadData.nodeHp = oldHeadAndCnt.node
242         _memoryFence
243         if (threadData.warning)
244             threadData.warning = false
245             goto cleanup
246         ann->oldHead = oldHeadAndCnt // Step 1 in Figure 1
247         if (CAS(&SQHead, oldHeadAndCnt, ann)) // Step 2 in Figure 1
248             break
249         cleanup:
250             threadData.nodeHp = NULL
251     threadData.nodeHp = NULL
252     ExecuteAnn(ann)
253     MM.Retire(ann)
254     return oldHeadAndCnt.node

```

Listing 18. ExecuteDeqsBatch

```

255 <unsigned int, Node*, Item*> ExecuteDeqsBatch()
256     while (true)
257         oldHeadAndCnt = HelpAnnAndGetHead()
258         threadData.nodeHp = oldHeadAndCnt.node
259         _memoryFence
260         if (threadData.warning)
261             threadData.warning = false
262             goto cleanup
263         newHeadNode = oldHeadAndCnt.node
264         successfulDeqsNum = 0
265         // Calculate new head and successful dequeues num:
266         repeat threadData.deqsNum times:
267             headNextNode = newHeadNode->next
268             _compilerFence
269             if (threadData.warning)
270                 threadData.warning = false
271                 goto cleanup
272             if (headNextNode == NULL)
273                 break
274             ++successfulDeqsNum
275             newHeadNode = headNextNode
276         if (successfulDeqsNum == 0)
277             lastDeqItem = NULL
278             break
279         lastDeqItem = newHeadNode->item
280         if (CAS(&SQHead, oldHeadAndCnt, <newHeadNode, oldHeadAndCnt.cnt +
281             successfulDeqsNum>))
282             break
283         cleanup:
284             threadData.nodeHp = NULL
285             threadData.nodeHp = NULL
286         return <successfulDeqsNum, oldHeadAndCnt.node, lastDeqItem>

```

Listing 19. ExecuteAnn

```

286 // ann is assured not to be reclaimed during ExecuteAnn run
287 ExecuteAnn(ann)
288 // Link items to tail and update ann
289 while (true)
290     tailAndCnt = SQTail
291     annOldTailAndCnt = ann->oldTail
292     if (annOldTailAndCnt.node != NULL)
293         threadData.nodeHp = annOldTailAndCnt.node
294         _memoryFence
295         if (threadData.warning)
296             threadData.warning = false
297         if (SQHead != ann)
298             threadData.nodeHp = NULL
299             return
300     break

```

```

301     threadData.nodeHp = tailAndCnt.node
302     _memoryFence
303     if (threadData.warning)
304         threadData.warning = false
305         if (SQHead != ann)
306             threadData.nodeHp = NULL
307         return
308     CAS(&tailAndCnt.node->next, NULL, ann->batchReq.firstEnq) // Step 3 in Figure 1
309     if (tailAndCnt.node->next == ann->batchReq.firstEnq)
310         // Step 4 in Figure 1:
311         ann->oldTail = annOldTailAndCnt = tailAndCnt
312         _memoryFence
313         break
314     else
315         CAS(&SQTail, tailAndCnt, <tailAndCnt.node->next, tailAndCnt.cnt+1>)
316         threadData.nodeHp = NULL
317     newTailAndCnt = <ann->batchReq.lastEnq, annOldTailAndCnt.cnt + ann->batchReq.
        enqsNum>
318     CAS(&SQTail, annOldTailAndCnt, newTailAndCnt) // Step 5 in Figure 1
319     threadData.nodeHp = NULL
320     UpdateHead(ann)
321
322 UpdateHead(ann)
323     oldQueueSize = ann->oldTail.cnt - ann->oldHead.cnt
324     successfulDeqsNum = ann->batchReq.deqsNum
325     if (ann->batchReq.excessDeqsNum > oldQueueSize)
326         successfulDeqsNum -= ann->batchReq.excessDeqsNum - oldQueueSize
327     if (successfulDeqsNum == 0)
328         CAS(&SQHead, ann, ann->oldHead) // Step 6 in Figure 1
329         return
330     threadData.nodeHp = ann->oldHead.node
331     _memoryFence
332     if (threadData.warning)
333         threadData.warning = false
334         if (SQHead != ann)
335             threadData.nodeHp = NULL
336         return
337     if (oldQueueSize > successfulDeqsNum)
338         newHeadNode = GetNthNode(ann->oldHead.node, successfulDeqsNum, ann)
339     else
340         newHeadNode = GetNthNode(ann->oldTail.node, successfulDeqsNum - oldQueueSize,
            ann)
341     if (newHeadNode == NULL)
342         threadData.nodeHp = NULL
343         return
344     lastDequeuedItem = newHeadNode->item
345     if (SQHead != ann)
346         threadData.nodeHp = NULL
347         return
348     ann->oldHead.node->item = lastDequeuedItem
349     threadData.nodeHp = NULL

```



```

350     CAS(&SQHead, ann, ⟨newHeadNode, ann->oldHead.cnt + successfulDeqsNum⟩) // Step 6 in
        Figure 1
351
352 Node* GetNthNode(node, n, ann)
353     repeat n times:
354         node = node->next
355         _compilerFence
356         if (threadData.warning)
357             threadData.warning = false
358         if (SQHead != ann)
359             return NULL
360     return node

```

Next, we present the implementation of the queue's interface methods.

Listing 20. Enqueue

```

361 Enqueue(item)
362     if (threadData.opsQueue.Empty())
363         EnqueueToShared(item)
364     else
365         Evaluate(FutureEnqueue(item))

```

Listing 21. Dequeue

```

366 Item* Dequeue()
367     if (threadData.opsQueue.Empty())
368         return DequeueFromShared()
369     else
370         return Evaluate(FutureDequeue())

```

Listing 22. FutureEnqueue

```

371 Future* FutureEnqueue(item)
372     AddToEnqsList(item)
373     ++threadData.enqsNum
374     return RecordOpAndGetFuture(ENQ)
375
376 AddToEnqsList(item)
377     node = MM.AllocateNode()
378     node.item = item; node.next = NULL
379     if (threadData.enqsHead == NULL)
380         threadData.enqsHead = node
381     else
382         threadData.enqsTail->next = node
383     threadData.enqsTail = node
384
385 Future* RecordOpAndGetFuture(futureOpType)
386     future = new Future()
387     threadData.opsQueue.Enqueue(⟨futureOpType, future⟩)
388     return future

```

Listing 23. FutureDequeue

```

389 Future* FutureDequeue()
390     ++threadData.deqsNum
391     threadData.excessDeqsNum = max(threadData.excessDeqsNum, threadData.deqsNum -
        threadData.enqsNum)
392     return RecordOpAndGetFuture(DEQ)

```

Listing 24. Evaluate

```

393 Item* Evaluate(future)
394     if (!future->isDone)
395         ExecuteAllPending()
396     return future->result
397
398 ExecuteAllPending()
399     if (threadData.enqsNum == 0)
400         // No enqueues. Execute a dequeues-only batch
401         <successfulDeqsNum, oldHeadNode, lastDeqItem> = ExecuteDeqsBatch()
402         PairDeqFuturesWithResults(oldHeadNode, successfulDeqNum, lastDeqItem)
403     else
404         // Execute a batch operation with at least one enq
405         oldHeadNode = ExecuteBatch(
406             <threadData.enqsHead,
407             threadData.enqsTail,
408             threadData.enqsNum,
409             threadData.deqsNum,
410             threadData.excessDeqsNum>)
411         PairFuturesWithResults(oldHeadNode)
412         threadData.enqsHead = NULL
413         threadData.enqsTail = NULL
414         threadData.enqsNum = 0
415         threadData.deqsNum = 0
416         threadData.excessDeqsNum = 0

```

Listing 25. PairFuturesWithResults

```

417 PairFuturesWithResults(oldHeadNode)
418     currentHead = oldHeadNode
419     shouldSetPrevDeqResult = false
420     lastSuccessfulDeqFuture = NULL
421     oldHeadItem = oldHeadNode->item
422     while true:
423         op = threadData.opsQueue.Dequeue()
424         op.future->isDone = true
425         if (op.type == ENQ)
426             break
427         else // op is DEQ
428             if (currentHead->next == threadData.enqsHead)
429                 // The queue is currently empty
430                 op.future->result = NULL
431             else
432                 nodePrecedingDeqNode = currentHead

```

```

433         currentHead = currentHead->next
434         MM.Retire(nodePrecedingDeqNode)
435         if (shouldSetPrevDeqResult)
436             lastSuccessfulDeqFuture->result = lastSuccessfulDeqItem
437         else
438             shouldSetPrevDeqResult = true
439             lastSuccessfulDeqFuture = op.future
440             lastSuccessfulDeqItem = currentHead->item
441     currentTail = threadData.enqsHead
442     while threadData.opsQueue is not empty:
443         op = threadData.opsQueue.Dequeue()
444         if (op.type == ENQ)
445             if (currentTail != NULL)
446                 currentTail = currentTail->next
447         else // op is DEQ
448             if (currentHead == currentTail)
449                 // The queue is currently empty
450                 op.future->result = NULL
451             else
452                 nodePrecedingDeqNode = currentHead
453                 currentHead = currentHead->next
454                 MM.Retire(nodePrecedingDeqNode)
455                 if (shouldSetPrevDeqResult)
456                     lastSuccessfulDeqFuture->result = lastSuccessfulDeqItem
457                 else
458                     shouldSetPrevDeqResult = true
459                     lastSuccessfulDeqFuture = op.future
460                     lastSuccessfulDeqItem = currentHead->item
461         op.future->isDone = true
462     if (shouldSetPrevDeqResult)
463         lastSuccessfulDeqFuture->result = oldHeadItem

```

Listing 26. PairDeqFuturesWithResults

```

464 PairDeqFuturesWithResults(oldHeadNode, successfulDeqsNum, lastDeqItem)
465     if (successfulDeqsNum > 0)
466         currentHead = oldHeadNode
467         repeat successfulDeqsNum-1 times:
468             nodePrecedingDeqNode = currentHead
469             currentHead = currentHead->next
470             MM.Retire(nodePrecedingDeqNode)
471             op = threadData.opsQueue.Dequeue()
472             op.future->result = currentHead->item
473             op.future->isDone = true
474             MM.Retire(currentHead)
475             op = threadData.opsQueue.Dequeue()
476             op.future->result = lastDeqItem
477             op.future->isDone = true
478         repeat threadData.deqsNum-successfulDeqsNum times:
479             op = threadData.opsQueue.Dequeue()
480             op.future->result = NULL
481             op.future->isDone = true

```

8 CORRECTNESS

In this section, we argue about the correctness of the algorithm. We start with an abstraction of the queue in Section 8.1, specify the linearization points in Section 8.2, and then prove the algorithm is linearizable in Section 8.3. We prove its lock-freedom in Section 8.4.

8.1 The Abstract State of the Queue

The abstract state of the queue is the sequence of items contained in the underlying list's nodes, starting with the item in the second node (i.e., the node succeeding the dummy node pointed to by the abstract head) if any, and ending with the node whose next pointer is NULL. The queue is empty iff the next pointer of the node pointed to by the abstract head is NULL. The tail pointer does not affect the state of the abstract queue.

The fine point is the definition of the abstract head of the queue:

- If no announcement is installed in *SQHead*, the abstract head is the same as *SQHead*. There is one exception to this rule: a dequeues-only batch operation that succeeds to dequeue at least one item.¹ Such an operation modifies the abstract head: when it reads the next pointer in line 153 (of Method *ExecuteDeqsBatch* in Listing 12) for the last time before *ExecuteDeqsBatch* returns, the abstract head is set to point to the last node dequeued by this batch operation. The pointer to this node is the value to which *SQHead* is set in line 160 in the same execution of *ExecuteDeqsBatch*.
- If there is an announcement installed in *SQHead* but the CAS that links its items to the tail (in line 44 in Listing 5) has not yet been performed successfully, then the abstract head is the same as *SQHead.ann->oldHead*, which is in practice *SQHead*'s value prior to the announcement's installation. Thus, installing an announcement does not change the abstract head.
- If there is an announcement installed in *SQHead* and the CAS that links its items to the tail has already succeeded, then the abstract head points to the node that is going to be the dummy node after all enqueues and dequeues of the batch operation have taken effect. *SQHead* is going to be set to the same value when the announcement is uninstalled. Thus, removing an announcement does not change the abstract head.

Hence, when a batch operation is announced (i.e., *SQHead* is set to point to the related announcement), the abstract state of the queue does not change. It remains the sequence of items currently contained in the nodes of the shared queue's list, starting with the node succeeding the node pointed to by the previous *SQHead*. The moment the CAS that links the batch's enqueued items to the tail (in line 44 in Listing 5) succeeds, the abstract queue's head is changed to point to the node to which *SQHead* will point after completing the announcement handling. Therefore, the whole batch, including both its enqueues and dequeues, takes effect instantaneously. From that moment, until another operation takes effect, the abstract state of the queue is the sequence of items in the list starting with the node succeeding the new dummy node, including the batch's linked items.

8.2 Linearization Points

From the preceding definitions of the abstract head and the abstract state of the queue, we derive linearization points for all operation types. These linearization points are detailed next.

We start the review with the single (non-batched) dequeue operation. A successful dequeue operation takes effect when it modifies the head pointer (and counter). An unsuccessful dequeue

¹In the short version of this work [23], we did not elaborate on the effect of a dequeues-only batch on the abstract head.

operation is linearized when it reads the next pointer of the dummy node (whose value is revealed to be NULL). Regarding a batch operation that contains only dequeue operations, all dequeues of such batch operation are linearized one after another, in the moment of reading the next pointer in line 153 (of Method *ExecuteDeqsBatch* in Listing 12) for the last time before *ExecuteDeqsBatch* returns. This reading ends the list traversal performed to calculate the new head, either due to encountering the end of the list (as detected in line 154) or due to completing a traversal of *deqsNum* items. The later advance of the head, which completes the dequeues batch, might intuitively seem like a simpler linearization point, yet nevertheless it is not a correct linearization point in all cases. Consider the following scenario. A thread *T* performs a dequeues-only batch. During the traversal over the nodes to be dequeued, it encounters the end of the nodes list, after traversing less than *deqsNum* items. Then, another thread enqueues an item, before *T* advances the head to point to the last dequeued node. In this scenario, the head modification by *T* cannot be considered the batch's linearization point, since it happens after the enqueue operation, while the batch operation does not dequeue the new item. However, the moment in which *T* read the next pointer in line 153 for the last time occurred appropriately before the enqueue.

We move to listing enqueue's linearization points, starting with the non-batched enqueue operation. An enqueue operation takes effect when the next pointer of the last node in the list is modified from NULL to point to a new node. Similarly, all enqueues and dequeues of a batch operation that enqueues at least one item take effect one after another when the next pointer of the last node is modified from NULL to point to the first node of the batch. Note that this is the only linearization step that may be carried out by a helping thread rather than the thread that invoked the operation.

To satisfy EMF-linearizability, the future history H_f , constructed from the original history H (as described in Definition 3.1), should be MF-linearizable. Thus far, we described linearization points relating to H . We set the same linearization points in H_f : the linearization point in H_f is the same as in H for a future operation call, and for a single operation call—we set the linearization point of the appropriate future call to the same moment as the linearization of the single call in H . Note that since the linearization points described for single operations occur during their method calls in H , they occur between the adequate future call's invocation and *Evaluate* call's response in H_f , which complies with MF-linearizability.

8.3 Linearizability Proof

In this section, we prove that our algorithm is linearizable. First, we examine all linearization points described in Section 8.2. We confirm that their operation on the abstract state of the queue, described in Section 8.1, complies with the sequential specification of a FIFO queue. We also explain why no operation may take effect twice, despite concurrent helping threads attempting to execute the same operation. To complete the proof, we show that the abstract state of the queue is not modified in any point in the algorithm other than the discussed linearization points. For simplicity, we ignore the nodes' memory reclamation in our proof.

8.3.1 Linearization Points Modifying the Abstract State of the Queue. We start with the non-batched dequeue operation. A successful dequeue's linearization point is in line 20 in Listing 2, where *SQHead* is updated to point to the next node. Note that the abstract head before and after the update is the same as *SQHead*. This is because no batch operations are involved. First, no announcement is installed in the head at this moment, since both the previous and new *SQHead* values contain *PtrCnt* objects. Second, no dequeues-only batch operation has advanced the abstract head. To do so, it needs to later succeed in advancing the current *SQHead*, which is impossible as the present dequeue is the one to succeed in performing a CAS of *SQHead*. Thus, the CAS in

line 20, which advances *SQHead* by one node, advances the abstract head as well. This translates to dequeuing the first item from the abstract queue.

Regarding a dequeues-only batch operation that takes effect when the queue is not empty—all its dequeues are linearized successively, in the moment of reading the next pointer in line 153 (of Method *ExecuteDeqsBatch* in Listing 12) for the last time before *ExecuteDeqsBatch* returns. We obtain *SQHead*'s value in line 149. We perform a CAS of *SQHead* from this value in the last time we execute line 160 before returning. During this whole time, the abstract head remains equal to the initially obtained *SQHead*'s value, as no announcement is installed in the head and no other dequeues batch succeeds to perform a CAS of the current head (as only one CAS from the same value may succeed, and ours does). If the last next pointer to be read in line 153 is NULL, then right before this read there are exactly *successfulDeqsNum* items in the abstract queue (because the abstract head is still the one we obtained in line 149 as explained earlier, and we traversed *successfulDeqsNum* nodes that are linked after this head). Right after that read, the abstract head is advanced by *successfulDeqsNum* nodes and the queue becomes empty. The rest of the dequeues in the batch fail. If, however, the last next pointer to be read in line 153 is not NULL, then right before this read there are at least *threadData.deqsNum* items in the queue. Right after the read, the abstract head is advanced by *threadData.deqsNum* nodes, which translates to an extraction of this amount of items from the beginning of the queue. In any case, *SQHead* is eventually advanced in line 160, in a CAS that does not affect the abstract state of the queue.

Moving to enqueue, and starting with the non-batched operation, an enqueue is linearized in line 5 in Listing 1, where the next pointer of the last node in the underlying list is modified from NULL to point to the new node that the enqueueer created. To show that the abstract state of the queue reflects the change—that is, the new item is appended to the abstract queue's items—we rely on the following observations.

OBSERVATION 8.1. *The next field of a node may change only once in the algorithm, from NULL to a pointer to a linked node.*

PROOF. During the *FutureEnqueue* method (Listing 8), a node intended to be enqueued is thread-local. Its next field's value is initially NULL. It might be set to a non-NULL value in line 93, and right after that the local queue's tail is advanced to the new linked node, so the current node will not be locally further modified. The only additional modifications of a node's next field are performed in lines 5 and 44 (in Listings 1 and 5, respectively), where it might be modified (using a CAS operation) from NULL to a non-NULL value, and thus such CAS may succeed only once per node. \square

COROLLARY 8.2. *Linking nodes twice to the same node is impossible.*

OBSERVATION 8.3. **SQTail* always points to a node that is contained in the underlying list of nodes starting with the initial dummy node of the shared queue. (We ignore the nodes' memory reclamation for simplicity.)*

PROOF. The claim holds initially since *SQTail* is initialized to point to the first dummy node, when it is the only node in the list. Based on Observation 8.1, the only changes applied to the queue's list of nodes are additions of nodes to its end. *SQTail* is updated in lines 7, 13, 50, and 52 to point to nodes of this list, so the claim prevails. \square

Consider a successful CAS in line 5 of the next field of the node pointed to by the obtained tail. According to Observation 8.3, this node (which was pointed to by *SQTail*) was part of the list of nodes starting with the initial node of the shared queue, and remains part of this list since nodes are not removed from this list according to Observation 8.1. In addition, the previous value of the above mentioned next pointer is NULL. Hence, this node, to which we link the new node, is the

last node in the underlying list of nodes. Specifically, it means that the abstract head points to this node or a prior node. Thus, the node that we link to it becomes one of nodes in the list that starts with the node succeeding the dummy node (pointed to by the abstract head), which means its item becomes part of the abstract queue in the linearization moment. Moreover, the next pointer of the enqueued node is NULL, and thus only this node's item is appended to the abstract queue's items in this linearization point.

Last, similarly to an enqueue operation, all enqueues and dequeues of a batch operation that enqueues at least one item are linearized one after another when the next pointer of the last node is modified from NULL to point to the first node of the batch in line 44 in Listing 5. The abstract state of the queue changes accordingly: the batch's items are appended to the abstract queue's items (the new items become part of the abstract queue due to similar arguments to the ones stated above for a single enqueue). In addition, items that the batch operation dequeues are omitted from the abstract queue in the linearization point, since from this moment the abstract head points to the new dummy node, which takes into account all enqueues and dequeues of the batch operation (using a calculation described in Section 5.2).

8.3.2 Linearization Points Not Modifying the Abstract State of the Queue. Linearization points that do not modify the abstract state of the queue occur during a single dequeue and a dequeues-only batch that are applied to an empty queue. We prove that these linearization points take place when the abstract queue is indeed empty, and thus the operations follow the sequential specification of a queue: they appropriately fail and return without affecting the state of the abstract queue. We will focus on a failing dequeue operation, and the same arguments apply for a dequeues-only batch applied to an empty queue.

Let t_{lin} be the linearization moment of an unsuccessful dequeue performed by a thread T —that is, the moment T executes line 17, reading the next pointer of its obtained dummy node before revealing in the next line that it is NULL. Let t_{read} be the moment in which T executed line 24 for the last time before t_{lin} . We will prove that the abstract queue is empty at t_{lin} .

The next pointer of the node pointed to by the head obtained at t_{read} is NULL at t_{lin} (according to t_{lin} 's definition). From Observation 8.1, we deduce that this next pointer was NULL also at any point earlier than t_{lin} , particularly at t_{read} .

At t_{read} , T obtains the queue's head from $SQHead$ when no announcement is installed. Thus, the abstract head is either equal to the obtained head or was equal to it at some previous moment after which a concurrent dequeues-only batch has advanced the abstract head. But the latter is impossible, because for a dequeues-only batch to succeed advancing the head, there must have been at least one node linked after the obtained head, but its next is NULL as mentioned earlier. Thus, the abstract head equals the obtained head at t_{read} .

The abstract head is not modified between t_{read} and t_{lin} : for it to change by a batch operation or a successful dequeue operation, nodes should have been linked—between t_{read} and t_{lin} —after the obtained head, but they have not, based on Observation 8.1. Thus, at t_{lin} , the abstract head still equals the obtained head. Since the next field of the dummy node pointed to by this head is NULL at t_{lin} , the abstract queue is empty at this moment.

8.3.3 No Recurring Linearization Points. We prove why each operation takes effect once, namely its linearization point occurs one time only. For each linearization point, the thread that performs it does not take any backward branches after that, and the operation completes without repeating the linearization point. We will now establish that other threads do not perform the linearization step again. A successful dequeue operation and a dequeues-only batch that succeeds to dequeue at least one item are achieved using a single operation on the shared queue, and thus no help is involved and the initiator thread is the only one to perform the linearization step. An enqueue

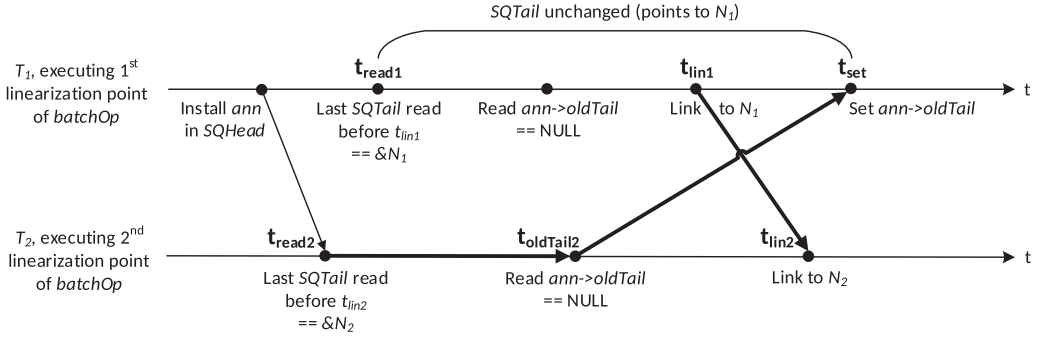


Fig. 2. The impossible scenario of two linearization points of the same batch.

operation may be assisted by other threads advancing the tail, but the linearization step is taken by the enqueueer only, so no helping thread may perform it and cause the operation to take effect twice. This is not the case for a batch operation that contains enqueues: a helping thread may perform its linearization step. Next, we explain how we prevent the linearization step of such an operation from occurring twice. The proof is also illustrated in Figure 2.

Let *batchOp* be a batch operation that contains at least one enqueue. The thread that initiates the batch installs an announcement in *SQHead* (in line 33 in Listing 4), which makes the batch public. Let t_{lin1} be the first time in which *batchOp*'s linearization step is performed (i.e., the CAS in line 44 in Listing 5 that links the batch's items to the tail is performed successfully for the first time for *batchOp*). Let T_1 be the thread that performed this linking. T_1 could either be the batch's initiator (as depicted in Figure 2) or a helping thread that encountered the announcement. Let t_{read1} be the last time in which T_1 obtained *SQTail*'s value in line 40 before performing the linearization step, and N_1 be the node pointed to by this value (i.e., the node to which *batchOp*'s items are linked at t_{lin1}). Let t_{set} be the first time that line 47, which sets the *oldTail* field of the announcement, is executed for *batchOp*. It could be executed by any thread; its execution by T_1 in Figure 2 is merely an example. $t_{set} > t_{lin1}$ since line 47 is executed only after the batch's items are linked.

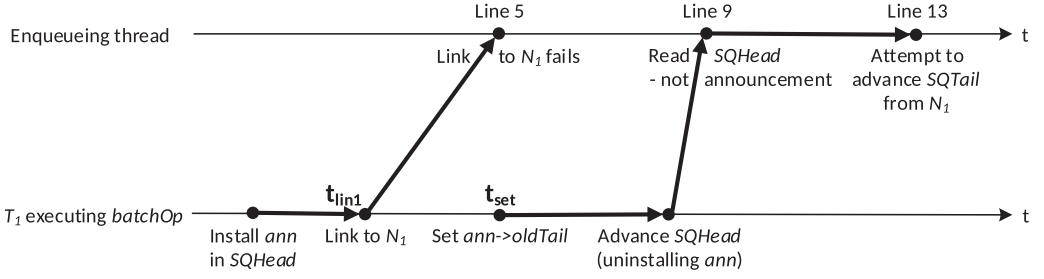
Assume a second linearization step of *batchOp* is carried out by a thread T_2 at t_{lin2} . Let t_{read2} be the last time in which T_2 obtained *SQTail*'s value in line 40 before performing the linearization step, and N_2 be the node pointed to by this value (i.e., the node to which T_2 linked *batchOp*'s items at t_{lin2}). After t_{read2} and before t_{lin2} , T_2 obtains the *oldTail* field of the announcement in line 41 at moment $t_{oldTail2}$. For T_2 to proceed to the linking, this field must be revealed (in line 42) to be NULL.

To complete the proof that a linearization of a batch operation that contains enqueues does not occur twice, we proceed to show how assuming a second linearization point results in a contradiction. This is an overview of the rest of the proof in a nutshell: *SQTail* points to N_1 during t_{read1} through t_{set} (Claim 8.5), and thus N_2 (the node pointed to by *SQTail* at t_{read2} , which happens before t_{set}) is either N_1 or a preceding node (Claim 8.7). Hence, $N_2 \rightarrow next$ is not NULL after t_{lin1} , so the CAS at t_{lin2} cannot succeed.

To prove that *SQTail* points to N_1 during t_{read1} through t_{set} , we need to establish that *SQTail* does not change in this time frame. To prove that, we will rely on the following lemma.

LEMMA 8.4. *For any CAS operation of *SQTail* that occurs between t_{read1} and t_{set} , the previous value passed to the CAS is not a pointer to N_1 .*

PROOF. We list all code lines that modify *SQTail* and explain why the claim holds for each of them.

Fig. 3. Enqueueer's attempt to advance $SQTail$ scenario.

In line 7, an enqueue operation advances $SQTail$ to point to the node it has just linked. $SQTail$ could not have pointed to N_1 prior to this change due to Corollary 8.2, as $batchOp$'s items are linked to N_1 . So a CAS in line 7 with a pointer to N_1 passed as the previous value is impossible.

In line 13, an enqueue operation attempts to assist a conflicting operation and advance $SQTail$ using a CAS. Suppose that the previous $SQTail$'s value passed to the CAS operation is a pointer to N_1 (Figure 3). We will show that this CAS must happen after t_{set} , so it particularly cannot take place between t_{read1} and t_{set} . If the enqueue operation reached line 13, it means that previously the CAS in line 5 that attempted to link a node to N_1 has failed, then $SQHead$ has not consisted of an announcement (when reading $SQHead$ in line 9). For the CAS in line 5 to fail, it must have happened after t_{lin1} (due to Observation 8.1), which in turn happened after the installation of $batchOp$'s announcement in $SQHead$. Thus, ann must have been uninstalled from $SQHead$ before line 9's execution. $SQHead$ that consists of an announcement is modified only in Method *UpdateHead* (in Listing 5), which is called in line 53 during the batch's execution. Therefore, line 53, which uninstalls the announcement from $SQHead$ and completes the $batchOp$'s execution, must have been executed for $batchOp$ before line 9. Prior to the batch completion, as part of $batchOp$'s execution, $ann \rightarrow oldTail$ was set at t_{set} . It follows that the above-mentioned CAS in line 13 happens after t_{set} , which is what we aimed to prove, before line 9's execution.

Another CAS of $SQTail$ in an attempt to assist a conflicting operation occurs in line 50. However, N_1 could not be the previous $SQTail$'s value passed to this CAS: to reach line 50 with N_1 as the previous value, the attempt to link an item to N_1 in line 44 must fail, which means N_1 's next field is not NULL. In addition, N_1 's next field does not point to the first node enqueued by $batchOp$, according to the check in line 45. Consequently, N_1 's next field must point to another node. But this is impossible, due to N_1 's definition as the node to which the batch's items were linked, and based on Corollary 8.2.

An additional modification of $SQTail$ happens during a batch execution in line 52. Suppose some thread advances $SQTail$ in line 52, and suppose that the previous value passed to the CAS operation is a pointer to N_1 . If the thread tries to carry out $batchOp$, it does not advance the tail between t_{read1} and t_{set} : to reach line 52, it has to break from the while loop, which could happen only after t_{set} (the first time $oldTail$ field of $batchOp$'s announcement was set). Otherwise, the thread tries to carry out another batch operation. To reach line 52 within the execution of this batch operation, it has to break from the while loop. This happens only after line 47 is carried out for this other batch and sets the $oldTail$ field of the batch's announcement to point to N_1 . It follows that the check in line 45 has passed, namely the first node that this batch wishes to enqueue has been linked to N_1 . But this is impossible due to N_1 's definition as the node to which the batch's items were linked, and based on Corollary 8.2. \square

CLAIM 8.5. $SQTail$ is not modified between t_{read1} and t_{set} .

PROOF. At t_{read1} , $SQTail$ points to N_1 (by t_{read1} 's definition). According to Lemma 8.4, no CAS of $SQTail$ may be the first to modify it from N_1 between t_{read1} and t_{set} . Thus, no successful CAS of $SQTail$ occurs during this time frame. \square

LEMMA 8.6. *Up to t_{set} , $SQTail$ points to either N_1 or a preceding node.*

When mentioning a *preceding* or *subsequent* node, we refer to the nodes' order in the queue's underlying list of nodes. We view this list as starting with the initial dummy node, so it contains all nodes that were ever enqueued. (In the proof we ignore the nodes' memory reclamation for simplicity, but anyhow in practice the threads do not hold pointers to reclaimed nodes.)

PROOF. According to Observation 8.3, $SQTail$ always points to a node in the queue's underlying list of nodes. Up to t_{lin1} , this list does not consist of any nodes subsequent to N_1 , so $SQTail$ must point to N_1 or a preceding node. Namely, up to t_{lin1} the claim holds. In view of Claim 8.5, $SQTail$ remains the same since t_{read1} , and in particular since t_{lin1} , until t_{set} . Hence, the claim prevails. \square

COROLLARY 8.7. *N_2 , the node pointed to by the tail obtained by T_2 at t_{read2} , is either N_1 or a preceding node.*

PROOF. T_2 's reading of $SQTail$ at t_{read2} happens before t_{set} , because after t_{read2} , the announcement's *oldTail* field is still NULL at $t_{oldTail2}$. The claim immediately follows from Lemma 8.6. \square

CLAIM 8.8. *At t_{lin2} , $N_2 \rightarrow next \neq NULL$.*

PROOF. $t_{lin2} > t_{lin1}$ based on t_{lin1} 's definition as *batchOp*'s first linearization point. Hence, by the time of t_{lin2} , T_1 has linked a node to N_1 . Nodes had been clearly previously linked to all preceding nodes as well. According to Corollary 8.7, N_2 is either N_1 or a preceding node, so a node has been linked to it before t_{lin2} . This implies that the *next* field's value of N_2 is not NULL at t_{lin2} . \square

Claim 8.8 yields a contradiction to the assumption of a second linearization point, as the CAS at t_{lin2} is destined to fail.

8.3.4 Modifications of the Shared Queue Not Affecting the Abstract State of the Queue. The abstract state of the queue may change only in the occasion of modifications of either *SQHead* or the next field of the last node in the queue. The latter is changed only in linearization points addressed earlier. *SQHead*, however, is modified in three additional occasions, but without affecting the abstract state of the queue. In line 33, an announcement is installed in *SQHead*, which does not change the abstract state of the queue according to its definition. Announcements' removal from the head in lines 61 and 67 do not modify the abstract state of the queue as well. In line 160, *SQHead* is advanced after a dequeues-only batch, but the abstract head is not affected after it has already reflected the change previously (in its linearization point).

8.4 Lock-Freedom Proof

In this section, we show that our algorithm ensures system-wide progress. In the algorithm of BQ , announcements are used to assist in constituting lock-freedom: a thread that wishes to perform a batch operation installs an announcement describing the batch in the shared queue's head. The purpose of the installation is to enable other threads to complete this batch operation so that they can thereafter proceed to perform their own operations, even if the thread that installed the announcement is delayed.

To prove that our algorithm is lock-free, we break each of the shared queue's operations down to a sequence of intermediate progress steps.

Definition 8.9. The completion of an interface method of the queue (one of *Enqueue*, *Dequeue*, *FutureEnqueue*, *FutureDequeue*, and *Evaluate*) is labeled a *full progress step*.

The following operations may be applied to the shared queue: enqueue, dequeue, batch with at least one enqueue, and dequeues-only batch. We will refer to them as the *shared queue's operations*. The methods *EnqueueToShared*, *DequeueFromShared*, *ExecuteBatch*, and *ExecuteDeqsBatch* apply these operations, respectively.

Definition 8.10. An *intermediate progress step* is a CAS operation that achieves progress toward achieving a full progress step. It might be executed either by the thread that initiated the operation or by a helping thread. It is a point of no return in the context of the current shared queue's operation: once a thread (either the initiator or a helping thread) that executes a shared queue's operation detects that an intermediate progress step has been completed for this operation, it may not branch back to a step in that shared queue's operation that is earlier than the completed intermediate progress step.

Definition 8.11. A *backward branch* refers to branching back to an earlier point in the execution of the same shared queue's operation, due to a test that failed.

In practice, after the test fails and before the jump backward, an attempt to assist the failing conflicting operation is made. Afterward, there is a branch back to the beginning of the current loop—a loop that appears right after the last intermediate step, or at the beginning of the operation if no intermediate progress step has been accomplished yet. The thread then starts another iteration in attempt to accomplish an intermediate progress step.

We now list the intermediate progress steps of all shared queue's operations:

- (1) In the *EnqueueToShared* method:
 - A CAS of the next pointer of the node pointed to by the shared queue's tail from NULL to the enqueued node.
 - A CAS of the shared queue's tail from the current tail to a pointer to the enqueued node.
- (2) In the *DequeueFromShared* method: In case the next field of the node pointed to by the obtained queue's head is NULL, *DequeueFromShared* finishes and returns NULL without performing any intermediate progress steps. Otherwise:
 - A CAS of the shared queue's head from the current head to a pointer to the next node.
- (3) In the *ExecuteDeqsBatch* method: In case the next field of the node pointed to by the obtained queue's head is NULL, *ExecuteDeqsBatch* finishes without performing any intermediate progress steps. Otherwise:
 - A CAS of the shared queue's head from the current head to a pointer to the last node dequeued by the batch operation.
- (4) For a batch with at least one enqueue (the first intermediate progress step is performed in *ExecuteBatch* method and the rest are performed in *ExecuteAnn* auxiliary method):
 - A CAS of the shared queue's head from the current head to the batch's announcement.
 - A CAS of the next field of the node pointed to by the shared queue's tail from NULL to a pointer to the first node enqueued by the batch operation.
 - A CAS of the shared queue's tail from the current tail to a pointer to the last node enqueued by the batch operation.
 - A CAS of the shared queue's head from the installed announcement to a pointer to the last node dequeued by the batch operation.

LEMMA 8.12. *Every up to $3n + 1$ system-wide intermediate progress steps, a new full progress step is accomplished.*

PROOF. The lemma is derived from the fact that each of the queue's interface methods executes at most one shared queue's operation, which is composed of up to four intermediate progress steps (as listed earlier). \square

Definition 8.13. An intermediate progress step s is the *root step* of a backward branch b , if s caused a failure of a test in the thread that executed b , a failure that resulted in taking the backward branch b .

Note that all backward branches are caused by conflicting intermediate progress steps, so a root step is necessarily an intermediate progress step. Moreover, a root step of a backward branch cannot be another step of the same thread, since this thread is aware of the step and will not attempt to apply conflicting operations.

CLAIM 8.14. *Let T be a thread executing a shared queue's operation. Suppose T takes a backward branch, after its test failed due to a root step of some queue operation that another thread has made. Then, the same root step can cause only one additional backward branch in the same code line in T 's run.*

PROOF. We list all backward branches and show that, as claimed, each of them is caused by a simultaneous conflicting operation that would cause at most one additional backward branch when the same thread executes the same line later.

Let T be a thread executing an operation on the queue. We review the backward branches according to the shared queue's operations. Note that while executing a certain operation, T might encounter conflicting operations and take backward branches while assisting them:

- (1) Enqueue (performed by *EnqueueToShared*, Listing 1): While executing an enqueue operation, T obtains the value of the tail in line 4. Let N be the node pointed to by this value. If T then fails to CAS the next pointer of N (in line 5), it is due to a conflicting operation that has linked a node to N in a step denoted s . As a result of the CAS failure, T attempts to assure the tail is advanced and then takes a backward branch due to s . The conflicting operation could be either an enqueue or a batch.
 - First, we analyze the case of a single enqueue operation $enq1$ executing the linking step s . We will show that after at most two backward branches of T due to linking failures, the tail no longer points to N (as either T or another thread has advanced the tail). Therefore, the next time T attempts to link to the tail, it might not fail again due to the item $enq1$ has enqueued.
 - If when T obtains the head (in line 9) it does not consist of an announcement, then T tries to advance the tail to point to the node linked by s . If this attempt fails, it implies that the tail has been already advanced by another thread. So in this case, the tail no longer points to N after a single backward branch.
 - However, the head value obtained by T might consist of an announcement. The following proof is illustrated in Figure 4. Let t_{fail1} be the moment T fails to CAS the next pointer of N when trying to perform an enqueue operation $enq2$, and t_{lin} be s 's execution moment, namely the moment in which a thread denoted T_2 links a new item to N while executing a conflicting enqueue operation $enq1$. We assumed that after the CAS fails, T obtains from the head a value pointing to an announcement. Let ann be this announcement, installed for a batch operation $batchOp$. Right after reading the head, T calls *ExecuteAnn* to assure the completion of $batchOp$'s execution.

Let t_{read} be the moment in which T_2 obtains the tail value for the last time before t_{lin} . $batchOp$'s step of advancing the tail could either happen before or after t_{read} .

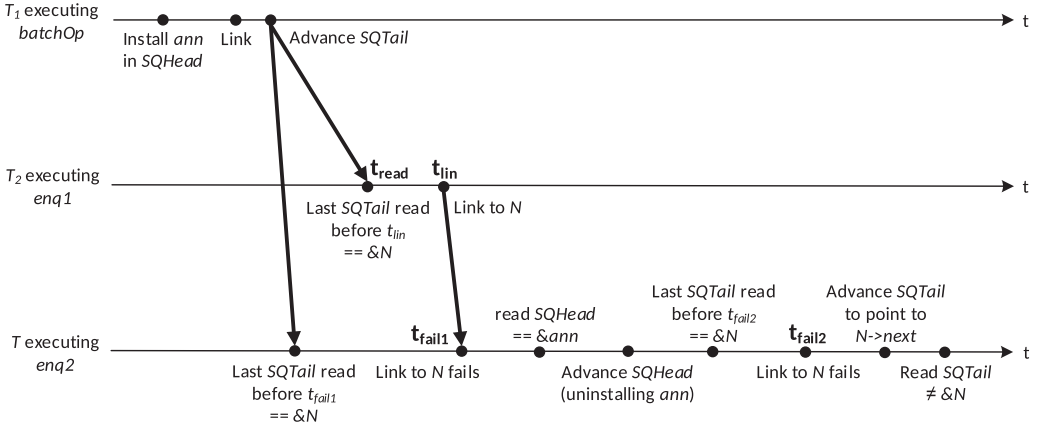


Fig. 4. *enq1* fails *enq2* while *ann* is installed in the head, after *batchOp* has advanced the tail.

- * Suppose *batchOp*'s step of advancing the tail has been accomplished (by the batch initiator as illustrated in Figure 4 or by a helping thread) before t_{read} . This is the scenario depicted in Figure 4. It is the only case in which a root step may cause two backward branches of the same thread in the same code line. So the tail has already been advanced for *batchOp* when T executes the *ExecuteAnn* method (since this happens after t_{fail1} , and $t_{read} < t_{lin} < t_{fail1}$). Therefore, the only remaining step *ExecuteAnn* applies to the shared queue is uninstalling *ann* from the head if it has not been uninstalled yet. All previous steps of the batch execution are already done. Then, T branches backward to start a second attempt. It is possible that no thread has advanced the tail yet from pointing to N . In such a case, T would obtain a pointer to N again when reading *SQTail*, and would again fail to link to N . The difference from the previous loop iteration is that now *ann* is no longer installed in the head. Thus, either no announcement is installed in the head when T reads it in line 9 and T makes sure in line 13 that the tail is advanced, or a new announcement is installed and T makes sure it is completed by calling *ExecuteAnn*, during which—prior to linking the batch's items—the tail is advanced in line 50 if it has not been advanced earlier. Consequently, in any case, when T reads the tail again after the second linking failure, the tail no longer points to N .
- * However, *batchOp*'s step of advancing the tail might happen after t_{read} (unlike depicted in Figure 4). This implies that *batchOp*'s items are linked to a node down the list, subsequent to N , after t_{lin} (because if they were linked before t_{lin} , the tail must have been advanced before t_{lin} , for T_2 to be able to link after the tail). Before linking *batchOp*'s items, the tail must be advanced from pointing to N to point to the next node. Thus, when T 's call to *ExecuteAnn* returns after the batch completion, the tail has already been advanced from pointing to N .
- Second, we analyze the case in which a batch operation *batchOp* that contains at least one enqueue is the one to execute the linking step s . We will show that by the time T branches backward, the tail will have already been advanced from pointing to N to point to the last node enqueued by *batchOp*. Therefore, if T fails to link to the tail again, it would be to a new tail to which another conflicting operation has linked an item, so s would not cause T to take another backward branch.

If the head does not consist of *batchOp*'s announcement when T obtains the head, then *batchOp*'s announcement must have been uninstalled, so *batchOp*'s execution has been

completed, including advancing the tail (and if the obtained head is not an announcement, then T would perform a CAS of the tail in line 13 but would fail). Otherwise, T calls *ExecuteAnn* to assure that *batchOp*'s execution is completed, including advancing the tail.

It remains to explain why the above mentioned advancing of the tail from the old tail value recorded in *batchOp*'s announcement is necessarily to the last node enqueued by *batchOp* and not to the first one. This is required to guarantee that s is not going to be the root step of additional backward branches of T , which could be the case if the tail were advanced node by node through all of *batchOp*'s enqueued nodes. The tail is indeed advanced to point to the last enqueued node, since in the two occasions (in lines 13 and 50) of attempting to assist a conflicting operation by advancing the tail by one node, if the attempt succeeds, then the assisted operation must be a single enqueue and not a batch. Claim 8.15 proves it for line 50. For the other occasion, of line 13: T reaches this line after reading the tail (in line 4), obtaining a pointer to a node N , and later reading the head when it does not consist of an announcement. Assume a batch operation has linked its items to N . Then it has advanced the tail after T read it and uninstalled its announcement from the head before T read the head. Namely, the batch's execution has been completed, including advancing the tail, before T read the head. Hence, the tail had been advanced before T performed the CAS of the tail with N as the previous value in line 13, which means this CAS would fail. Consequently, the CAS may succeed only if the conflicting operation is a single enqueue, as required.

- (2) Dequeue (performed by *DequeueFromShared*, Listing 2) has two possible backward branches:
 - *DequeueFromShared* first calls the *HelpAnnAndGetHead* auxiliary method. In case *HelpAnnAndGetHead* encounters an announcement installed in the shared queue's head by a conflicting batch operation, it assists the related batch by calling *ExecuteAnn* (in line 27). Then, after the batch's execution is assured to be over, including uninstalling its announcement, the dequeuer branches backward and will certainly not encounter the same announcement installed in the head again.
 - If the CAS of the shared queue's head (in line 20) fails, the thread executing *DequeueFromShared* restarts the operation. Such failure implies that the shared queue's head has changed. Thus, a pointer to another node would be obtained the next time this thread obtains the head, and the thread may not encounter the same conflicting operation again.
- (3) Dequeues-only batch (performed by *ExecuteDeqsBatch*, Listing 12): Its backward branches are similar to those of *DequeueFromShared*.
- (4) A batch containing at least one enqueue (performed by *ExecuteBatch*, Listing 4): It has the same backward branches as *DequeueFromShared*, and one more backward branch during its call to the auxiliary method *ExecuteAnn* (Listing 5) for the current batch (in line 35):

ExecuteAnn branches to its beginning in case it fails to CAS the next pointer of the node pointed to by the tail, to point to the first enqueued node of the batch (in line 44), and no other thread has accomplished this modification. Before restarting, *ExecuteAnn* helps the conflicting operation and advances the queue's tail by one node, in case it has not yet been advanced by another thread (in line 50). We will prove in Claim 8.15 that the conflicting operation must be a single enqueue and not a batch operation, thus advancing the tail by one node completes the conflicting operation. This way, after *ExecuteAnn*'s restart, a new tail would be obtained, pointing to a node to which the above-mentioned conflicting operation did not link a node. Therefore, the same root step would not cause a second CAS failure in line 44. (If the conflicting operation were a batch, its linearization step could be the root step of several backward branches after line 50: *ExecuteAnn* might advance the tail node by

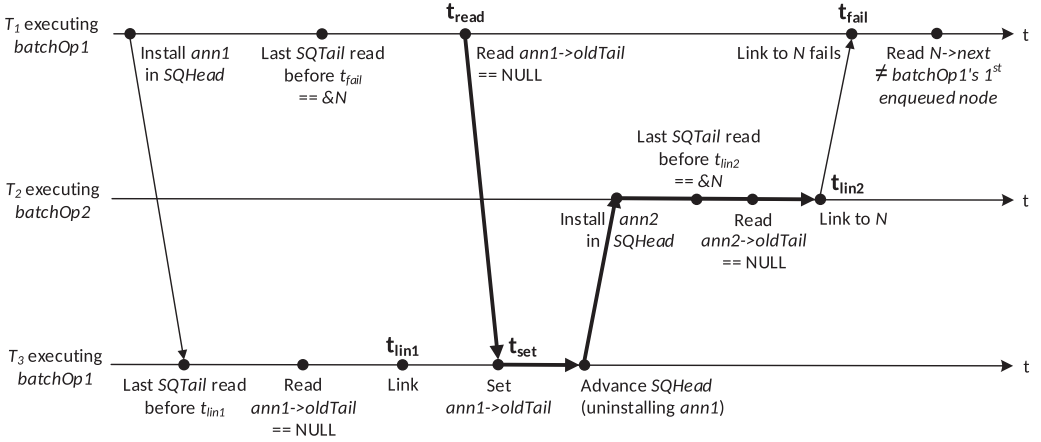


Fig. 5. The impossible scenario of one batch failing another.

node, one node down the list before each backward branch, until the tail would point to the conflicting batch's last enqueued node.) \square

We shall prove the following claim and lemmas to complete the last proof.

CLAIM 8.15. *When attempting to advance the tail in line 50 in `ExecuteAnn` after an attempt to link a batch's items to a node N has failed, the conflicting operation that linked to N must be a single enqueue and not a batch operation.*

PROOF. See Figure 5 for an illustration of the proof. Let $batchOp1$ be a batch operation containing at least one enqueue. Let T_1 be a thread that attempts at t_{fail} to link $batchOp1$'s items to N , the node it obtained from $SQTail$, and fails. T_1 then checks in line 45 whether another thread has linked $batchOp1$'s items to N and caused T_1 to fail performing the CAS. Assume T_1 finds out that this is not the case—that is, an item of another operation has been linked to N . Hence, T_1 needs to take a backward branch. Assume the conflicting operation whose root step caused T_1 's CAS to fail was another batch operation that contains at least one enqueue, denoted $batchOp2$. We will show that this assumption leads to a contradiction, and hence the conflicting operation must be a single enqueue.

Let T_2 be the thread that executed the above-mentioned root step, linking $batchOp2$'s items to N at t_{lin2} prior to t_{fail} . Let H be the history described in the proof (and in Figure 5). Let t_{read} be the moment in which T_1 obtained the $oldTail$ field of $ann1$, $batchOp1$'s announcement, in line 41. The obtained value must be NULL since T_1 proceeded to a linking attempt. We will establish in Lemma 8.16 that $t_{read} < t_{lin2}$, so at $t_{lin2} - \epsilon$, T_1 has passed line 41. It has not passed line 44, since in H it executes this line at t_{fail} , which happens after t_{lin2} . So at $t_{lin2} - \epsilon$, T_1 's next step is either line 42 or 44.

Now, consider an alternative history H' , which starts with the same prefix as H until $t_{lin2} - \epsilon$, but then T_1 is scheduled to run rather than T_2 . The result of T_1 executing line 42 is predetermined: the value obtained from $ann1 \rightarrow oldTail$ is NULL. So whether T_1 executes this line in H' at $t_{lin2} - \epsilon$ or earlier, it will proceed to attempt a CAS in line 44, trying to link $batchOp1$'s items to N . This CAS would succeed, since in H , T_2 succeeded performing a CAS of the tail at the same moment, which means that the current value of the tail's next pointer must be NULL.

To reach a contradiction, we will prove in Lemma 8.17 that a linearization step has been carried out for $batchOp1$ in H at t_{lin1} , prior to t_{lin2} . Therefore, it was carried out in H' as well. This implies that two linearization steps have been performed for $batchOp1$ in H' : both at t_{lin1} and in

the new suffix of H' . This contradicts what we proved in Section 8.3.3—that no operation has two linearization points—and concludes our proof. \square

LEMMA 8.16. t_{read} happens before t_{lin2} .

PROOF. For T_2 to link $batchOp2$'s items at t_{lin2} , an announcement $ann2$ for this batch must first be installed in $SQHead$ (either by another thread or by T_2 as illustrated in Figure 5). Prior to the installation, $ann1$ must be uninstalled from $SQHead$ (by a thread executing $batchOp1$, which could be another thread T_3 as depicted in Figure 5, and could also be T_2 itself, helping completing $batchOp1$'s execution). Before $ann1$ is uninstalled as the last step of $batchOp1$'s execution, $ann1 \rightarrow oldTail$ is set as part of the batch's execution at t_{set} (by T_3 or by another thread executing $batchOp1$). This must happen after t_{read} , when the value of $ann1 \rightarrow oldTail$ is still NULL. \square

LEMMA 8.17. $batchOp1$'s items were linked at t_{lin1} earlier than t_{lin2} .

PROOF. Before $ann1 \rightarrow oldTail$ is set at t_{set} , $batchOp1$'s items must have been linked as part of the batch's execution (by T_3 or by another thread executing $batchOp1$), at moment $t_{lin1} < t_{set}$. In addition, in the proof of Lemma 8.16, we argued why $t_{set} < t_{lin2}$. Consequently, $t_{lin1} < t_{lin2}$. \square

We continue with the lock-freedom proof.

ASSUMPTION 8.18. There is a bounded number of threads operating simultaneously on the shared queue, denoted n .

LEMMA 8.19. Each intermediate progress step may be the root step of up to $2B(n - 1)$ backward branches, where B is the number of code lines in which backward branches may occur.

PROOF. From Claim 8.14, it follows that a root step may cause at most two backward branches per backward branch code line per thread. Moreover, as noted before, a root step may not cause a backward branch in the thread that carried it out, since this thread is aware of the step and will not attempt to apply conflicting operations. \square

We will perform an amortized analysis to show that the complexity does not depend on batch sizes if viewed over a sequence of operations that starts when there are no pending operations in any thread. To translate this to worst case analysis, the complexity should be multiplied by a factor of the current batch size (i.e., the current maximum number of pending operations in any of the threads).

OBSERVATION 8.20. Each of the queue's interface methods, denoted IM , wraps zero or one internal method that applies a shared queue's operation. Other than the call to this internal operation method, IM executes amortized $O(1)$ computational steps (under a sequence of operations starting when there are no pending operations).

PROOF. *Enqueue* calls either *EnqueueToShared* or *Evaluate* that in turn calls an internal method as detailed in the following. *Dequeue* calls either *DequeueFromShared* or *Evaluate*. *FutureEnqueue* and *FutureDequeue* do not call any of the queue's internal methods. *Evaluate* calls either *ExecuteBatch* or *ExecuteDeqsBatch*.

Other than these calls to internal methods, all interface methods execute $O(1)$ computational steps (with no backward branches), with the exception of the *Evaluate* method. This method, called either by the user or by *Enqueue* or *Dequeue*, calls *PairFuturesWithResults* or *PairDeqFuturesWithResults*. These result-pairing methods make $O(ops)$ computational steps, where ops is the current batch size. These $O(ops)$ computational steps are performed after $ops - 1$ calls to *FutureEnqueue* and *FutureDequeue* followed by a call to *Enqueue* or *Dequeue*, or after ops

calls to *FutureEnqueue* and *FutureDequeue* followed by a call to *Evaluate*. Therefore, *Evaluate* carries out amortized $O(1)$ computational steps under a sequence of operations starting when there are no pending operations. \square

OBSERVATION 8.21. *Each of the shared queue's operation methods makes amortized $O(1)$ computational steps (under a sequence of operations starting when there are no pending operations) if no backward branches are taken.*

PROOF. If *EnqueueToShared* does not take backward branches, it means it succeeded to complete without encountering conflicting operations, so in particular it did not call *ExecuteAnn*. Thus, it performed $O(1)$ computational steps. The same is true for *DequeueFromShared*.

ExecuteDeqsBatch carries out $O(deqs)$ computational steps, where *deqs* is the number of dequeues in the batch, since it calculates the new head by traversing the dequeued nodes (in lines 152 through 157). However, *ExecuteDeqsBatch* performs amortized $O(1)$ computational steps under a sequence of operations starting when there are no pending operations, since it is called after *deqs* *FutureDequeue* operations (or *deqs* - 1 *FutureDequeue* operations followed by a single *Dequeue* operation). *ExecuteBatch* performs $O(deqs)$ computational steps in the *GetNthNode* auxiliary method (called in line 64 or 66) for calculating the new head, where *deqs* is the number of dequeue operations in the batch. It executes amortized $O(1)$ computational steps under a sequence of operations starting when there are no pending operations, since it is called after at most *deqs* *FutureDequeue* calls. \square

LEMMA 8.22. *Every amortized $O(Bn)$ system-wide computational steps (for sequences of operations starting when there are no pending operations), some intermediate progress step or full progress step is accomplished.*

PROOF. From Observations 8.20 and 8.21, it follows that in every thread that operates on the shared queue, every amortized $O(1)$ computational steps either a full progress step is completed (the interface method returns to the user) or a backward branch is taken. Combining this with Lemma 8.19, we get that every amortized $O(Bn)$ system-wide computational steps, either an intermediate progress step or a full progress step is achieved. \square

LEMMA 8.23. *Every amortized $O(Bn \cdot (3n + 1)) = O(Bn^2)$ system-wide computational steps (of the shared queue's methods), a new full progress step is accomplished, namely a queue's interface method returns to the user.*

PROOF. Derived directly from Lemmas 8.12 and 8.22. \square

In the worst case, from a moment t , a new full progress step is accomplished after $O(Bn^2P)$ system-wide computational steps, where P is the maximum number of pending operations in a thread at t .

COROLLARY 8.24. *BQ is lock-free.*

9 PERFORMANCE

We compared the proposed BQ to two queue algorithms: the original MSQ that executes one operation at a time, and the queue by Kogan and Herlihy [17] that satisfies MF-linearizability, henceforth denoted KHQ. KHQ executes pending operations in batches of homogeneous operations: it executes each subsequence of enqueues-only together by linking nodes to the end of the queue, and each subsequence of dequeues-only by unlinking nodes from the head of the queue.

We implemented the shared parts of the different queue versions identically to filter any unrelated performance difference. All queues use the optimistic access scheme for memory management. The implementations were coded in C++ and compiled using the g++ compiler version 6.5.0 with a $-O3$ optimization level.

We conducted our experiments on a machine running Linux (Ubuntu 18.04) equipped with four AMD Opteron 6376 2.3-GHz processors. Each processor has 16 cores, resulting in 64 threads overall. The number of threads in each experiment varied from 1 to 128. Each thread was attached to a different core, except for the experiment that ran 128 threads, in which two threads were attached to each core. The machine used 64 GB of RAM, an L1 data cache of 16 KB per core, an L2 cache of 2 MB for every 2 cores, and an L3 cache of 6 MB for every 8 cores.

The queues were empty in the beginning of all experiments. In each experiment testing n -long batches on BQ or KHQ, our workloads performed repeatedly n future operations followed by an *Evaluate* call for the last future operation. Our workloads for MSQ performed standard operations only. We ran three workloads:

- (1) Random operations without delay, in which operations (standard or future) were randomly chosen to be enqueue or dequeue (50-50 uniform distribution), and each thread executed its operations consecutively one right after another, for 2 seconds.
- (2) Random operations with delay, in which operations were randomly chosen, and random artificial delays were inserted between queue operation calls to simulate work done between calls, for 2 seconds. The inserted random delay between operations follows [28] and is intended to avoid artificial long run scenarios, in which a thread completes a series of operations when it has the shared object in its L1 cache without being interrupted by other threads.
- (3) Enqueues without delay, in which each thread consecutively performed (standard or future) enqueues only, for 1 second.

Each data point $[x, y]$ in the graphs in Figures 6 through 8 represents the average result of 10 experiments. In each experiment, x threads performed operations concurrently for 1 or 2 seconds as detailed previously. The graphs depict the throughput in each case—that is, the number of operations (*Enqueue/Dequeue/FutureEnqueue/FutureDequeue*) applied to the shared queue per second by the threads altogether, measured in million operations per second. The BQ curve appears along with the MSQ and KHQ curves for different batch sizes.

In the random operations workloads (see Figures 6 and 7), BQ demonstrates a significant performance improvement over both competitors for batches of 16 operations or more. Indeed, for batches containing less operations, MSQ and KHQ are preferable. The overhead of executing a batch operation makes small batches less worthwhile. However, for longer batches, and when at least three threads operate on the queue, BQ performs better. BQ exploits parallelism better as execution of operations in batches reduces contention substantially: instead of accessing the shared queue for every operation, each thread interacts with the shared queue throughout the execution of a single batch operation. Later, it performs local work to pair futures applied by the batch operation with results. BQ performs better than KHQ as well, since it applies each batch at once to the shared queue, whereas KHQ applies each batch operation using several homogeneous batch executions.

BQ performs better also in an enqueues-only workload (see Figure 8) for long enough batches. BQ and KHQ perform similarly for 16-long batches, and BQ outperforms KHQ for larger batches. KHQ performs better than BQ for short enqueue batches since it applies all enqueues of each batch at once, without incurring the overhead of executing a batch using an announcement. But for larger batches, the obstructing batch operations slow each other down substantially in KHQ: when some batch succeeds to link its nodes to the tail, all concurrent enqueue batches try to help it advance

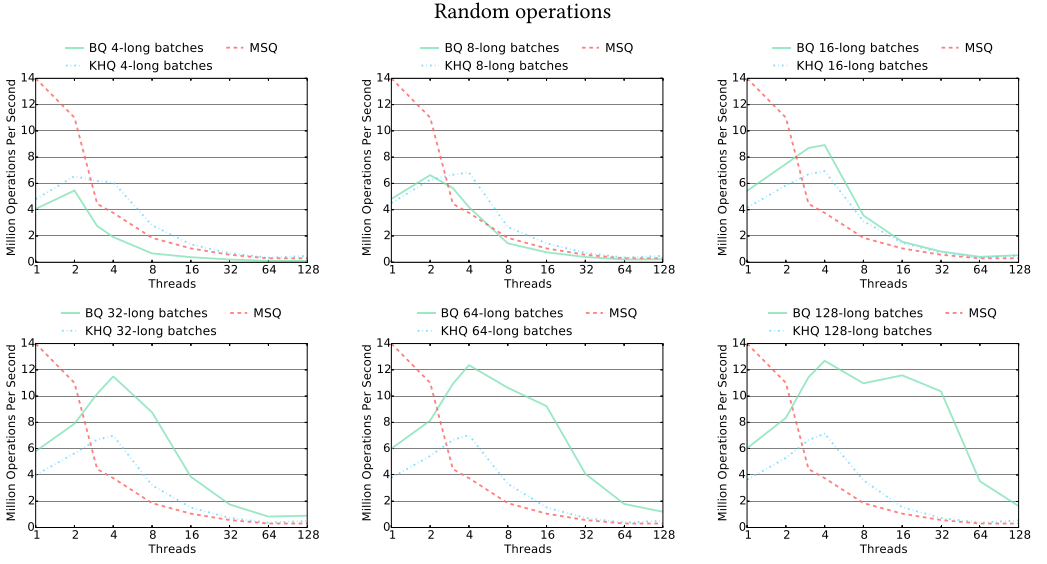


Fig. 6. Throughput for 4, 8, 16, 32, 64, and 128 long batches respectively of random operations without delay.

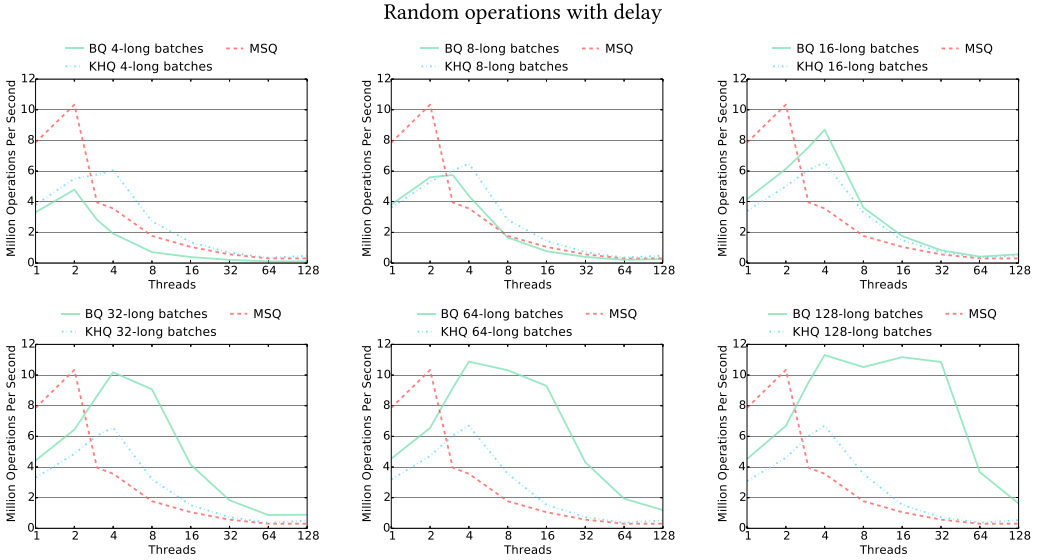


Fig. 7. Throughput for 4, 8, 16, 32, 64, and 128 long batches respectively of random operations with delay.

the tail node by node, whereas in the BQ scheme, helpers know the address of the new tail (it is the address of the last enqueued node, detailed in the batch's announcement) and attempt to advance the tail to point at this node without needing to traverse the newly enqueued nodes.

The more operations a batch contains, the greater the performance gap between BQ and the other queues becomes. BQ performs better as batch size increases since the reduction in contention more than compensates for the greater overhead. Therefore, BQ is an excellent choice for a lock-free queue when future operations can be employed.

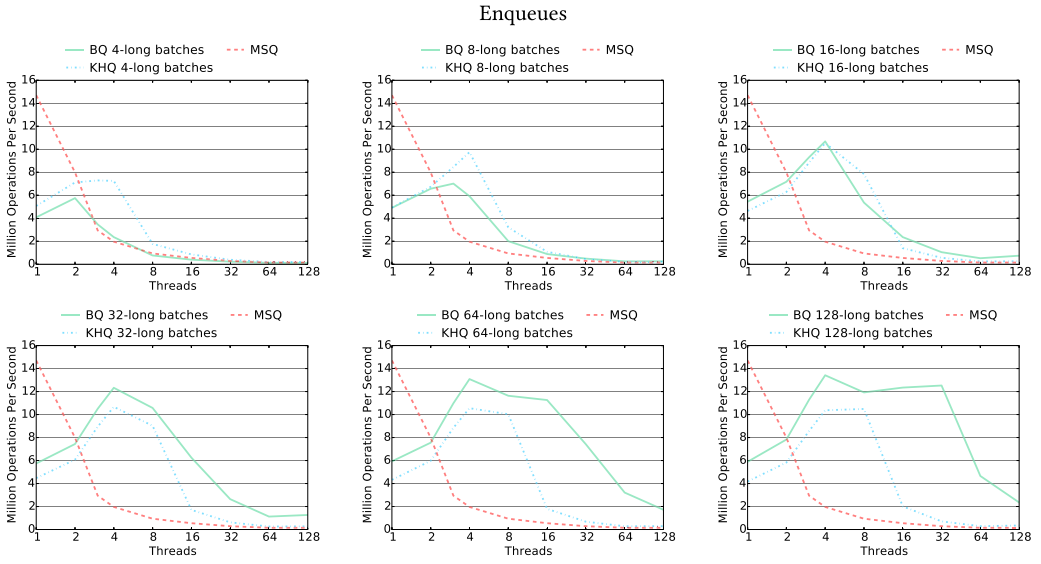


Fig. 8. Throughput for 4, 8, 16, 32, 64, and 128 long batches respectively of enqueues without delay.

9.1 Batched Michael-Scott Queue against Other Non-Batched Queues

BQ is an extension of the Michael-Scott queue (MSQ) that handles batches to improve concurrent performance. The MSQ is often used in practice due to its portability: it only uses the CAS primitive, which is widely available in most platforms. There exist faster queues that make novel use of the fetch-and-add primitive [25, 28]. An interesting open problem is to apply the batching ideas in this article to allow batches in these queues and improve performance even further. The MSQ was chosen for this first study due to its simplicity and portability. In this section, we provide a comparison between the batched MSQ to these other more efficient queues. Although these measurements may look like comparing apples to oranges, they come to show that if fetch-and-add is available, then the faster queues are still worth using, especially when batches are of smaller size. These measurements also motivate extending these faster queues with batches to potentially achieve superb performance.

In what follows, we compare BQ to two different non-batched queue algorithms that build on the additional fetch-and-add primitive. We consider the wait-free queue of Yang and Mellor-Crummey [28] (specifically, its version called *WF-10* in their work [28]), denoted *wfQ*, and the lock-free queue of Morrison and Afek [25], denoted *LCRQ*. We used their C implementation from <https://github.com/chaoran/fast-wait-free-queue>. We ran these queues on the same machine under the same workloads detailed earlier, where our workloads for *wfQ* and *LCRQ* performed standard operations only. The results appear in Figure 9. BQ outperforms these queues only for larger batches (e.g., for eight threads—for batches longer than 32).

10 AVOID USING DOUBLE-WIDTH CAS

To make the algorithm portable to platforms that do not implement double-width CAS, the algorithm may be modified to use a single-word CAS only. Currently, *SQHead* is a *PtrCntOrAnn* object and *SQTail* is a *PtrCnt* object. Both of them are double-word wide, so an atomic update of them requires a double-width CAS. To avoid it, *SQHead* and *SQTail* may become pointers only.

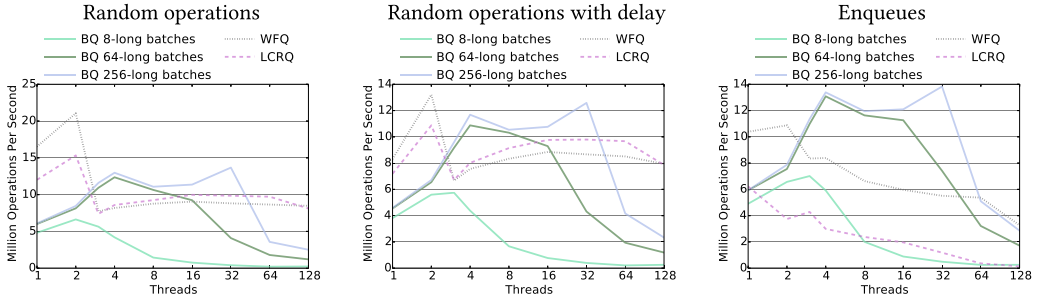


Fig. 9. Comparison to fetch-and-add based queues: throughput for random operations, random operations with delay, and enqueues, respectively.

The dequeue and enqueue counters associated with the head and tail, respectively, are still required, because we use them to calculate the queue's size when a batch is operated to figure out the new head. As we eliminate them from *SQHead* and *SQTail*, we place a counter field in the *Node* object as a substitute, and we use it when needed (as elaborated next) to indicate the node's position among all the queue's nodes from the beginning of time. In other words, a node's counter may be used to indicate the serial number of the enqueue or future enqueue operation that inserted this node among all enqueues ever applied to the queue (i.e., if the counter is c , then the node was the c^{th} node to be enqueued). The algorithm should be adjusted in terms of both updating the counters and using them.

Starting with their usage, in the new scheme, instead of reading the counters in *SQHead* and *SQTail*, we read the counters in the nodes pointed to by them. This yields the same value as in the original scheme: the head and tail counters in the original scheme bear the exact same value as the node counter of the node pointed to by the head and tail in the new scheme—which is the serial number of the enqueue or future enqueue operation that inserted this node among all enqueues ever applied to the queue. In detail, the tail counter in the original scheme counted the total number of enqueues, which equals to the number of nodes enqueued up to the current tail, and the head counter counted the total number of dequeues, which equals to the number of nodes enqueued up to the current head (as each dequeue advances the head forward by one enqueued node).

Proceeding to the update of the nodes' counters, the counter field does not need to be set in every node. It is sufficient to set the counter in nodes that might be pointed to by the head or tail, since only the counters in these nodes might be read by later operations—either batch operations that will use the counters for calculating the new head or operations that rely on this counter to update the counter of later nodes. We do not set the counter in all nodes, to make batch execution more simple and efficient, as otherwise during a batch execution, one should also set the counters of all enqueued nodes after succeeding to link them to the tail. We will set a node's counter right before an attempt to update the queue's head or tail to point to this node. This occurs in the following cases:

- (1) A single dequeue operation will update *head->next.count* before performing a CAS of the head. Similarly, a dequeues-only batch operation will update the counter of the node pointed to by the new head before performing a CAS of the head to point to this node. The new counter value in this case is the amount of dequeues in the batch accumulated to the counter of the node currently pointed to by the head.
- (2) A single enqueue operation will update the counter of the new node before linking it to the tail.
- (3) When a thread carries out a batch operation that contains at least one enqueue operation, it will update the counter of its last enqueued node, which is about to be pointed to by the

new tail. The new counter value equals the amount of enqueues in the batch summed to the counter of the node pointed to by the current tail. This update shall be performed right before the CAS of the tail to point to the last enqueued node. If other threads try to execute this batch operation simultaneously, they may also perform this update, as the amount of enqueues in the batch is detailed in the announcement.

If the batch operation contains at least one successful dequeue, it will also update the counter of the node that is about to be pointed to by the head, right before performing the head's CAS that completes the batch execution.

Note that writing the counter does not require a CAS: the written value is the serial number of the enqueue that inserted this node to the queue, which is unambiguous. Therefore, under no circumstances may two threads try to write different values.

An additional adaptation is required to distinguish whether *SQHead* points to a node or an announcement. Currently, the least significant bit of *SQHead.tag*, which overlaps *SQHead.ptrCnt.node*, is set to indicate that *SQHead* contains an announcement. In the new suggested design, we would take a similar approach and use the least significant bit of the *SQHead* pointer as a mark.

11 CONCLUSION

We presented BQ, a novel lock-free extension to MSQ that supports future operations. Unlike KHQ, BQ supports single operations as well, according to EMF-linearizability. BQ exploits batching to reduce contention and improve scalability. It enables a fast application of a mixed sequence of enqueue and dequeue operations all at once to the shared queue. Thus, it significantly reduces accesses to the shared queue and overall processing in it, in comparison to both MSQ and KHQ. BQ demonstrates a substantial performance improvement of more than an order of magnitude compared to MSQ and KHQ (depending on the batch lengths and the number of threads).

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Received April 2019; revised April 2021; accepted November 2021