Mathematical Analysis 1
Exam duration: 60 minutes
Number of questions: 20

Part 1. Domain and range of a function (2 questions)
Part 2. Properties of a function (2 questions)
Part 3. Limit of a sequence. Limit of a function (4 questions)
Part 4. Derivative of a function (4 questions)
Part 5. Applications of the derivative of a function (3 questions)
Part 6. Indefinite integral (5 questions)

Part 1. Domain and Range of a Function (2 questions)

1. Find the domain of the function $y = 15x + ln(6 - 3x)$
2. Find the domain of the function $f(x) = x + arcsin(2x + 3)$
3. Find the range of the function $f(x) = x^3$
4. Find the range of the function $f(x) = \frac{\pi}{2} + arctanx$
5. Find the range of the function $f(x) = \frac{sin3x}{2}$

Part 2. Properties of a Function (2 questions)

6. If $f(x) = \frac{x-1}{x}$ and $g(x) = 1 - x$, then g(f(x)) =
7. Which of the following functions is bounded on $(-\infty, +\infty)$?
   - $y = x^2$
   - $y = 2^x$
   - y = x
   - $y = e^x$
   - $y = sinx$
8. Which of the following functions is unbounded on $(-\infty, +\infty)$?
   - $y = arctanx$
   - $y = 2sinx$
   - $y = x$
   - $y = sinx$

- $y = cosx$

9. Which of the following functions is even?

- $y = tanx + 2$
- $y = xsinx$
- $y = xcosx$
- $y = x^2 + sinx$
- $y = x^3 - x$

10. Find the inverse of the function $y = 2^x + 1$

Part 3. Limit of a Sequence. Limit of a Function (4 questions)

11. $\lim\limits_{n\to\infty} \frac{3^n - 2^n}{3^{n+2} - 2^n}$

12. Find $\lim\limits_{x\to -1} \frac{3x^2+7x+4}{x^2+2x+1}$

13. Find $\lim\limits_{n\to\infty} \frac{2n^4 - n^3 + 1}{3n^4 + 5n^2 - n + 2}$

14. Find $\lim\limits_{n\to\infty} \left(1 - \frac{3}{2n}\right)^n$

15. Find $\lim\limits_{n\to\infty} \left(\frac{n+4}{5n-1}\right)^n$

16. Which of the following sequences is infinitely large?

- $x_n = 5$
- $x_n = n$
- $x_n = 1/2^n$
- $x_n = (-1)^n$
- $x_n = 1/n$

17. A function f(x) is called infinitely small as $x \to x_0$ if

18. A function f(x) is called infinitely large as $x \to x_0$ if

19. Find $\lim\limits_{x\to 0} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2}$

20. The first remarkable limit is written as

21. The second remarkable limit is written as

22. Find $\lim\limits_{x\to 0} \frac{3arcsinx}{7x^2}$

Part 4. Derivative of a function (4 questions)

23. f′(x) is defined as:

24. Let u = u(x) be a function of x. What is the derivative of the function u^{α}, where α is a real number?

- $\alpha \cdot u^{\alpha-1}$
- $\alpha \cdot (u')^{\alpha-1}$
- $u^{\alpha} \cdot u'$
- $u^{\alpha} ln\alpha \cdot u'$
- $\alpha u^{\alpha-1} \cdot u'$

25. Which of the following is false?

- $(a^x)' = a^x \, ln\, a$
- $(log_a\, x)' = \frac{1}{x\, ln\, a}$
- $(tan\, x)' = \frac{1}{cos^2 x}$
- $(ln\, x)' = \frac{1}{x\, lg\, e}$
- $(cot\, x)' = -\frac{1}{sin^2 x}$

26. Write the equation of the tangent line to the curve $y = sin\, 2\, x + cos\, 2\, x$ at the point $x_0 = 0$:

27. Find the derivative of the function $f(x) = tan^2\, x + lncos^2\, x$

28. If $y = 2 + \sqrt{x}$ for all x, then the domain of $f'(x)$ is

29. If $f(x) = (x^2 + 1)^{(2-3x)}$, then $f'(0) =$

30. Find the derivative y'_x of the implicit function $x + y - x\, sin\, y = 0$

31. Find $y^{(n)}$ if $y = e^{2x}$

Part 5. Applications of the derivative of a function (3 questions)

32. If the derivative of a function *f* (*x*) on an interval (*a*, *b*) is positive then

33. If the second order derivative of a function *f* (*x*) on an interval (*a*, *b*) is negative then

- *f* (*x*) decreases on (*a*, *b*)

- $f(x)$ increases on (a, b)
- $f(x)$ is bounded on (a, b)
- $f(x)$ is constant on (a, b)
- $f(x)$ is concave down on (a, b)

34. Using the L'Hospital's rule, find the limit $\lim_{y\to 0} \frac{e^y+\sin y-1}{\ln(1+y)}$

35. Find the intervals on which the function $y = \frac{5}{x}$ is decreasing

36. Find the inflection point of the function $y = xe^x$

37. Find the intervals on which the function $y = x^4 - 6x^2 + 5$ is concave down

38. Find the asymptotes of the function $y = \frac{x^2}{x+2}$

39. Find the maximum value of the function $y = \frac{1}{3}x^3 - 4x^2 + 15x - 1$

40. Find the oblique asymptote of the function $y = \frac{x^2}{x-1}$

Part 6. Indefinite integral (5 questions)

41. If $F(x)$ is an antiderivative of a function $f(x)$ then the following holds:

- $\int f(x)dx = F(x)$
- $\int f(x)dx = f(x)$
- $\int F(x)dx = f(x) + C$
- $\int F(x)dx = f(x)$
- $\int f(x)dx = F(x) + C$

42. $(\int f(x)\,dx)' = ?$

- $f(x)dx$
- $f(x)$
- $F'(x) + C$
- $F(x)dx$
- $f'(x)$

43. $\int F'(x)dx = ?$

- $f'(x)$
- $f(x)dx$
- $F'(x)$

- $F(x) + C$
- $f(x) + C$

44. Which of the following statements about indefinite integral are true?

I. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$; II. $\int f(x)g(x)dx = \int f(x)dx \cdot \int g(x)dx$; III. $\int f'\big(g(x)\big)g'(x)dx = f\big(g(x)\big) + C$; V. $\int [f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + C$

- only I and II are true
- only I and III are true
- only I and IV are true
- only I, II and IV are true
- only I, III and IV are true

45. $\int \frac{dx}{\sqrt{9-x^2}} =?$

46. $\int arctan\, x\, dx =?$

47. What substitution can be used to find the integral $\int \sqrt{a^2 + x^2}dx$?

- $a^2 + x^2 = t^2$
- $x^2 = y$
- $x = a\, tan\, t$
- $x = at$
- $x = a\, sin\, t$

48. What substitution can be used to find the integral $\int \sqrt{16 - x^2}dx$?

- $16 - x^2 = t^2$
- $x^2 = y$
- $x = 16\, cos\, t$
- $x = 4t$
- $x = 4\, sin\, t$

49. What substitution can be used to find the integral $\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx$?

- $t = \sqrt[6]{x+1}$
- $t = \sqrt[6]{x}$
- $t = \sqrt{x+1}$
- $t = x^6$

- $t = \sqrt[3]{x+1}$

50. $\int \frac{xdx}{x^2+3} =?$

51. $\int \frac{dx}{x^2+2x+5} =?$

52. The integral $\int x \sin x \, dx$ can be found by
- making the substitution $u = x$
- making the substitution $u = \sin x$
- using integration by parts, with $u = \sin x$ and $dv = xdx$
- using integration by parts, with $u = x$ and $dv = \sin x \, dx$
- using integration by parts, with $u = \cos x$ and $dv = xdx$

53. The partial fraction decomposition of $\frac{2x-3}{(x-1)(x^2+4)}$ has the following form:
- $\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+2}$
- $\frac{A}{x-1} + \frac{B}{x^2+4}$
- $\frac{A}{x-1} + \frac{Bx}{x^2+4}$
- $\frac{A}{x-1} + \frac{Bx+C}{x^2+4}$
- $\frac{A}{x-1} + \frac{Bx+\mathrm{C}}{(x+2)^2}$

54. The partial fraction decomposition of $\frac{x^3+5x-1}{(x^4-1)^2}$ has the following form:
- $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$
- $\frac{A}{x-1} + \frac{Bx+C}{x^2-1}$
- $\frac{A}{x-1} + \frac{B}{x+1}$
- $\frac{A_1}{x-1} + \frac{A_2}{(x-1)^2} + \frac{B_1}{x+1} + \frac{B_2}{(x+1)^2} + \frac{C_1x+D_1}{x^2+1} + \frac{C_2x+D_2}{(x^2+1)^2}$
- $\frac{A_1}{x^2-1} + \frac{A_2}{(x^2-1)^2} + \frac{B_1}{x^2+1} + \frac{B_2}{(x^2+1)^2}$

55. $\int \sin^2 \frac{x}{2} dx =?$