Dynamic Pricing on Online Marketplaces: Strategies and Applications

Demand Probabilities

Dr. Rainer Schlosser

Hasso Plattner Institute (EPIC)

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Project A: Multiple Features (Optimization)

- Estimate sales probabilities

$$\pi(1,\vec{x}(a,\vec{s})) \text{ (sale) and } \pi(0,\vec{x}(a,\vec{s})) = 1-\pi(1,\vec{x}(a,\vec{s})) \text{ (no sale).}$$

- Use logistic regression: $\pi(1,\vec{x}(a,\vec{s})) = \dfrac{\exp(\vec{\beta}'\vec{x})}{1+\exp(\vec{\beta}'\vec{x})} \in (0,1)$.

- Model: time homogeneous

  infinite horizon

  single product

  real-life data

Project B: Multiple Products

- Assume sales observations for two products, e.g.,

$$y_t^{(1)} \sim \left(25 \; -0.4 \cdot (a_t^{(1)})^{1.2} + 0.3 \cdot (a_t^{(2)})^{0.9} + 0.3 \cdot (p_t^{(1)})^{1.1} + 0.1 \cdot (p_t^{(2)})^{0.8} + U(-5,5)\right)^+$$

$$y_t^{(2)} \sim \left(15 \; +0.3 \cdot (a_t^{(1)})^{0.7} - 0.4 \cdot (a_t^{(2)})^{1.1} + 0.2 \cdot (p_t^{(1)})^{0.8} + 0.3 \cdot (p_t^{(2)})^{0.9} + U(-5,5)\right)^+$$

- The competitors' prices can be defined by $p_1^{(1)} = 45$, $p_1^{(2)} = 35$,

$$p_t^{(i)} = p_{t-1}^{(i)} + 1_{U(0,1)<0.2} \cdot U(-2,2), \quad i = 1,2, \quad \text{for } t = 1,\ldots,100 .$$

- Our prices are defined by $a_1^{(1)} = 45$, $a_1^{(2)} = 35$,

$$a_t^{(j)} = \mathit{if}\; U(0,1) < 0.5 \;\mathit{then}\; a_{t-1}^{(j)} \;\mathit{else}\; \left(p_{t-1}^{(j)} + U(-3,3)\right), \quad j = 1,2, \quad \text{for } t = 1,\ldots,100 .$$

- Use least squares to estimate the sales rates: $\lambda^{(j)}(\vec{x}^{(j)}(\vec{a},\vec{s})) = \vec{\beta}^{(j)\prime}\vec{x}^{(j)}$.

- Use explanatory variables, $j=1,2$, $I=J=2$, $\vec{a}=(a^{(1)},a^{(2)})$, $\vec{s}=(p^{(1)},p^{(2)})$,

(i)	$x_{t,1}^{(j)}(\vec{a},\vec{s})=1$	constant / intercept
(ii)	$x_{t,2}^{(j)}(\vec{a},\vec{s}):=a_t^{(j)}-\min\limits_{k=1,\ldots,J,k\neq j}\{a_t^{(k)}\}$	price gap between $a_t^{(j)}$ and the best own price
(iii)	$x_{t,3}^{(j)}(\vec{a},\vec{s}):=a_t^{(j)}-\min\limits_{i=1,\ldots,I}\{p_t^{(i)}\}$	price gap between $a_t^{(j)}$ and the best competitors' price
(vi)	$x_{t,4}^{(j)}(\vec{a},\vec{s}):=x_{t,2}^{(j)}(\vec{a},\vec{s})^2$	squared version of variable $x_{t,2}^{(j)}(\vec{a},\vec{s})$

- Use sales probabilities (based on the rates $\lambda^{(j)}(\vec{a},\vec{s};\vec{\beta}^{(j)*})$, $j=1,2$):

$$P_t^{(j)}(i,\vec{a},\vec{s}) := \frac{\lambda^{(j)}\left(\vec{a},\vec{s};\vec{\beta}^{(j)*}\right)^i}{i!} \cdot e^{-\lambda^{(j)}\left(\vec{a},\vec{s};\vec{\beta}^{(j)*}\right)}, \ i=0,1,2,\ldots \quad \text{(probability of } y^{(j)}=i\text{)}$$

- Sales observations can also be simulated by $\lambda^{(j)}(\vec{x}^{(j)}(\vec{a},\vec{s})) = \vec{\beta}^{(j)\prime}\vec{x}^{(j)}$, where

j	$\beta_1^{(j)}$	$\beta_2^{(j)}$	$\beta_3^{(j)}$	$\beta_4^{(j)}$
1	20.43	0.92	-0.73	-0.042
2	7.08	-1.09	-0.49	-0.038

- Alternative (Binary case): Use sales probabilities (sale or no sale), $j=1,2$,

$$\pi^{(j)}(1,\vec{x}^{(j)}(\vec{a},\vec{s})) = 1-\exp\left(-\lambda^{(j)}\left(\vec{x}^{(j)}(\vec{a},\vec{s})\right)\right) \quad \text{(probability of } y^{(j)}=1\text{)}$$

Project C: Perishable Products

- Use explanatory variables:

(i)	$x_1^{(i,t)} = 1$	constant / intercept
(ii)	$x_2^{(i,t)} := r^{(i,t)}$	rank of price $a^{(i,t)}$ within the prices $\vec{p}^{(i,t)}$
(iii)	$x_3^{(i,t)} := a^{(i,t)} - \min_{k=1,\ldots,K^{(i,t)}} \{p_k^{(i,t)}\}$	price gap between $a^{(i,t)}$ and the best competitors' price
(iv)	$x_4^{(i,t)} := K^{(i,t)}$	total number of competitors for product *i* in period *t*
(v)	$x_5^{(i,t)} := (a^{(i,t)} + \sum_k p_k^{(i,t)}) / (1 + K^{(i,t)})$	average price level for product *i* in period *t*
(vi)	$x_6^{(i,t)} := t^2$	impact of time/period *t*

- Consider the sales probability: $\pi_t\left(\vec{x}(a,\vec{s})\right) = \dfrac{\exp(\vec{\beta}'\vec{x})}{1+\exp(\vec{\beta}'\vec{x})} \in (0,1)$, $\vec{s} = (t, \vec{p})$.

- Use coefficients, e.g., $\vec{\beta}^* = (\beta_1^*, \beta_2^*, \beta_3^*, \beta_4^*, \beta_5^*, \beta_6^*) = (-3.89,\ -0.56,\ -0.01,\ 0.07,\ -0.02,\ 0.01)$

- Use sales probabilities (based on $\pi_t\left(\vec{x}(a,\vec{s})\right)$):

(i) Poisson probabilities, $i = 0,1,2,\ldots$

$$P_t(i,a,\vec{s}) := \frac{\left(10 \cdot \pi_t\left(\vec{x}(a,\vec{s})\right)\right)^i}{i!} \cdot e^{-10\cdot\pi_t\left(\vec{x}(a,\vec{s})\right)} \qquad \text{(probability of } y = i\text{)}$$

or

(ii) Binomial probabilities, $i = 0,1,2,\ldots,m$, where, e.g., $m = 10$,

$$P_t(i,a,\vec{s}) := \binom{m}{i} \cdot \pi_t\left(\vec{x}(a,\vec{s})\right)^i \cdot \pi_t\left(\vec{x}(a,\vec{s})\right)^{m-i} \qquad \text{(probability of } y = i\text{)}$$

Simulation of observation data

- The competitors' prices can be defined by $p_1^{(1)} = 45$, $p_1^{(2)} = 35$,

$$p_t^{(i)} = p_{t-1}^{(i)} + 1_{U(0,1)<0.2} \cdot U(-2,2), \quad i = 1,2, \text{ for } t = 1,...,100.$$

- Our prices are defined by $a_1^{(1)} = 45$, $a_1^{(2)} = 35$,

$$a_t = \mathit{if}\ U(0,1) < 0.5 \ \mathit{then}\ a_{t-1} \ \mathit{else}\ \left(a_{t-1} + U(-3,3)\right), \text{ for } t = 1,...,100.$$

- Simulate sales observations according to the sales probabilities above:

$$y_t \sim P_t(i, a, \vec{s})$$

Project D: Response Strategies

- Use explanatory variables:

(i)	$x_1(a,p)=1$	constant / intercept
(ii)	$x_2(a,p)=r(a,p):=1+0.5\cdot\left(1_{\{p<a\}}+1_{\{p\leq a\}}\right)$	price rank
(iii)	$x_3(a,p)=a-p$	price gap
(iv)	$x_4(a,p):=1$	total number of competitors
(v)	$x_5(a,p)=(a+p)/2$	average price level

- Consider the sales probability: $\pi\left(\vec{x}(a,p)\right)=\dfrac{\exp(\vec{\beta}'\vec{x})}{1+\exp(\vec{\beta}'\vec{x})}\in(0,1)$.

- Use coefficients, e.g., $\vec{\beta}^* = (\beta_1^*, \beta_2^*, \beta_3^*, \beta_4^*, \beta_5^*, \beta_6^*) = (-3.89,\ -0.56,\ -0.01,\ 0.07,\ -0.02,\ 0.01)$

- Define sales probabilities (based on $\pi\left(\vec{x}(a,p)\right)$) using:

(i) Poisson probabilities, $i = 0,1,2,\ldots$

$$P(i,a,p) := \frac{\left(10 \cdot \pi\left(\vec{x}(a,p)\right)\right)^i}{i!} \cdot e^{-10 \cdot \pi\left(\vec{x}(a,p)\right)} \qquad \text{(probability of } y = i\text{)}$$

or

(ii) Binomial probabilities, $i = 0,1,2,\ldots,m$, where, e.g., $m = 10$,

$$P(i,a,p) := \binom{m}{i} \cdot \pi\left(\vec{x}(a,p)\right)^i \cdot \pi\left(\vec{x}(a,p)\right)^{m-i} \qquad \text{(probability of } y^{(j)} = i\text{)}$$

Simulation of observation data

- The competitors' prices can be defined by $p_1 = 45$,

  $$p_t = p_{t-1} + 1_{U(0,1)<0.2} \cdot U(-2,2), \quad i = 1,2, \text{ for } t = 1,...,100.$$

- Our prices are defined by $a_1 = 45$,

  $$a_t = \mathit{if}\ U(0,1) < 0.5\ \mathit{then}\ a_{t-1}\ \mathit{else}\ \left(p_{t-1} + U(-3,3)\right), \text{ for } t = 1,...,100.$$

- Simulate sales observations according to the sales probabilities above:

  $$y \sim P(i, a, p)$$

Overview

2	April 19	Demand Estimation
3	April 26	Optimization Techniques
4	May 3	Extensions / Projects A-D
5	May 10	Assign Projects to Teams
6	May 17	no Meeting
7	May 24	Workshop / Group Meetings
8	May 31	Workshop / Group Meetings
9	**June 7**	**Presentations (First Results)** <<<
10	June 14	Workshop / Group Meetings
11	June 21	Workshop / Group Meetings
12	June 28	Workshop / Group Meetings
13	July 5	no Meeting
14	July 12	Presentations (Final Results), Discussions, Feedback